

This Question Paper consists of 45 questions and 9 printed pages + Graph Sheet.
इस प्रश्न-पत्र में 45 प्रश्न तथा 9 मुद्रित पृष्ठ + ग्राफ शीट हैं।

Roll No.
अनुक्रमांक

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Code No. 71/SS/311
कोड नं.

SET/सेट-

A1

MATHEMATICS गणित (311)

Day and Date of Examination
(परीक्षा का दिन व दिनांक)

1. _____

Signature of Invigilators :
(निरीक्षकों के हस्ताक्षर)

2. _____

General Instructions :

1. Candidate must write his/her Roll Number on the first page of the Question Paper.
2. Please check the Question Paper to verify that the total pages and total number of questions contained in the Question Paper are the same as those printed on the top of the first page. Also check to see that the questions are in sequential order.
3. Making any identification mark in the Answer-Book or writing Roll Number anywhere other than the specified places will lead to disqualification of the candidate.
4. Write your Question Paper Code No. 71/SS/311, Set - A1 on the Answer-Book.
5. (a) The Question Paper is in English/Hindi medium only. However, if you wish, you can answer in any one of the languages listed below :
English, Hindi, Urdu, Punjabi, Bengali, Tamil, Malayalam, Kannada, Telugu, Marathi, Oriya, Gujarati, Konkani, Manipuri, Assamese, Nepali, Kashmiri, Sanskrit and Sindhi.
You are required to indicate the language you have chosen to answer in the box provided in the Answer-Book.
(b) If you choose to write the answer in the language other than Hindi and English, the responsibility for any errors/mistakes in understanding the question will be yours only.

सामान्य अनुदेश :

1. परीक्षार्थी प्रश्न-पत्र के पहले पृष्ठ पर अपना अनुक्रमांक अवश्य लिखें।
2. कृपया प्रश्न-पत्र को जाँच लें कि प्रश्न-पत्र के कुल पृष्ठों तथा प्रश्नों की उतनी ही संख्या है जितनी प्रथम पृष्ठ के सबसे ऊपर छपी है। इस बात की जाँच भी कर लें कि प्रश्न क्रमिक रूप में हैं।
3. उत्तर-पुस्तिका में पहचान-चिह्न बनाने अथवा निर्दिष्ट स्थानों के अतिरिक्त कहीं भी अनुक्रमांक लिखने पर परीक्षार्थी को अयोग्य ठहराया जायेगा।
4. अपनी उत्तर-पुस्तिका पर प्रश्न-पत्र का कोड नं 71/SS/311, सेट - A1 लिखें।
5. (क) प्रश्न - पत्र केवल हिंदी/अंग्रेजी माध्यम में है। फिर भी, यदि आप चाहें तो नीचे दी गई किसी एक भाषा में उत्तर दे सकते हैं :
अंग्रेजी, हिंदी, उर्दू, पंजाबी, बाँगला, तमिल, मलयालम, कन्नड़, तेलुगु, मराठी, उड़िया, गुजराती, कोंकणी, मणिपुरी, असमिया, नेपाली, कश्मीरी, संस्कृत और सिंधी।
कृपया उत्तर-पुस्तिका में दिए गए बॉक्स में लिखें कि आप किस भाषा में उत्तर लिख रहे हैं।
(ख) यदि आप हिंदी एवं अंग्रेजी के अतिरिक्त किसी अन्य भाषा में उत्तर लिखते हैं तो प्रश्न को समझने में होने वाली त्रुटियों / गलतियों की जिम्मेदारी केवल आपकी होगी।

MATHEMATICS

(311)

Time : 3 Hours]

[Maximum Marks : 100

- Notes :**
- (i) This question paper consists of **45** questions in all.
 - (ii) All questions are **compulsory**.
 - (iii) Marks are given against each question.
 - (iv) **Section - A** consist of
 - (a) **Q.No. 1 to 20** - Multiple Choice Type Questions (MCQs) carrying **1** mark each. Select and write the most appropriate option out of the four options given in each of these questions.
 - (b) **Q.No. 21 to 29** - Objective Type Questions. **Q.No. 21 to 24** carries **2** marks each (with **2** sub-parts of **1** mark each). **Q.No. 25 to 28** carries **4** marks each (with **4** sub-parts of **1** mark each) and **Q.No. 29** carries **6** marks (with **6** sub-parts of **1** mark each). Attempt these questions as per the instructions given for each of the questions **21 to 29**.
 - (v) **Section - B** consists of
 - (a) **Q.No. 30 to 38** - Very Short Answer Type Questions carrying **2** marks each.
 - (b) **Q.No. 39 to 43** - Short Answer Type Questions carrying **4** marks each.
 - (c) **Q.No. 44 and 45** - Long Answer Type Questions carrying **6** marks each.

An internal choice has been provided in some of these questions in **Section - B**. You have to attempt **only one** of the given choices in such questions.

NOTE :

- (1) Answers of all questions are to be given in the Answer-Book given to you.
- (2) 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 02.15 p.m. From 02.15 p.m. to 02.30 p.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

SECTION - A

Q. No. 1 to 20 are the Multiple Choice Questions (MCQs) of 1 mark each.

1. Slope of a line which cuts off intercepts of equal length on the positive sides of two axes is : 1
 (A) -1 (B) 1 (C) 0 (D) 2
2. The coordinates of the foot of the perpendicular drawn from the point $(2, 3)$ on the line $y = 3x + 4$ is : 1
 (A) $\left(\frac{37}{10}, \frac{-1}{10}\right)$ (B) $\left(\frac{-1}{10}, \frac{37}{10}\right)$ (C) $\left(\frac{10}{37}, -10\right)$ (D) $\left(\frac{2}{3}, \frac{-1}{3}\right)$
3. If a circle passes through the points $(0, 0)$, $(a, 0)$ and $(0, b)$, then the centre of the circle is : 1
 (A) (a, b) (B) $\left(\frac{-a}{2}, \frac{-b}{2}\right)$ (C) $(-a, -b)$ (D) $\left(\frac{a}{2}, \frac{b}{2}\right)$
4. The length of latus rectum of an ellipse $3x^2 + y^2 = 12$ is : 1
 (A) 8 (B) 4 (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{\sqrt{3}}$
5. The equations of a parabola whose vertex is $(-3, 0)$ and directrix is $x + 5 = 0$, is : 1
 (A) $x^2 = 8(y + 3)$ (B) $y^2 = 8(x + 3)$
 (C) $x^2 = -8(y + 3)$ (D) $y^2 = -8(x + 3)$
6. If $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$, then 1
 (A) $x = 1, y = 3, z = 4$ (B) $x = 1, y = 4, z = 3$
 (C) $x = 3, y = 1, z = 4$ (D) $x = 3, y = 4, z = 1$
7. In the matrix $A = (a_{ij}) = \begin{bmatrix} 0 & 6 & 3 \\ -6 & -3 & 4 \\ 2 & -2 & 0 \end{bmatrix}$, the value of $a_{13} a_{23} + a_{31}$ is : 1
 (A) 0 (B) 12 (C) -4 (D) 14
8. The cofactor of a_{23} in the determinant $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$ is : 1
 (A) 1 (B) -1 (C) 2 (D) 4
9. If $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x^4$, then 1
 (A) f is one-one and onto (B) f is many-one and onto
 (C) f is one-one but not onto (D) f is neither one-one nor onto

10. If $f(x) = x^2 + 2$ and $g(x) = 1 - \frac{1}{1-x}$ ($x \neq 1$), then $\text{fog}(x)$ is : 1

- (A) x (B) $x^4 + 4x^2 + 6$ (C) $\frac{x^2}{(1-x)^2} + 2$ (D) $\frac{x^2 + 2}{x^2 + 1}$

11. The relation $\mathbf{R}$ in the set $A = \{1, 2, 3\}$ given by $\mathbf{R} = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is : 1

- (A) reflexive only (B) symmetric only
(C) transitive only (D) symmetric and transitive

12. If a binary operation $*$ is defined on the set $\mathbf{R}^+$ as $a * b = \frac{ab}{3}$ for all $a, b \in \mathbf{R}^+$, then the value of $(2 * 3) * 5$ is : 1

- (A) 7 (B) 10 (C) $\frac{13}{3}$ (D) $\frac{10}{3}$

13. If $y = \log(2x + 3)$, then $\frac{dy}{dx}$ is : 1

- (A) $\frac{1}{2x + 3}$ (B) $(2x + 3)^2$ (C) $\frac{2}{2x + 3}$ (D) $\frac{1}{2(2x + 3)}$

14. The derivable of $y = \sin^3 x$ w.r.t. x is : 1

- (A) $\cos^3 x$ (B) $3\sin^2 x$ (C) $\sin^2 x \cos x$ (D) $3\sin^2 x \cos x$

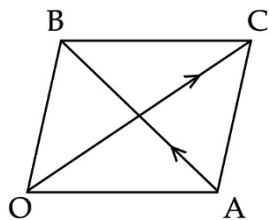
15. Which of the following is a vector quantity ? 1

- (A) Distance (B) Velocity (C) Time period (D) Work done

16. A unit vector in the direction of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ is : 1

- (A) $\hat{i} + \hat{j} + \hat{k}$ (B) $5(2\hat{i} + 3\hat{j} + \hat{k})$
(C) $\frac{1}{\sqrt{6}}(2\hat{i} + 3\hat{j} + \hat{k})$ (D) $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$

17. If OACB is a parallelogram with $\vec{OC} = \vec{a}$ and $\vec{AB} = \vec{b}$, then $\vec{OA}$ is equal to : 1



- (A) $\vec{a} + \vec{b}$ (B) $\vec{a} - \vec{b}$
(C) $\frac{1}{2}(\vec{a} - \vec{b})$ (D) $\frac{1}{2}(\vec{b} - \vec{a})$

18. The degree of the differential equation $\frac{d^3y}{dx^3} + 3\left(\frac{dy}{dx}\right)^2 = y^2 \log\left(\frac{d^2y}{dx^2}\right)$ is : 1
- (A) 2 (B) 3 (C) 5 (D) not defined

19. $\int_0^1 \frac{x}{x^2+1} dx$ is equal is : 1
- (A) $\frac{1}{2}$ (B) $\log 2$ (C) $2 \log 2$ (D) $\frac{1}{2} \log 2$

20. Converse of the statement “if a number is divisible by 9, then it is divisible by 3” is : 1
- (A) if a number is divisible by 3, then it is divisible by 9
 (B) if a number is divisible by 9, then it is not divisible by 3
 (C) if a number is not divisible by 3, it is not divisible by 9
 (D) if a number is not divisible by 9, then it is also not divisible by 3

Q. No. 21 to 24 are the objective type questions of 2 marks each :

21. Match **Column - I** statement with the correct option of **Column - II** : 1 × 2

A 2×3 matrix $[a_{ij}]$, whose elements are given by $a_{ij} = \frac{i-j}{i+j}$

Column - I	Column - II
(a) a_{23} is	(P) $\frac{1}{5}$
	(Q) $\frac{-1}{5}$
(b) a_{12} is	(R) $\frac{1}{3}$
	(S) $\frac{-1}{3}$

22. Fill in the blanks : 1 × 2

- (a) The distance between the planes $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) + 4 = 0$ and $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ is _____ units.
- (b) The order do the differential equation $y = \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1}$ is _____.

23. Write “True” for correct statement and “False” for incorrect statement : 1 × 2

- (a) The function $f: \mathbf{R} \rightarrow \mathbf{R}$ defined as $f(x) = \begin{cases} 1, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$
- (b) The function $f: \mathbf{R} \rightarrow \mathbf{R}$ defined as $f(x) = |x|$ is continuous at all points of $\mathbf{R}$.

24. Write the negation of each at the following statements : 1 × 2

- (a) The capital of Gujarat is Ahmedabad.
- (b) All circles are congruent.

Q. No 25 to 28 are the objective type questions of 4 marks each.

25. Fill in the blanks. 1 × 4

- (a) The function f defined as $f(x) = x - \cos x$ is _____ function for all x .
- (b) The local minima of the function $f(x) = 2x^3 - 3x^2 - 12x + 8$ exists at _____.
- (c) The slope of the normal to the curve $x^3 + x^2 + 3xy + y^2 - 5 = 0$ at $(1, 1)$ is _____.
- (d) The tangent to a curve $y = f(x)$ at the point (x_1, y_1) is parallel to x - axis, if _____.

26. Fill in the blanks : 1 × 4

- (a) If $y = e^{\sin^{-1}x}$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
- (b) If $x = at^2, y = 2at$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
- (c) The number of arbitrary constants in the particular solution of a differential equation of third order is _____
- (d) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx = \underline{\hspace{2cm}}$

27. Write “True” for correct statement and “False” for incorrect statement. 1 × 4

- (a) Two matrices are comparable if each one of them contains as many rows and columns as the other.
- (b) Matrix A is said to singular if $|A| \neq 0$
- (c) The adjoint of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ is $\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$
- (d) The inverse of a skew symmetric matrix of order 3 is a diagonal matrix.

28. Answer the following questions : 1 × 4

- (a) Check if the relation $\mathbf{R}$ in the set $A = \{1, 2, 3\}$ given by $\mathbf{R} = \{(1, 2), (2, 1)\}$ is an equivalence relation.
- (b) Check if the function $f: \mathbf{R} \rightarrow \mathbf{R}$ defined as $f(x) = 3 - 4x$ is one - one and onto.
- (c) Find : $\int \sin^{-1}(\cos x) dx$
- (d) Using differentials, find the approximate value of $\sqrt{26}$ upto one place of decimal.

29. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. then

1 × 6

- Find $\vec{a} \times \vec{b}$
- Find the angle between $\vec{a}$ and $\vec{b}$
- Find the projection of $\vec{a}$ on the vector $\vec{b}$
- Check, if $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$
- Find the area of the parallelogram having $\vec{a}$ and $\vec{b}$ as its diagonals
- Check if $(\vec{a} \times \vec{b})^2 = (\vec{a})^2 (\vec{b})^2 - (\vec{a} \cdot \vec{b})^2$

SECTION - B

Questions 30 to 38 carry 2 marks each.

30. Using matrix method, solve the following system of linear equations :

2

$$5x - 7y = 3$$

$$x + 3y = 5$$

31. Simplify : (i) $\cos(\sin^{-1}x)$

2

OR

(ii) $\cot(\operatorname{cosec}^{-1}x)$

32. Find : (i) $\lim_{x \rightarrow 3} \left(\frac{x^4 - 81}{2x^2 - 5x - 3} \right)$

2

OR

Find : (ii) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

33. Find the coordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of the conic.

2

$$(i) \quad \frac{x^2}{16} - \frac{y^2}{9} = 1$$

OR

$$(ii) \quad 4x^2 + 9y^2 = 36$$

34. Without expanding the determinant, prove that $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0.$ 2

35. Using elementary row transformation ; find the inverse of the matrix $A = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ 2

36. Find : $\int \frac{1}{x^2-9} dx$ 2

OR

Evaluate : $\int_0^2 3^x dx$

37. Find the value of k for which the line $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$ is parallel to x-axis. 2

38. Find the Cartesian equation of the line which passes through the point $(1, -1, 5)$ and parallel to the line $\frac{x-2}{3} = \frac{y-5}{-2}, z = -1.$ 2

Questions 39 to 43 carry 4 marks each.

39. Evaluate : $\int_{-5}^5 |x+2| dx$ 4

OR

Determine the area enclosed by the curves $y = x^2$ and $y = x + 2$, using integration.

40. Find the Cartesian equation of the plane passing through the points. $(1, 1, 0)$, $(1, 2, 1)$ and $(-2, 2, -1)$. Also, write the vector equation of the plane. 4

41. Show that the relation $\mathbf{R}$ defined in the set A of all triangles as $\mathbf{R} = \{(T_1, T_2) : T_1 \text{ is similar to } T_2, \text{ where } T_1, T_2 \in A\}$ is an equivalence relation. 4

42. Show that the tangents to the curve $6y = 3x^5 + 2x^3 - 3x$ at the points $x = \pm 3$ are parallel. 4

OR

Find the intervals in which $f(x) = 2x^3 - 3x^2 - 12x + 6$ is increasing or decreasing.

43. Show that the four points A, B, C and D with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ respectively are coplanar. 4

Q.No. 44 and 45 are each of 6 marks.

44. Using the properties of determinants, prove that
$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3 \quad 6$$

OR

If $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix}$, then find A^{-1}

Using A^{-1} , solve the following system of linear equations.

$$x + y + z = -2$$

$$x + 2y - 3z = 12$$

$$2x - y + 3z = -13$$

45. Find the maximum and minimum values of $z = x + 2y$ subject to the following constraints : 6
 $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x \geq 0, y \geq 0$ graphically.

OR

A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available is prepared mix packets from two different suppliers, S and T. Supplier S had a packet of mix of 4 units of chemical A and Z units of chemical B and costs Rs 10. The supplier T had a packet of mix of 1 unit of A and 1 unit of B and costs Rs 4. Formulate this problem as a L.P.P so that the company honour the contract requirement at minimum cost. Also, obtain the feasible region corresponding to the constraints.

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Roll No.

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