SAMPLE PAPER

CLASS – X | CBSE | MATHEMATICS

Time: 3 hours **Maximum Marks : 80**

GENERAL INSTRUCTIONS

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

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SECTION - A

1. Which term of the A.P $6, 13, 20, 27........$ is 97

(A) 14 (B) 13

(C) 15 (D) 12

Ans. C

Sol. The A.P is $6, 13, 20, 27......$

$a = 6,\ d = 7$

$97 = 6 + (n-1)7$

$\frac{97-6}{7} + 1 = n$

$\therefore n = 14$

2. Find the ratio of the total surface area & lateral surface area of a cube.

(A) 6:2 (B) 3:2

(C) $2\sqrt{3} : 1$ (D) $4:3$

Ans. B

Sol. T.S.A = $6a^2$

L.S.A = $4a^2$

Ratio $\Rightarrow \frac{6a^2}{4a^2} \Rightarrow 3:2$

3. Find the value of x, if 3 – x, x + 2, 2x + 1 are in A.P.

(A) 0 (B) 2

(C) 1 (D) –1

Sol. $x + 2 - (3 - x) = 2x + 1 - x - 2$

$2x - 1 = x - 1$

$x = 0$

4. A bag contain 5 red balls & some blue balls. If probability of drawing a blue ball is double that of red ball. Find number of blue balls in the bag ?

(A) 5 (B) 3

(C) 10 (D) 6

Ans. C

Sol. Let $\times$ blue balls

Total balls $5 + x$

$P(B) = 2P(R)$

$\frac{x}{5+x} = 2\left(\frac{5}{5+x}\right)$

$x = 10$

5. One card is drawn from a well shuffled deck of 52 cards. The probability that it is black queen is

(A) $\frac{1}{26}$ (B) $\frac{1}{13}$

(C) $\frac{1}{52}$ (D) $\frac{2}{13}$

Sol. $\frac{1}{26}$

6. If $\sin\theta = \frac{1}{3}$, find the value of $2\cot^2\theta + 2$.

(A) 15 (B) 16

(C) 17 (D) 18

Sol. $\sin\theta = \frac{1}{3} = \frac{P}{H}$

So, $B = \sqrt{H^2 - P^2} = \sqrt{9-1} = \sqrt{8}$

$\cot\theta = \frac{B}{P} = \frac{\sqrt{8}}{1} \Rightarrow \cot^2\theta = 8$

$\Rightarrow\ 2\cot^2\theta + 2 \Rightarrow 2\times 8 + 2 \Rightarrow 18$

7. For what value of k is –4 a zero of the polynomial $f(x) = x^2 - x - (2k+2)$

(A) 6 (B) –6

(C) 9 (D) –9

Ans. C

Sol. $f(x) = x^2 - x - (2k+2)$

Substituting $x = -4$,

$f(-4) = (-4)^2 - (-4) - (2k+2) = 0$

$\Rightarrow 18 - 2k = 0$

$\Rightarrow k = 9$

8. The value of k for which the system of equation $x + y - 4 = 0$ and $2x + ky = 3$ has no solution is

(A) –2 (B) $\neq 2$

(C) 3 (D) 2

Ans. D

Sol. $x + y - 4 = 0$

$2x + ky - 3 = 0$

For no solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$

$k = 2$; $k \neq \frac{3}{4}$

9. A wheel makes 1000 revolutions in covering a distance of 88 km. The radius of the wheel is,

(A) 11 m (B) 14 m

(C) 12 m (D) 10 m

Ans. B

Sol. Distance covered in 1 revolution $=\frac{88\times1000}{1000}\text{m}=88\text{ m}$

$\Rightarrow\ 2\pi R=88\text{ m}$

$2\times\frac{22}{7}\times R=88$

R = 14 m

10. The numerical value of $\left(\frac{1}{\cos\theta}+\frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta}-\frac{1}{\cot\theta}\right)$ is,

(A) 0
(B) –1
(C) 1
(D) 2

Ans. C

Sol. $\left(\frac{1}{\cos\theta}+\frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta}-\frac{1}{\cot\theta}\right)=\frac{1}{\cos^2\theta}-\frac{1}{\cot^2\theta}$

$=\sec^2\theta-\tan^2\theta=1$

11. Given that $\sin\theta=\frac{a}{b}$ then $\cos\theta$ is equal to

(A) $\frac{b}{\sqrt{b^2-a^2}}$
(B) $\frac{b}{a}$
(C) $\frac{\sqrt{b^2-a^2}}{b}$
(D) $\frac{a}{\sqrt{b^2-a^2}}$

Ans. C

Sol. $\sin\theta=\frac{a}{b}$

$\cos\theta=\sqrt{1-\sin^2\theta}$

$=\sqrt{1-\frac{a^2}{b^2}}$

$=\frac{\sqrt{b^2-a^2}}{b}$

12. Abscissa of centroid of triangle having vertices (a, b – c) ; (b, c – a) ; (c, a – b) is

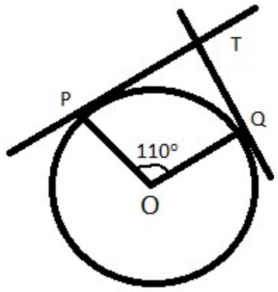
(A) 0
(B) $\frac{a+b+c}{3}$
(C) a + b + c
(D) $\frac{2a+2b+2c}{3}$

Ans. B

Sol. Co-ordinates of centroid of a triangle $=\left(\frac{x_1+x_2+x_3}{3}\ ;\ \frac{y_1+y_2+y_3}{3}\right)$

$\therefore$ Abscissa $=\frac{x_1+x_2+x_3}{3}=\frac{a+b+c}{3}$

13. In the given figure, if TP and TQ are tangents to a circle with centre O, so that $\angle POQ = 110°$, then $\angle PTQ$ is



(1) 110° (2) 90°

(3) 80° (4) 70°

Ans. D

Sol. 70°

14. The sum of the zeroes of the polynomial $2x^2 - 8x + 6$ is

(A) – 3 (B) 3

(C) – 4 (D) 4

Ans. D

Sol. 4

15. The distance of the point P (3, - 4) from the origin is

(A) 7 units (B) 5 units

(C) 4 units (D) 3 units

Ans. B

Sol. 5 units

16. The mid point of the line segment joining the points (- 5, 7) and (- 1, 3) is

(A) (-3, 7) (B) (-3, 5)

(C) (-1, 5) (D) (5, -3)

Ans. B

Sol. (- 3, 5)

17. The number of decimal places after which the decimal expansion of the rational number 14587/1250 will terminate is

(A) 1 (B) 2

(C) 3 (D) 4

Ans. D

Sol. $\frac{14587 \times 8}{1250 \times 8} = \frac{14587 \times 8}{10000} = 116696 \times 10^{-4}$

18. If sin (A+B) = sin A cos B + cos A sin B then find sin 75° =

(A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (B) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{2}$ (D) $\frac{\sqrt{3}+1}{\sqrt{2}}$

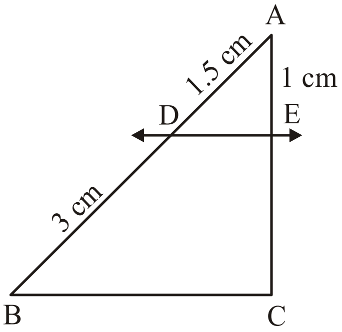
Ans. A

Sol. $\sin(45+30)^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

19. In the given figure, DE || BC. What is the value of EC ?



(A) 4 (B) 5

(C) 6 (D) 2

Ans. D

Sol. Since, DE || BC

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2\,\text{cm}$$

20. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time tower casts the shadow 40 m long on the ground. Find height of tower ?

(A) 60 m (B) 40 m

(C) 80 m (D) 100 m

Ans. A

Sol. $\frac{12}{x} = \frac{8}{40} \Rightarrow x = 60\,\text{m}$

SECTION - B

1. Find the four angles of cyclic quadrilateral ABCD in which

$\angle A = (2x-1)^\circ$ $\angle B = (y+5)^\circ$ $\angle C = (2y+15)^\circ$ $\angle D = (4x-7)^\circ$

Sol. $\angle A + \angle C = 180^\circ$

$\angle B + \angle D = 180^\circ$

$2x - 1 + 2y + 15 = 180$ $y + 5 + 4x - 7 = 180$

$2x + 2y = 166$ $4x + y = 182$

$x = 33$ $y = 50^\circ$

$\angle A = 65^\circ$ $\angle B = 55^\circ$

$\angle C = 115^\circ$ $\angle D = 125^\circ$

2. If a, b, c are in AP, then find the roots of $ax^2 + 2bx + c = 0$

Sol. Given that a, b, c are in AP

$2b = a + c$

$ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + (a + c)x + c = 0$

$\Rightarrow ax^2 + ax + cx + c = 0$

$\Rightarrow ax(x+1) + c(x+1) = 0$

$\Rightarrow (x+1)(ax+c) = 0$

$\Rightarrow x + 1 = 0 \text{ or } ax + c = 0$

$\Rightarrow x = -1 \text{ or } x = \frac{-c}{a}$

(OR)

Find the mode of following data :

Classes	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250
Frequency	12	13	15	8	12

Sol. Here maximum frequency is 15 and corresponding class is 100 - 150. So, 100 - 150 is the modal class Such that

$l = 100$, $h = 50$, $f = 15$, $f_1 = 13$, $f_2 = 8$

$$\text{Mode } = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 100 + \frac{15 - 13}{30 - 13 - 8} \times 50$$

$$= 100 + \frac{2}{9} \times 50$$

$$= 100 + \frac{100}{9} = \frac{1000}{9} = 111.11$$

3. Find the probability that a leap year selected at random will contain 53 sundays

Sol. There are 366 days in a leap year that contain 52 sundays and 2 days

These 2 days can be

(Mon, Tue)

(Tue, Wed)

(Wed, Thu)

(Thu, Fri)

(Fri, Sat)

(Sat, Sun)

In order to have 53 Sundays we should have Either {Sat, Sun} or {Sun, Mon}

No. of sample spaces $= 7$

No. of events that gives 53^{rd} Sunday in leap year $= 2$

(Sun, Mon) Required probability $= \frac{2}{7}$

4. If $2\sin(A+B) = \sqrt{3}$ and $\tan B = 1$, find sin2A

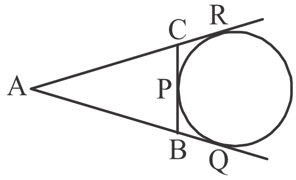
Sol. $\sin(A+B) = \frac{\sqrt{3}}{2}$ | $\tan B = 1$

$A + B = 60$ ——① | $B = 45$ ——②

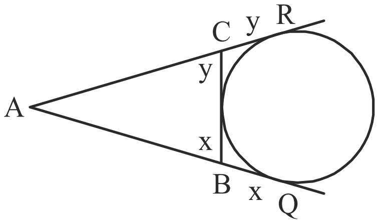
From ① & ② A=15

$\therefore \sin 2A = \sin 30 = \frac{1}{2}$

5. In the given figure a circle touches the sides BC of ΔABC at P and AB and AC are produced at Q and R respectively. If AQ = 15 cm. Find the perimeter of ΔABC



Sol.



$AC + y + AB + x = \text{perimeter}$

$AR + AQ = 30$

(OR)

The perimeter of a sector of circle with central angle 90° is 25 cm. Find the radius of a circle

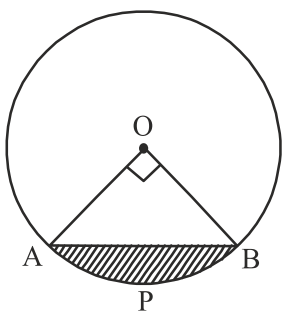
Sol. Perimeter of sector with central angle $90° = \frac{2\pi r}{4} + 2r$

$\frac{2\pi r}{4} + 2r = 25$

$2r\left(\frac{22}{28} + 1\right) = 25$

$2r \times \frac{50}{28} = 25$

$2r = 14 \Rightarrow r = 7$



SECTION - C

1. Find 31st term of an A.P., whose 11th term is 38 and 16th term is 73.

Sol. Let first term of an A.P. is a and common difference is d.

Now,

$$a_{11} = 38 \qquad \text{...............(i)}$$

$$\Rightarrow \quad a + 10d = 38$$

$$a_{16} = 73$$

$$\Rightarrow \quad a + 15d = 73 \qquad \text{...............(ii)}$$

On subtracting equation (i) from equation (ii),

$$15d - 10d = 73 - 38$$

$$\Rightarrow \quad 5d = 35$$

$$\Rightarrow \quad d = 7$$

Put $d = 7$ in equation (i),

$$a + 10 \times 7 = 38$$

$$\Rightarrow \quad a = 38 - 70 = -32$$

So, 31st term $= a + 30d$

$$= -32 + 30 \times 7$$

$$= -32 + 210$$

$$= 178$$

(OR)

Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Sol. Let $x = \sqrt{2} + \sqrt{3} = \text{rational number}$

$$\therefore x^2 = 2 + 3 + 2\sqrt{6}$$

$$\sqrt{6} = \frac{x^2 - 5}{2}$$

As x, 2, 5 are rational $\Rightarrow \frac{x^2 - 5}{2}$ is also rational.

i.e., $\sqrt{6} = \frac{x^2 - 5}{2}$ is rational

which is contradiction of the fact that $\sqrt{6}$ is irrational.

$\therefore$ Our assumption is wrong.

$\sqrt{2} + \sqrt{3}$ is irrational.

2. A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream

Sol. Let the speed of the stream be x km/hr

Given, the speed of the boat = 18 km/hr

Speed of the boat in upstream = (18 - x) km/hr

Speed of the boat in downstream = (18 + x) km/hr

Distance travelled in both the cases = 24 km

Time taken to go upstream $= \frac{24}{18-x}$ hours

Time taken to go downstream $= \frac{24}{18+x}$ hours

According to the problem,

$$\frac{24}{18-x} - \frac{24}{8+x} = 1$$

$$\Rightarrow \frac{24(18+x)-24(18-x)}{(18-x)(18+x)} = 1$$

$$\Rightarrow \frac{48x}{324-x^2} = 1$$

$$\Rightarrow 48x = 324 - x^2$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow (x-6)(x+54) = 0$$

x = 6 or x = - 54

Since the speed of the boat never be negative x = -54 is ignored

The speed of the stream = 6 km/hr

3. Water flows at the rate of 10m/min, through a cylindrical pipe having its diameter as 5mm. How much time will it take to fill a conical vessel whose diameter of base is 40cm and depth 24cm.

Sol. Diameter of pipe = 5mm = $\frac{5}{10}$ cm $= \frac{1}{2}$ cm

Radius of pipe = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ cm

In 1 min, the length of water column in a cylindrical pipe = 10m = 1000 cm.

$\therefore$ Volume of water that flows out of pipe in 1 min $= \pi \times \frac{1}{4} \times \frac{1}{4} \times 1000\,\text{cm}^3$

volume of cone = $\frac{1}{3}\pi \times 20 \times 20 \times 24$ cm^3 So times needed to fill up conical vessel

$$= \frac{1}{3}\pi \times 20 \times 20 \times 24 / \pi \times \frac{1}{4} \times \frac{1}{4} \times 1000 \text{ min}$$

$$\Rightarrow \frac{256}{5} \text{ min} \Rightarrow 51.2 \text{ min}$$

4. The mean of the following frequency table is 50. But the frequency f_1 and f_2 in class 20-40 and 60-80 are missing. Find the missing frequencies.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total
Frequency	17	f_1	32	f_2	19	120

Sol. Let the assumed mean be A = 50 and h = 20

Class	Frequency (f_i)	Mid values (x_i)	$u_i = \frac{x_i - A}{h}$	$f_i \cdot \mu_i$
0 – 20	17	10	–2	–34
20 – 40	f_1	30	–1	$-f_1$
40 – 60	32	50	0	0
60 – 80	f_2	70	1	f_2
80 – 100	19	90	2	38

$N = \Sigma f_i = 68 + f_1 + f_2$ $\qquad\qquad$ $\Sigma f_i \cdot u_i = 4 - f_1 + f_2$

Now : $68 + f_1 + f_2 = 120$ [given]

$f_1 + f_2 = 52 \qquad$ ...(i)

Now : Mean = 50

$$\Rightarrow A + h\left[\frac{1}{N}\Sigma f_i \cdot u_i\right] = 50$$

$$\Rightarrow 50 + 20\left\{\frac{4 - f_1 + f_2}{120}\right\} = 50$$

$$\Rightarrow \frac{4 - f_1 + f_2}{6} = 0$$

$\Rightarrow f_1 - f_2 = 4 \qquad$ ...(ii)

Solving (i) and (ii)

$f_1 = 28$, $f_2 = 24$

5. The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units, and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length of breadth of the rectangle.

Sol. Let length = x

Breadth = y

Area = xy sq. units

Now according to question

$\Rightarrow xy - 9 = (x - 5)(y + 3)$

$\Rightarrow xy - 9 = xy + 3x - 5y - 15$

$\Rightarrow 3x - 5y - 6 = 0 \qquad$ ...(i)

Now in second condition

$xy + 67 = (x + 3)(y + 2)$

$\Rightarrow xy + 67 = xy + 2x + 3y + 6$

$\Rightarrow 2x + 3y - 61 = 0$ ...(ii)

Now we have to simultaneous equation

$3x - 5y - 6 = 0$

$2x + 3y - 61 = 0$

Solving these equations

x = 17, y = 9

(OR)

If α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4,$ find the value of

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

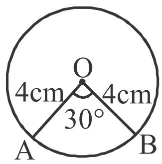
Sol. Since α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4.$

$$\therefore \quad \alpha + \beta = \frac{-(-6)}{3} = 2 \text{ and } \alpha\beta = \frac{4}{3}$$

We have $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) + 3\alpha\beta$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + \frac{2(\alpha + \beta)}{\alpha\beta} + 3\alpha\beta = \frac{(2)^2 - 2 \times \frac{4}{3}}{\frac{4}{3}} + \frac{2 \times 2}{\frac{4}{3}} + 3 \times \frac{4}{3} = 8$$

6. Find the area of sector of circle with radius 4 cm and of angle $30°$. Also find the area of the corresponding major sector.[Take $\pi = 3.14$]



Sol. $\frac{\theta}{360} \times \pi r^2 = \frac{30}{360} \times 3.14 \times 4 \times 4 \text{cm}^2$

$\frac{3.14 \times 4}{3} = 4.186 \text{ cm}^2$

$\pi r^2 - \pi r^2 \left(\frac{\theta}{360}\right)$

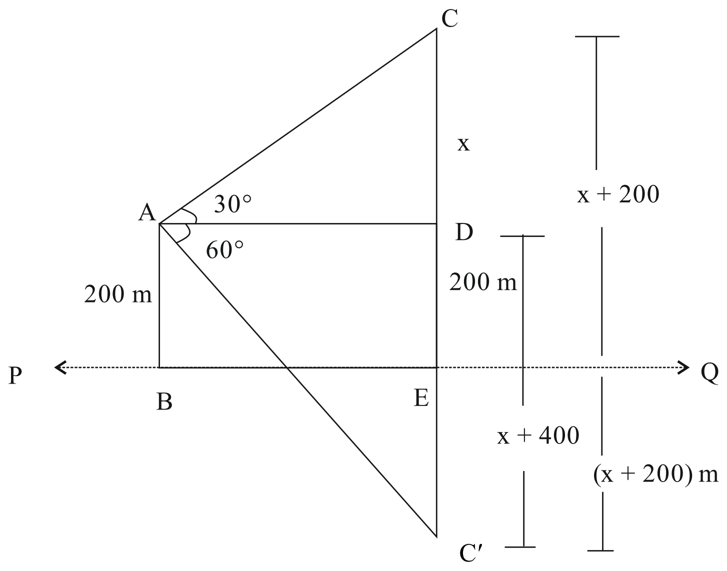
$\pi r^2 - \pi r^2 \frac{\theta}{360}$

$\pi r^2 \left(1 - \frac{\theta}{360}\right) = \frac{3.14 \times 44}{3} = 46.05 \text{cm}^2$

SECTION - D

1. The angle of elevation of a cloud from a point 200 metres above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud.

Sol.



Let AB be the height point of observation = 200 m

Let PQ be the surface of the lake

Let CE be the height of the cloud (x + 200) m

In $\Delta ADC, \angle ADC = 90°$

$$\tan 30° = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{AD}$$

$$\Rightarrow AD = \sqrt{3}x$$

In $\Delta ADC'', \angle ADC' = 90°$

$$\tan 60° = \frac{DC'}{AD}$$

$$\sqrt{3} = \frac{x + 400}{\sqrt{3}x}$$

$$\Rightarrow 3x = x + 400$$

$x = 200$

The height of the cloud $= x + 200$

$= 400$ mts

(OR)

If the polynomial $f(x)=x^4-6x^3+16x^2-26x+10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be a, find k & a.

Sol.

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & \cancel{x^4}-6x^3+16x^2-26x+10 \\
 & \cancel{x^4}-2x^3+kx^2 \\
 & \scriptstyle{-\quad +\quad -} \\
\hline
 & \cancel{-4x^3}+x^2(16-k)-26x+10 \\
 & \cancel{-4x^3}+8x^2 \qquad -4kx \\
 & \scriptstyle{+\quad -\qquad +} \\
\hline
 & x^2\cancel{(8-k)}-x(4k-26)+10 \\
 & x^2\cancel{(8-k)}-x(16-2k)+k(8-k) \\
 & \scriptstyle{-\qquad +\qquad -} \\
\hline
 & x(2k-10)+k^2-8k+10
\end{array}
$$

Rem. is a so, x coeff = 0

$2k - 10 = 0 \Rightarrow k = 5$

So Rem. $a = k^2 - 8k + 10$

$= 25 - 40 + 10 \Rightarrow -5$

2. If $\sec\theta+\tan\theta=P$, show that $\frac{P^2-1}{P^2+1}=\sin\theta$

Sol. We have

$$\text{LHS}=\frac{P^2-1}{P^2+1}=\frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$$

$$\Rightarrow \text{LHS}=\frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}$$

$$\Rightarrow \text{LHS}=\frac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec\theta+2\sec\theta\tan\theta+(1+\tan^2\theta)}$$

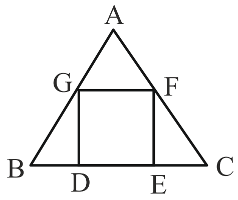
$$\Rightarrow \text{LHS}=\frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+2\sec\theta\tan\theta+\sec^2\theta}$$

$$\Rightarrow \text{LHS}=\frac{2\tan^2\theta+2\tan\theta\sec\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$$

$$\Rightarrow \text{LHS}=\frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)}$$

$$\Rightarrow \text{LHS}=\frac{\tan\theta}{\sec\theta}=\frac{\sin\theta}{\cos\theta.\sec\theta}=\sin\theta=\text{RHS}$$

3. In figure, DEFG is a square and $\angle BAC = 90°$



Prove that

(i) $\Delta AGF \sim \Delta DBG$

(ii) $\Delta AGF \sim \Delta EFC$

(iii) $\Delta DBG \sim \Delta EFC$

(iv) $DE^2 = BD \times EC$

Sol. (i) In ΔAGF and ΔDBG

$\angle GAF = \angle BDG$ [Each $90°$]

$\angle AGF = \angle DBG$ [Corresponding angles]

$\Delta AGF \sim \Delta DBG$ [By AA-criterion of similarity]

(ii) In ΔAGF and ΔEFC, we have

$\angle FAG = \angle CEF$ [Each equal to $90°$] $\angle AFG = \angle ECF$ [Corresponding angles]

$\Delta AGF \sim \Delta EFC$ [AA-criterion of similarity]

(iii) Since $\Delta AGF \sim \Delta DBG$ and $\Delta AGF \sim \Delta EFC$ $\therefore$ $\Delta DBG \sim \Delta EFC$

(iv) We have

$\Delta DBG \sim \Delta EFC$ $\frac{BD}{EF} = \frac{DG}{EC}$

$\frac{BD}{DE} = \frac{DE}{FC}$ [DEFG is square $\therefore$ $EF = DE,\ DG = DE$] $\Rightarrow DE^2 = BD \times EC$

4. Find the cordinates of the centre of the circle passing through the points (0,0),(-2,1)and(-3,2). Also find its radius ?

Ans. *Let centre is* $P(x, y)$

$A(0,0), B(-2,1), c(-3,2)$ $\therefore PA^2 = PB^2 = PC^2$ $PA^2 = PB^2$

$\Rightarrow x^2 + y^2 = x^2 + y^2 + 4x - 2y + 5$ $\therefore 4x - 2y + 5 = 0$ _____(1)

$PA^2 = PC^2$ $x^2 + y^2 = (x+3)^2 + (y-2)^2$ $\therefore 6x - 4y + 13 = 0$ _______(2)

by solving (1) ,(2) $x = \frac{3}{2}$ $y = \frac{11}{2}$ $\therefore$ *Centre* $(3/2, 11/2)$

$radius = AP = \sqrt{x^2 + y^2} = \sqrt{\frac{9}{4} + \frac{121}{4}} = \frac{1}{2}\sqrt{130}$

(OR)

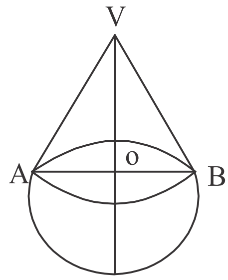
(i) A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1cm, and the height of the cone is equal to it's radius. Find the volume of solid in terms of π

Ans. We have

r_1 = radius of care =1cm

r_2 = radius of hemisphere 1 cm

h_1 = height of the cone =1cm



$\therefore$ Volume of the solid = Volume of cone + Volume of hemisphere

$$=\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3 \qquad =\frac{1}{3}\pi(1)^2(1)+\frac{2}{3}\pi(1)^3=\pi \text{cm}^3$$

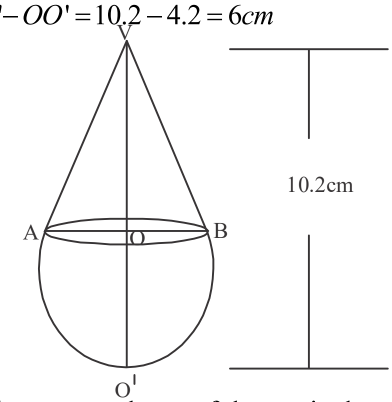
(ii) A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm find the volume of the wooden toy ?

Ans. We have VO' =10.2 cm , OA = OO' = 4.2cm

Let 'r' be the radius of the hemisphere and 'h' be the height of the conical part

Then r =OA =4.2cm

$h=VO=VO'-OO'=10.2-4.2=6cm$



Volume of the toy = volume of the conical part + volume of the hemisphere part

$$=\left(\frac{1}{3}\pi r^2 h+\frac{2\pi}{3}r^3\right)$$

$$=\frac{\pi r^2}{3}(h+2r)=\frac{1}{3}\times\frac{22}{7}\times 4.2\times 4.2\times 14.4=266.11cm^3$$

SECTION - E

1. A farmer wants to dig a well in his house premises for his household purpose. He dug a well 30m deep and 7m diameter. The earth from digging the well can be used construction of his house. For this, he wants to prepare bricks of size $15\text{cm}\times 8\text{cm}\times 5\text{cm}$ each.

Study the above data and answer the following

(i) Find the surface area of each brick.

(ii) Find the number of bricks he prepared with the earth which he obtained during digging the well

Sol. (i) Length of the brick $l = 15$ cm

Breadth of the brick , b = 8 cm

Height of the brick h = 5 cm

Surface area of each brick = $2\ (lb + bh + hl)$

$$= 2(15 \times 8 + 8 \times 5 + 5 \times 15)$$

$$= 2(120 + 40 + 75)$$

$$= 2 \times 235$$

$$= 470\text{cm}^2$$

(ii) Volume of the soil dug $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 30$$

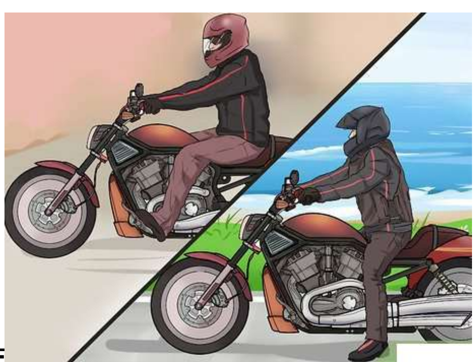
$$= 77 \times 15\text{m}^3 = 1155\text{m}^3$$

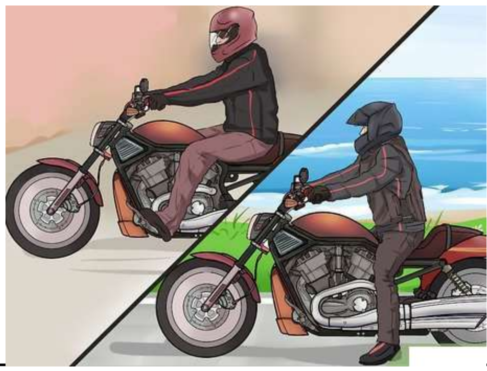
Volume of each brick = lbh

$$= \frac{15}{100} \times \frac{8}{100} \times \frac{5}{100} = \frac{3}{5000}\text{m}^3$$

Number of bricks $= \dfrac{1155 \times 5000}{3} = 385 \times 5000 = 1925000$

2. Amar bought a motor cycle. He does not know how to ride it. So he decided to learn every day for 1 hour. After getting same practice he would like to increase the speed day by day. First day he drove with the speed of 10 km/hr, second day 15 km/hr, third day 20 km/hr, 4th day 25 km/hr, 5th day 30 km/hr and so on. He decided to learn within 12 days.





Study the data and answer the following.

1. Find the distance travelled by Amar on each day.
2. Is this data forms AP
3. If yes, find the total distance travelled by Amar in 12 days.

Sol. Distance travelled on 1st day = 10 km

Distance travelled on 2nd day = 15 km

Distance travelled on 3rd day = 20 km

Here 10, 15, 20, ... forms AP

because common difference is same

No.of days he learnt , n = 12

First term, $a_1 = a = 10$

Second term, $a_2 = 15$

Common difference, $d = a_2 - a_1 = 15 - 10 = 5$

Total distance he travelled in 12 days $= \frac{n}{2}\left[2a + (n-1)d\right]$

$$= \frac{12}{2}\left[2 \times 10 + (12-1)5\right] \quad = 6\left[20+55\right] = 6 \times 75 = 450 \text{ kms}$$

3. A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

(i). In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number of participants that can be accommodated in each room are

(ii). What is the minimum number of rooms required during the event?

(iii). The LCM of 60, 84 and 108 is

(iv). The product of HCF and LCM of 60, 84 and 108 is

Sol. (i) Maximum number of participants will be HCF of 60, 80, 108

HCF(60, 84, 108) =12

(ii) Minimum number of rooms required = $\frac{\text{Total participants}}{\text{Max participants in one room}}$

$$= \frac{252}{12} = 21$$

(iii) LCM(60, 84,100)= 3780

(iv) $\text{HCF} \times \text{LCM} = 12 \times 3780 = 45360$