ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)
path to success

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2022 - 2023)

JEE (Main)
Sample Practice Paper

JEE(Main) : SAMPLE TEST PAPER - 01

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	B	B	A	D	D	A	D	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	C	A	B	B	D	A	C	C	D
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	6	1	4	1	0	64	83	2	500	6

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	B	A	D	D	D	B	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	A	D	A	B	D	B	C	D	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	54	1107	14	20	5	1	1	1	2	0

PART-3 : MATHEMATICS

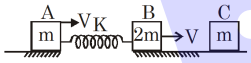
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	D	C	A	B	C	B	A	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	A	B	A	B	C	A	A	A
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	2	36	49	11	5	2	-1	1	1

HINT – SHEET

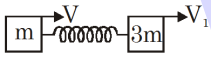
PART-1 : PHYSICS

SECTION-I

1. Ans (A)



before collision



Just after collision

$$v_{CM} = \frac{mv + 2mv}{4m} = \frac{3v}{4}$$

for block 'A'

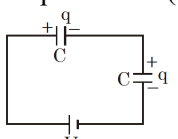
$$-\frac{v}{4} \leqslant v_{A/CM} \leqslant \frac{v}{4}$$

$$\frac{v}{2} \leqslant v_A \leqslant \frac{v}{4} + \frac{3v}{4}$$

$$(v_A)_{min} = (v_A)_{min} = \frac{v}{2} = 5m/s$$

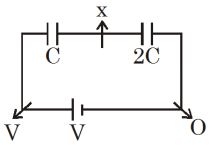
2. Ans (B)

For position---(1)



$q = \frac{C}{2}v$

For position---(2)



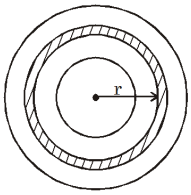
$$(x - V)C + (x - 0)2C = -q = -\frac{CV}{2}$$

$$3x - V = -\frac{V}{2}$$

$$x = \frac{V}{6}$$

$$q_{2C} = 2C(x - 0) = \frac{CV}{3}$$

3. Ans (B)



Using guass law

$$\int \vec{E}.\vec{ds} = \frac{q_{in}}{\epsilon_0} \quad .....(1)$$

$$q_{in} = q + \int_{R_1}^{r} \frac{b}{r} 4\pi r^2 dr$$

$$= q + 4\pi b \frac{(r^2 - R_1^2)}{2}$$

from equation -1

$$E\, 4\pi r^2 = \frac{1}{\epsilon_0}\left(q + \frac{4\pi b\,(r^2 - R_1^2)}{2}\right)$$

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{(q - 2\pi b R_1^2)}{r^2} + 2\pi b\right)$$

as $\vec{E}$ is constant, therefore

$$q - 2\pi b R_1^{\,2} = 0$$

$$b = \frac{q}{2\pi R_1^2} = \frac{16\times 10^{-6}}{2\pi \times \frac{4}{\pi}\times 10^{-6}} = 2$$

4. Ans (B)

Time of flight $= \frac{2u}{g} = \frac{50}{10} = 5$ sec

$$(\vec{r}_f)_{football} = \left(2\hat{i} + 5\hat{j}\right)\times 5 = 10\hat{i} + 25\hat{j}$$

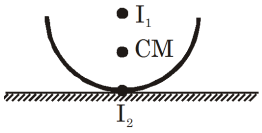
$$(\vec{r}_f)_{player} = \left(5\hat{i} + 8\hat{j} + 2\hat{i} + 4\hat{j} + 6\hat{j}\right) + \left(2\hat{i} + 3\hat{j}\right)$$

$$= 9\hat{i} + 21\hat{j}$$

distance $= \sqrt{1^2 + 4^2} = \sqrt{17}$

5. Ans (A)

$$r_{CM} = \frac{R}{2}$$



$$I_1 = I_{CM} + m\left(\frac{R}{2}\right)^2 = \frac{2}{3}mR^2$$

$$I_2 = I_{CM} + m\left(\frac{R}{2}\right)^2 = \frac{2}{3}mR^2$$

$$W = \Delta K$$

$$mg\frac{R}{2} = \frac{1}{2}\frac{2}{3}mR^2\omega^2 \Rightarrow \omega^2 = \frac{3g}{2R}$$

using $F = ma_{CM}$

$$N - mg = m\frac{\omega^2 R}{2}$$

$$N = mg + \frac{3mg}{4} = \frac{7mg}{4}$$

6. Ans (D)

$$v_{wave} = \frac{30\times 10}{60} = 5 m/s$$

when boat is moving, suppose n waves are striking per minute

$$n = \frac{(v_{rel} \times time)}{\lambda} = \frac{10\times 60}{10} = 60$$

7. Ans (D)

$$\frac{R}{x} = \frac{16}{100 - x}$$

$$R = \frac{16x}{100 - x}$$

$$\log R = \log 16 + \log x - \log (100 - x)$$

$$\frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{\Delta x}{100 - x}$$

$$\frac{\Delta R}{R} = \frac{0.1}{25} + \frac{0.1}{75}$$

$$\% \text{ error} = 0.4 + \frac{0.4}{3} = \frac{1.6}{3} = 0.53$$

8. Ans (A)

Initially no. of active nuclei of $P_{15}^{32} = N_1$

no. of active nuclie of $P_{15}^{33} = N_2$

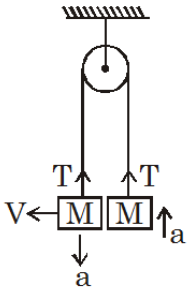
$\frac{\lambda_1 N_1}{\lambda_2 N_2} = 9$ .....(1)

finally $\frac{\lambda_1 N_1 e^{-\lambda_1 t}}{\lambda_2 N_2 e^{-\lambda_2 t}} = \frac{1}{9}$

$e^{-(\lambda_2-\lambda_1)t} = 81$

$t = \frac{\ell n 81}{-(\lambda_2 - \lambda_1)} = \frac{4\ell n 3}{\left(\frac{\ell n 2}{T_1} - \frac{\ell n 2}{T_2}\right)}$

9. Ans (D)



before collision with pan

$V_0 = \sqrt{2g\frac{\ell}{2}} = \sqrt{g\ell}$

Just after collision

$\frac{M}{2}V_0 = \left(\frac{M}{2} + M\right)V$

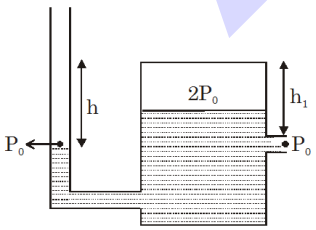
$V = \frac{V_0}{3}$

for 'A' $-T + Mg + \frac{MV^2}{\ell} = Ma$ ....(1)

for 'B' $T - Mg = Ma$ ...(2)

(1) + (2) $\Rightarrow a = \frac{V^2}{2\ell} = \frac{g}{18}$

10. Ans (D)



Pressure at same height is same, therefore

$h = h_1$

11. Ans (A)

$E_2 - E_1 = \frac{hc}{\lambda_\alpha}$ & $E_1 = -23.5$

$E_2 + 23.5 = \frac{12400}{0.71} \Rightarrow E_2 = -6keV$

energy of atom = $|E_2| = 6keV$

12. Ans (C)

$L = 1 \times 4 \times 0.2 + \frac{1 \times 4 \times 10^{-2}}{2} \times 10$

$= 0.8 + 0.2 = 1$

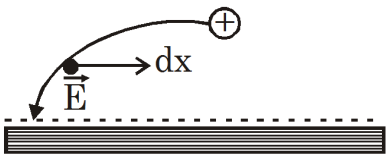
13. Ans (A)

$\frac{\sqrt{a^2+h^2}}{V_S} < \frac{a}{V_P} + \frac{h}{V_S}$

$\frac{a^2+h^2}{V_S^2} < \frac{a^2}{V_P^2} + \frac{h^2}{V_S^2} + \frac{2ah}{V_P V_S}$

$a < \frac{\frac{2h}{V_P V_S}}{\left(\frac{1}{V_S^2} - \frac{1}{V_P^2}\right)} < \frac{2h\left(\frac{V_P}{V_S}\right)}{\left(\frac{V_P}{V_S}\right)^2 - 1}$

14. Ans (B)



$W_{elec} = \int q\vec{E} \cdot dx$

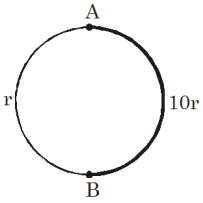
$= \int qE\,dx\cos\theta$

& θ is obtuse angle

$W_{Fext} + W_{elec} = 0$

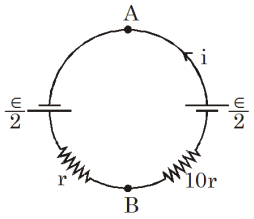
$W_{ext} = +ve$

15. Ans (B)



$$\in = \pi R^2 \frac{dB}{dt} = \pi R^2 b$$

$$i = \frac{\pi R^2 b}{11r}$$



$$\Delta V_{AB} = E\pi R = \frac{\in}{2} - ir$$

$$E\pi R = \frac{\pi R^2 b}{2} - \frac{\pi R^2 b}{11}$$

$$E = \frac{9Rb}{22}$$

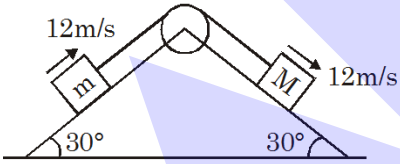
16. Ans (D)

$$F = -\frac{dU}{dx} = -20 \sin 2x$$

$a = -40\ x$ for small 'x'

$$T = 2\pi\sqrt{\frac{1}{40}} = \frac{\pi}{\sqrt{10}}$$

17. Ans (A)



$$a_M = g \sin 30° - \frac{g \cos 30°}{\sqrt{2}} = 5 - 6 = -1 m/s^2$$

$$a_m = -(g\sin 30° + \frac{g \cos 30°}{\sqrt{2}}) = -11\ m/s^2$$

as limiting friction is greater than mgsinθ,

block 'm' will not move after $t = \frac{12}{11}$ sec.

$v_M = 12 - 1 \times 2 = 10$ m/s

18. Ans (C)

$a = \omega^2 x$

$$V^2 = \omega^2 (A^2 - x^2)$$

$$V^2 = \omega^2 \left(A^2 - \frac{a^2}{\omega^4}\right)$$

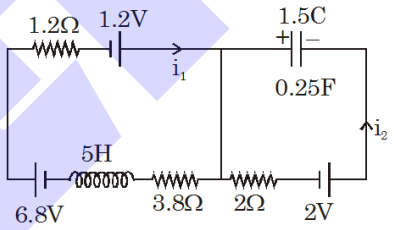
$$\text{Slope} = -\frac{1}{\omega^2} = -\frac{\ell}{g}$$

$$\tan 30° = \frac{\ell}{g_0}$$

$$\tan 60° = \frac{\ell\left(1 + \frac{h}{R}\right)^2}{g_0}$$

$$\Rightarrow \left(1 + \frac{h}{R}\right)^2 = 3 \Rightarrow h = 0.73R$$

19. Ans (C)



Current through key

$i = i_1 + i_2$

$$i = \frac{8}{5}(1 - e^{-t}) + \frac{2}{2}(e^{-2t}) + \frac{6}{2}e^{-2t}$$

for minimum current

$$\frac{di}{dt} = 0 \Rightarrow \frac{8}{5}e^{-t} - 8e^{-2t} = 0 \Rightarrow e^{-t} = \frac{1}{5}$$

$$i_{min} = \frac{32}{25} + \frac{4}{25} = \frac{36}{25} \text{amp.}$$

20. Ans (D)

$$f' = \frac{(332 - 2)}{332} \times \left(\frac{332}{332 + 2}\right) \times 334$$

$f' = 330$ Hz

PART-1 : PHYSICS

SECTION-II

1. Ans (6)

$$r = \frac{h}{\sqrt{\mu^2 - 1}}$$

$$= \frac{8}{\sqrt{\left(\frac{5}{3}\right)^2 - 1}}$$

= 6 meter

2. Ans (1)

$E = a\cos\omega_0 t + a\cos\omega t\cos\omega_0 t$

$E = a\cos\omega_0 t +$

$$\frac{a}{2}[\cos(\omega + \omega_0)t + \cos(\omega - \omega_0)t]$$

$$\nu_{max} = \frac{\omega + \omega_0}{2\pi} = \frac{4.8}{2\pi} \times 10^{15}$$

$KE_{max} = h\nu - \phi$

$$= \frac{6.6 \times 10^{-34} \times \frac{4.8}{2\pi} \times 10^{15}}{1.6 \times 10^{-19}} - 2.15$$

$$= \frac{6.3}{2} - 2.15 = 1eV$$

3. Ans (4)

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_{max}} = eV$$

$$\frac{hc}{2\lambda} - \frac{hc}{\lambda_{max}} = \frac{eV}{3} = \frac{1}{3}\left(\frac{hc}{\lambda} - \frac{hc}{\lambda_{max}}\right)$$

$$\frac{1}{2\lambda} - \frac{1}{\lambda_{max}} = \frac{1}{3\lambda} - \frac{1}{3\lambda_{max}}$$

$$\frac{1}{6\lambda} = \frac{2}{3\lambda_{max}} \Rightarrow \frac{\lambda_{max}}{\lambda} = 4$$

4. Ans (1)

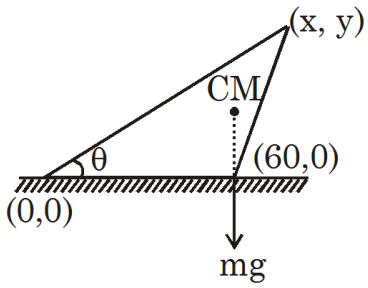
$$\nu_{PJ} = (i\lambda_{30}) = \left(\frac{10}{R+1}\right) R \text{ for } 1^{st} \text{ case}$$

$$\nu_{PJ} = i\lambda 10 = \left(\frac{5R}{R+2}\right) \text{ for } 2^{nd} \text{ case}$$

taking ratio

$$3 = \frac{2(R+2)}{(R+1)} \Rightarrow R = 1$$

5. Ans (0)



Just to prevent from toppling, mg must pass through toppling point.

$$x_{CM} = 60 = \frac{0 + 60 + x}{3} \Rightarrow x = 120 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times \frac{60}{100} \times y = 1 \Rightarrow y = \frac{10}{3}\text{m}$$

$$\cot\theta = \frac{x}{y} = \frac{120}{100 \times 10} \times 3 = 0.36$$

6. Ans (64)

$$\text{ratio} = n = \frac{8^2}{9^2}$$

$$n = \frac{64}{81}$$

7. Ans (83)

$$I_C = \frac{5}{2}\text{mA}$$

$$I_B = \frac{2.3}{R_B}$$

$$\beta = 90 = \frac{2.5 \times 10^{-3}}{2.3} R_B$$

$$R_B = \frac{90 \times 23}{25}\text{k}\Omega$$

= 82.80 kΩ

8. Ans (2)

$$B = \frac{\mu_0 \varepsilon_0 A}{2\pi r}\frac{dE}{dt}$$

$B = 1.85 \times 10^{-18}$ T

9. Ans (500)

$\Delta I_E = \Delta I_B + \Delta I_C$

$2.1 = \Delta I_B + 2 \Rightarrow \Delta I_B = 0.1$ mA

$v_{in} = \Delta I_B R_{in}$

$0.05 = 10^{-4} R_{in}$

$R_{in} = 500\ \Omega$

10. Ans (6)

$I = \frac{\vec{M}}{V} = \frac{5}{10^{-6}}$ $H = 0.5 \times 10^4$

$B = \mu_0 (H + I)$

$= 4\pi \times 10^{-7} (0.5 \times 10^4 + 5 \times 10^6)$

$= 6.28$

PART-2 : CHEMISTRY

SECTION-I

1. Ans (C)

$H_3PO_3 + 2NaOH \rightarrow Na_2HPO_3 + 2H_2O$

50 ml 1M

1M V = ?

$\Rightarrow \frac{n_{NaOH}}{n_{H_3PO_3}} = \frac{2}{1}$

$\Rightarrow \frac{1 \times V}{50 \times 1} = \frac{2}{1} \Rightarrow \boxed{V_{NaOH} = 100 \text{ ml}}$

$H_3PO_2 + 2NaOH \rightarrow NaH_2PO_2 + H_2O$

100 ml 1M

2M V = ?

$\Rightarrow \frac{n_{NaOH}}{n_{H_3PO_3}} = \frac{1}{1}$

$\Rightarrow \frac{1 \times V}{2 \times 100} = \frac{1}{1} \Rightarrow \boxed{V_{NaOH} = 200 \text{ ml}}$

3. Ans (B)

$\frac{K_{sp}}{Ka} = \frac{s^2}{(H^+)}$; $s = \sqrt{\frac{K_{sp}}{Ka}(H^+)}$

$s = \sqrt{\frac{2.2 \times 10^{-16}}{6.2 \times 10^{-10}} \times 10^{-3}}$

$s = 1.9 \times 10^{-5}$

Hence answer is (B)

4. Ans (A)

(a) Water $\xrightarrow{0°C}$ ice; ΔS = –ve

(b) Water $\xrightarrow{-10°C}$

(c) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$; DS = –ve

(d) Adsorption; ΔS = –ve

(e) $NaCl(s) \rightarrow Na^+(aq) + Cl^-(aq)$; ΔS = +ve

6. Ans (D)

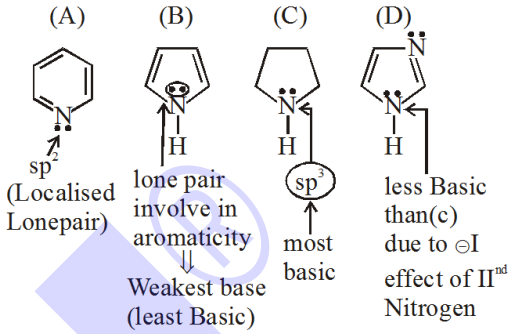
Cheese → liquid in solid

Pumice stone → gas in solid

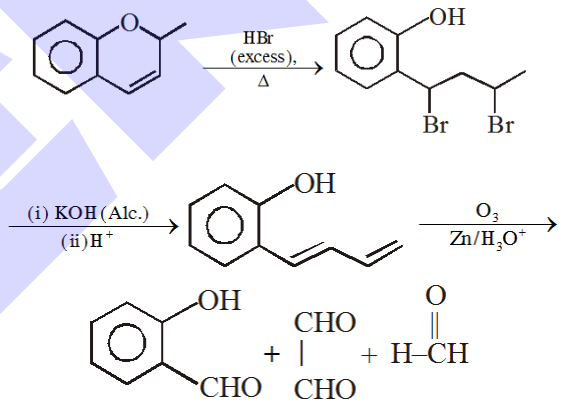
Hair cream → liquid in liquid

Cloud → liquid in gas

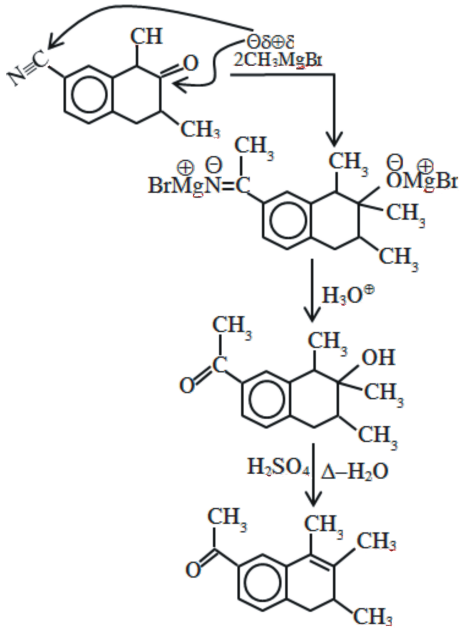
7. Ans (D)



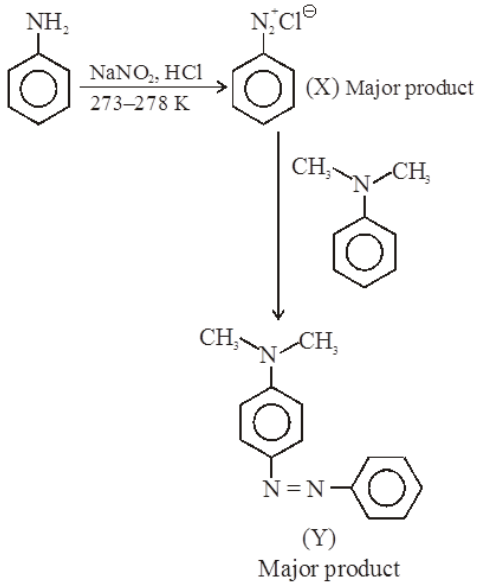
8. Ans (B)



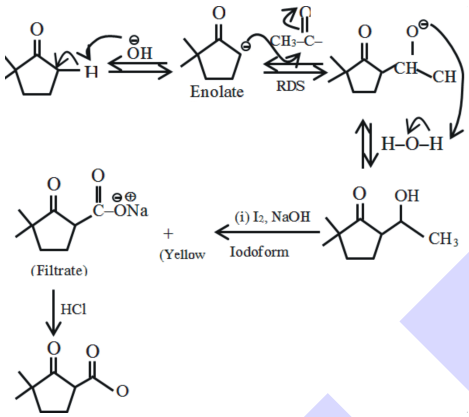
9. Ans (A)



10. Ans (B)



11. Ans (D)



12. Ans (A)

Modulation index: $m = \frac{A_m}{A_c}$

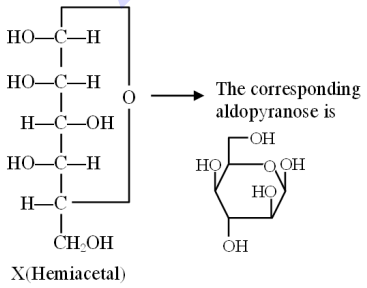
Given $2A_m = 8$

$A_m + A_c = 9 \Rightarrow A_c = 5$

$\therefore m = \frac{4}{5} = 0.8$

13. Ans (D)

Correct pyranose structure is



14. Ans (A)

According to question all the complexes are low spin.

Complex	Configuration	No. of unpaired electrons
$[V(CN)_6]^{4-}$	$t_{2g}^3 e_g^0$	3
$[Cr(NH_3)_6]^{2+}$	$t_{2g}^4 e_g^0$	2
$[Ru(NH_3)_6]^{3+}$	$t_{2g}^5 e_g^0$	1
$[Fe(CN)_6]^{4-}$	$t_{2g}^6 e_g^0$	0

Correct option : (1)

15. Ans (B)

$3Cl_2 + 6\ OH^- \rightarrow 5Cl^- + ClO_3^- + 3H_2O$

16. Ans (D)

Calcination in carried out for carbonates and oxide ores in absence of oxygen. Roasting is carried out mainly for sulphide ores in presence of excess of oxygen.

17. Ans (B)

Lewis base : Chemical species which has capability to donate electron pair.

In NF_3, SF_4, ClF_3 central atom (i.e. N, S, Cl) having lone pair therefore act as lewis base.

In PCl5 central atom (P) does not have lone pair therefore does not act as lewis base.

18. Ans (C)

O_2 (16 electrons)

$\sigma_{1s}^2,\ \sigma_{1s}^{*2},\ \sigma_{2s}^2,\ \sigma_{2s}^{*2},\ \sigma_{2p_z}^2$

$\pi_{2p_x}^2 = \pi_{2p_y}^2,\ \pi_{2p_x}^{*1} = \pi_{2p_y}^{*1},\ \sigma_{2p_z}^*$

Bond order of $O_2 \Rightarrow 2$

Bond order of $O_2^- \Rightarrow 1.5$

Bond order of $O_2^{2-} \Rightarrow 1$

Bond order of $O_2^+ \Rightarrow 2.5$

19. Ans (D)

Highest ionic mobility corresponds to lowest extent of hydration and highest size of gaseous ion. Hence Sr^{2+} has the highest ionic mobility in its aqueous solution

20. Ans (C)

$AgCl + 2NH3 \rightarrow [Ag(NH3)2]^+Cl^-$

soluble

PART-2 : CHEMISTRY

SECTION-II

1. Ans (54)

$\pi = C.R.T$

$7.47 = C \times 0.083 \times 300$

$C = 0.3\ M$

$= 0.3 \times 180\ gL^{-1}$

$= 54\ gL^{-1}$

2. Ans (1107)

Given : 2 mole of N_2 gas was present as inert gas.

Equilibrium pressure = 2.46 atm

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

t = 0	5	0	0
$t = Eq^m$	5 – x	x	x

from ideal gas equation

$PV = nRT$

2.46×200

$= (5 - x + x + x + 2) \times 0.082 \times 600$

$x = 3$

$$K_P = \frac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_5}} \times \left[\frac{P_{total}}{n_{total}}\right]$$

$$\frac{3 \times 3}{2} \times \frac{2.46}{10}$$

$= 1.107 = 1107 \times 10^{-3}$

3. Ans (14)

weight of organic compound = 0.2g

mass of $N_2(g)$ evolved $= \frac{22.4 \times 10^{-3}}{22.4} \times 28$

$= 28 \times 10^{-3} g$

$$\% \text{ of } N = \frac{28 \times 10^{-3}}{0.2} \times 100 = 14$$

4. Ans (20)

$Fe^{3+} + 3e^- \longrightarrow Fe$

$3F \longrightarrow$ 1 mole Fe is deposited

For 56 g $\longrightarrow$ 3×96500 (required charge)

For 1g $\longrightarrow$ $\frac{3 \times 96500}{56}$ (required charge)

For 0.3482 g $\longrightarrow$ $\frac{3 \times 96500}{56} \times 0.3482$

$= 1800.06$

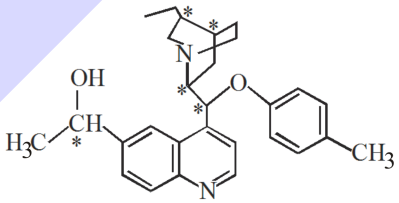
$Q = it$

$1800.06 = 1.5\ t$

$t = 20$ min

5. Ans (5)

No. of chiral centres



6. Ans (1)

Ofloxacin

7. Ans (1)

$$DU = 4 + 1 - \left(\frac{5-1}{2}\right) = 3$$

$H_3C-CH=CH-C\equiv N$

sp^3

or

$CH_2 = C = CH = CH = NH$

Zero sp^3 carbon

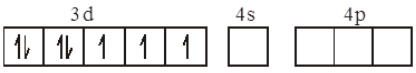
9. Ans (2)

Diamagnetic species are: N_2, O_2^{2-}

10. Ans (0)

$[Co(H_2O)_6]^{2+}$

Co^{+2}



number of unpaired $e^- = 3$

$\mu = \sqrt{15}BM$

$[Cr(H_2O)_6]^{3+}$

Cr^{+3}

number of unpaired $e^- = 3$

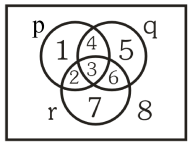
$\mu = \sqrt{15}BM$

Difference in spin only magnetic moment $= 0$

PART-3 : MATHEMATICS

SECTION-I

1. Ans (A)



$p \Rightarrow q \equiv 4, 3, 5, 6, 7, 8$

$p \Rightarrow q \equiv 1, 2, 3, 6, 7, 8$

$(p \Rightarrow q) \vee (q \Rightarrow r)$

$\equiv 1, 2, 3, 4, 5, 6, 7, 8$

Which is universal set

$\therefore$ Tautology

2. Ans (D)

$S_1 = \Sigma \tan\theta_1 = \sin 2\beta,$

$S_2 = \Sigma \tan\theta_1 \tan\theta_2 = \cos 2\beta$

$S_3 = \Sigma \tan\theta_1 \tan\theta_2 \tan\theta_3 = \cos\beta,$

$S_4 = -\sin\beta$

$$\tan(\theta_1+\theta_2+\theta_3+\theta_4) = \frac{S_1 - S_3}{1 - S_2 + S_4}$$

$$= \frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}$$

$$= \frac{\cos\beta(2\sin\beta - 1)}{\sin\beta(2\sin\beta - 1)} = \cot\beta$$

4. Ans (A)

$\therefore \sum a^2 + \sum ab \leq 0 \Rightarrow (a+b)^2 + (b+c)^2 + (c+a)^2 \leq 0$

$\Rightarrow a + b = 0, b + c = 0, c + a = 0$

$\Rightarrow a = b = c = 0$

$$\therefore \begin{vmatrix} 4 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 65$$

5. Ans (B)

$|A - \lambda I| = 0$

6. Ans (C)

$${}^9C_6 \times {}^6C_3 = \frac{9!}{6!3!} \times \frac{6!}{3!3!} = 1680$$

7. Ans (B)

$$\frac{z + \bar{z}}{2} = |z - 1|$$

$\Rightarrow x^2 = (x-1)^2 + y^2$ or $y^2 = 2x - 1$

Let $z_1\left(\frac{t_1^2+1}{2}, t_1\right)$ & $z_2\left(\frac{t_2^2+1}{2}, t_2\right)$

$$\because \arg(z_1 - z_2) = \frac{\pi}{3} \Rightarrow \frac{t_2 - t_1}{\frac{t_2^2 - t_1^2}{2}} = \sqrt{3}$$

$$\Rightarrow \frac{2}{t_1 + t_2} = \sqrt{3}$$

$$\Rightarrow \text{Im}(z_1 + z_2) = \frac{2}{\sqrt{3}}$$

8. Ans (A)

$P(S) + P(F) = 1$

$P(S) = 2P(F) \Rightarrow P(F) = \frac{1}{3}$, $P(S) = \frac{2}{3}$

Required probability

$$= {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1$$

$$+ {}^6C_6\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0 = \frac{496}{729}$$

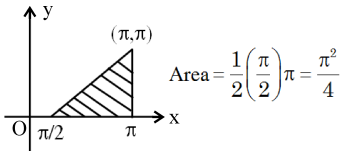
9. Ans (A)

$$y = \left|\frac{\pi}{2} - \sin^{-1}(\sin x)\right| + \left|\frac{\pi}{2} - \cos^{-1}(\cos x)\right|$$

$$y = \left|\frac{\pi}{2} - (\pi - x)\right| + \left|\frac{\pi}{2} - x\right|$$

$$y = \left|x - \frac{\pi}{2}\right| + \left|\frac{\pi}{2} - x\right|$$

$$= 2x - \pi \; \forall x \in \left[\frac{\pi}{2}, \pi\right]$$



$$\text{Area} = \frac{1}{2}\left(\frac{\pi}{2}\right)\pi = \frac{\pi^2}{4}$$

10. Ans (C)

$$A = \int_0^{\pi} \frac{\sin x}{x^2} dx$$

$$= \left(\sin x\left(-\frac{1}{x}\right)\right)_0^{\pi} + \int_0^{\pi} \frac{\cos x}{x}\, dx$$

$$A = 0 - (-1) + \int_0^{\pi} \frac{\cos x}{x}\, dx$$

Put $x = 2y$

$$A = 1 + \int_0^{\pi/2} \frac{\cos 2x}{x}\, dx$$

$$\int_0^{\pi/2} \frac{\cos 2x}{x} dx = A - 1$$

11. Ans (A)

Let $y + 1 = Y$

$$\therefore \quad \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$$

Put $-\frac{1}{Y} = k$

$$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$$

$$I.F. = e^{-\frac{x^2}{2}}$$

$$\therefore \quad k = (x + c)e^{x^2/2}$$

Put $k = -\frac{1}{y+1}$

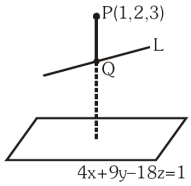
$$\therefore \quad y + 1 = -\frac{1}{(x+c)\,e^{x^2/2}} \quad ...(i)$$

when $x = 2$, $y = 0$, then $c = -2 - \frac{1}{e^2}$

differentiate equation (i) & put $x = 1$

we get $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{(1+e^2)^2}$

12. Ans (D)



Any point on line is

$Q(1 - 2\lambda,\ -1 + 3\lambda,\ 5 + 4\lambda)$

$\therefore$ Direction ratios of PQ

are $(-2\lambda,\ 3\lambda - 3,\ 4\lambda + 2)$

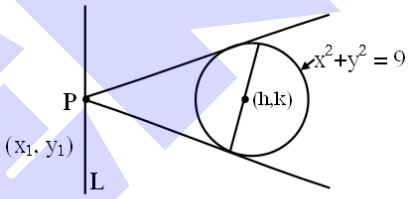
PQ is perpendicular to the plane

$\Rightarrow$ PQ is parallel to normal of the plane

$$\Rightarrow \frac{-2\lambda}{4} = \frac{3\lambda - 3}{9} = \frac{4\lambda + 2}{-18}$$

$$\Rightarrow \lambda = \frac{2}{5}$$

13. Ans (A)



$4x - 5y = 20$

Let mid point chord is p(h, k)

Equation of chord $T = S_1$

$hx + ky - 9 = h^2 + k^2 - 9$

$hx + ky = h^2 + k^2$ .....(1)

Let (x_1, y_2) is a point on line $4x - 5y = 20$ .... (2)

Equation at chord is

$T = 0$

Equation of chord of contact with respect to P

$xx_1 + yy_1 = 9$ .....(2)

(1) & (2) represent same line

$$\frac{x_1}{h} = \frac{y_1}{k} = \frac{9}{h^2 + k^2}$$

$$x_1 = \frac{9h}{h^2 + k^2},\ y_1 = \frac{9k}{h^2 + k^2}$$

$(x_1 y_1)$ lies on L.

$$\frac{4 \times 9h}{h^2 + k^2} - \frac{5.9k}{h^2 + k^2} = 20$$

Locus of $p(h_1 k)$ is

$20(x^2 + y^2) - 36x + 45y = 0$

14. Ans (B)

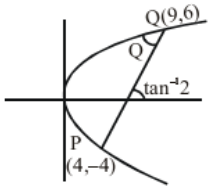
Normal slope m = 2

$\therefore$ Point P (4, – 4)

$\Rightarrow$ Point Q is (9, 6)

Slope of tangent at point Q is $\frac{1}{3}$

$\therefore \tan\alpha = \left|\frac{2-\frac{1}{3}}{1+\frac{2}{3}}\right| = 1$



$\Rightarrow \alpha = \frac{\pi}{4}$

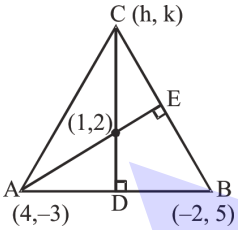
15. Ans (A)

Here and 2ae = 8m ; $e = \frac{4}{5}$, a = 5m

$\therefore$ $b^2 = a^2(1-e^2) = 9$ $\Rightarrow$ b = 3

Thus, required area = πab = 15π sq.metre.

16. Ans (B)



Let third vertex is c (h, k)

$\because$ AB $\perp$CD

$\Rightarrow \frac{k-2}{h-1} \times \frac{5-(-3)}{-2-4} = -1$

$\Rightarrow$ 3h – 4k + 5 = 0 ...(1)

similarly AE $\perp$ BC

$\Rightarrow \frac{-3-2}{4-1} \times \frac{k-5}{h-(-2)} = -1$

$\Rightarrow$ 3h – 5k + 31 = 0 ...(2)

solving both, gives h = 33, k = 26

17. Ans (C)

$f'(x) = 3x^2 - 12x + 9$

$= 3(x-1)(x-3)$

for maximum or minimum, f' (x) = 0

$\therefore$ x + 1, 3 but 3 $\in$ [0, 2]

$\therefore$ only one critical point in [0, 2] is x = 1

$\therefore$ Greatest value l = max. {f(0), f(1), f(2)}

= max. {1, 5, 3} = 5

Least value m = 1

$\therefore$ $\lambda^4 + \mu^8 = 5^4 + 1^8 = 626$

19. Ans (A)

$f(x) = \lim_{n\to\infty}\left(\cos\frac{x}{2}\cdot\cos\frac{x}{2^2}\cdot\cos\frac{x}{2^3}\ldots\cos\frac{x}{2^n}\right)$

$= \lim_{n\to\infty}\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)} = \frac{\sin x}{x}$

20. Ans (A)

$= \sin\left(\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right)\right)$

$+ \cos\left(\tan^{-1}\left(\frac{1}{2}\right) + \cot^{-1}\left(\frac{1}{2}\right)\right)$

$= \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right)$

= 1 + 0 = 1

PART-3 : MATHEMATICS

SECTION-II

1. Ans (3)

$2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha - 2 = 0$

$\Rightarrow (\sin\alpha - 1)(2\sin^2\alpha - 5\sin\alpha + 2) = 0$

$\Rightarrow (\sin\alpha - 1)(2\sin\alpha - 1)(\sin\alpha - 2) = 0$

$\Rightarrow \sin\alpha = 1$ & $\sin\alpha = \frac{1}{2}$

$\Rightarrow \alpha = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ Ans. 3

2. Ans (2)

$\sigma_{x_i} = \sigma_{(x_i-5)} = \sqrt{\frac{\Sigma(x_i-5)^2}{n} - \left(\frac{\Sigma(x_i-5)}{n}\right)^2}$

$= \sqrt{\frac{45}{9} - \left(\frac{9}{9}\right)^2} = \sqrt{5-1} = 2$

4. Ans (49)

Let T_{l+1} is max.

$$T_{l+1} = \frac{{}^{100}c_r}{(r+1)(r+2)(r+3)(r+4)}$$

$$= \frac{{}^{104}c_{r+4}}{101 \cdot 102 \cdot 103 \cdot 104} \text{ is max, when}$$

$r + 4 = 52$

$r = 48$

so term is 49^{th}

5. Ans (11)

$$A_{10} + A_{12} = \int \tan^{10} x dx + \int \tan^{12} x dx$$

$$= \int \left(\tan^{10} x + \tan^{12} x\right) dx$$

$$= \int \tan^{10} x \left(1 + \tan^{2} x\right) dx$$

$$= \int \tan^{10} x . \sec^2 x dx$$

Let $\int \tan x = t \Rightarrow \sec^2 x dx = dt$

$$A_{10} + A_{12} = \int t^{10} dt = \frac{t^{11}}{11} + \lambda$$

$$= \frac{\tan^{11} x}{11} + \lambda$$

6. Ans (5)

$$I = \int (\sin 100x . \cos x + \cos 100x . \sin x) \sin^{99} x . dx$$

$$I = \int \sin 100x . \cos x . \sin^{99} x dx + \int \cos 100x \sin^{100} x \, dx$$

$$I = \frac{\sin(100x)\sin^{100} x}{100} - \frac{100}{100}$$

$$\int \cos(100x)\sin^{100} x dx +$$

$$\int \cos(100x) . \sin^{100} x . dx$$

$$I = \frac{\sin(100x)\sin^{100} x}{100} + C$$

7. Ans (2)

Given, $\frac{(\vec{b} \cdot \vec{a})\vec{a}}{|\vec{a}|^2} = \frac{4}{3}(\hat{i} - \hat{j} - \hat{k})$

$$\Rightarrow \frac{\{(\lambda\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j} - \hat{k})\}(\hat{i} - \hat{j} - \hat{k})}{(1+1+1)}$$

$$= \frac{4}{3}(\hat{i} - \hat{j} - \hat{k})$$

$\Rightarrow (\lambda + 3 - 1)$

$(\hat{i} - \hat{j} - \hat{k}) = 4(\hat{i} - \hat{j} - \hat{k})$

$\Rightarrow (1 + 2)$

$(\hat{i} - \hat{j} - \hat{k}) = 4(\hat{i} - \hat{j} - \hat{k})$

On equating the coefficient of , we get

$\lambda + 2 = 4 \Rightarrow \lambda = 2$

8. Ans (-1)

Homogenisation

$5x^2 + 12xy - 6y^2 + (4x - 2y)(x + ky) + 3(x + ky)^2 = 0$

Because pair of st. line equally inclined with the x-axis.

So, coefficient of $xy = 0$

$12 + 4k - 2 + 6k = 0$

$10k = -10$

$k = -1$

9. Ans (1)

$f(0) = 0, f'(0) = 1, f''(0 = 1, f'''(0) = 2$

$g(f(x)) = x \quad \Rightarrow \quad g'(f(x)) f'(x) = 1$

$$\Rightarrow \quad g''(f(x)) = \frac{-f'(x)}{(f'(x))^3}$$

$$\Rightarrow \quad g'''(f(x))f'(x)$$

$$= -\left[\frac{(f'(x))^3 \cdot f'(x) - 3(f''(x))^2 (f'(x))^2}{(f'(x))^6)}\right]$$

Put $x = 0$

$$g'''(0)\ 1 = -\left[\frac{1 \times 2 - 3 \times 1}{1}\right] = 1$$

10. Ans (1)

$$f(0) = \lim_{x \to 0} \frac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$$

$$= \lim_{x \to 0} e^x \frac{(e^{\tan x - x} - 1)}{\tan x - x} + \lim_{x \to 0} \ln \frac{(\sec x + \tan x) - x}{\tan x - x}$$

$$= 1 + \lim_{x \to 0} \frac{\sec x - 1}{\sec^2 x - 1} = 1 + \frac{1}{2} = \frac{3}{2}$$