TRIGONOMETRIC EQUATION

1. समीकरण, $\sin^7x + \cos^7x = 1$ के $x \in [0, 4\pi]$ में हलों की संख्या है –

(1) 11 (2) 7

(3) 5 (4) 9

2. $[0, 2\pi]$ में x के सभी मानों, जिनके लिए $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$ है, का योग है :

(1) 8π (2) 11π

(3) 12π (4) 9π

3. यदि $\sin\theta + \cos\theta = \frac{1}{2}$, है, तो

$16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ बराबर है :

(1) 23 (2) –27 (3) –23 (4) 27

4. समीकरण $\frac{\cos x}{1+\sin x} = |\tan 2x|$,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ के हलो का योग है :

(1) $-\frac{11\pi}{30}$ (2) $\frac{\pi}{10}$

(3) $-\frac{7\pi}{30}$ (4) $-\frac{\pi}{15}$

5. माना $[0, 4\pi]$ में समीकरण $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ के सभी हलों (रेडियन में) का योग S है। तो $\frac{8S}{\pi}$ बराबर है _____।

6. समीकरण $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \le x \le \frac{\pi}{4}$ के हलों की संख्या है :

(1) 3 (2) 1

(3) 0 (4) 2

7. $\theta \in [0, 2\pi]$ के सभी संभव मान, जिनके लिए $\sin 2\theta + \tan 2\theta > 0$ है, निम्न में से किस में हैं ?

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

8. यदि $0 < x, y < \pi$ तथा $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, है, तो $\sin x + \cos y$ बराबर है :

(1) $\frac{1}{2}$ (2) $\frac{1+\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

9. 'k' के पूर्णांकीय मानों, जिनके लिए समीकरण $3\sin x + 4\cos x = k + 1$ का एक हल है, $k \in R$ है, की संख्या है_____।

10. समीकरण $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3} - 1\right)\cos x + 1$, जबकि $x \in \left[0, \frac{\pi}{2}\right]$, के हलों की संख्या है_____।

11. यदि $x \in \left(0, \frac{\pi}{2}\right)$ के लिए, $\log_{10}\sin x + \log_{10}\cos x = -1$ तथा $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$ हैं, तो n का मान बराबर है :

(1) 20 (2) 12 (3) 9 (4) 16

12. अंतराल $[0, \pi]$ में समीकरण $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ के मूलों की संख्या है :

(1) 3 (2) 4 (3) 8 (4) 2

13. अंतराल $[0, 2\pi]$ में समीकरण $|\cot x| = \cot x + \frac{1}{\sin x}$ के हलों की संख्या है_____।

SOLUTION

1. Official Ans. by NTA (3)

Sol. $\sin^7x \le \sin^2x \le 1$ …(1)

and $\cos^7x \le \cos^2x \le 1$ …(2)

also $\sin^2x + \cos^2x = 1$

$\Rightarrow$ equality must hold for (1) & (2)

$\Rightarrow \sin^7x = \sin^2x$ & $\cos^7 = \cos^2x$

$\Rightarrow \sin x = 0$ & $\cos x = 1$

or

$\cos x = 0$ & $\sin x = 1$

$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow$ 5 solutions

2. Official Ans. by NTA (4)

Sol. $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$$\Rightarrow 2\sin\frac{5x}{2}\left\{\cos\frac{3x}{2}+\cos\frac{x}{2}\right\}=0$$

$$\Rightarrow 2\sin\frac{5x}{2}\left\{2\cos x\cos\frac{x}{2}\right\}=0$$

$$2\sin\frac{5x}{2}=0\Rightarrow\frac{5X}{2}=0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi,\ 5\pi$$

$$\Rightarrow x=0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5},2\pi$$

$$\cos\frac{x}{2}=0\Rightarrow\frac{x}{2}=\frac{\pi}{2}\Rightarrow x=\pi$$

$$\cos x=0\Rightarrow x=\frac{\pi}{2},\frac{3\pi}{2}$$

So sum $= 6\pi + \pi + 2\pi = 9\pi$

3. Official Ans. by NTA (3)

Sol. $\sin\theta+\cos\theta=\frac{1}{2}$

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{1}{4}$$

$$\sin 2\theta=-\frac{3}{4}$$

Now :

$$\cos 4\theta=1-2\sin^2 2\theta$$

$$=1-2\left(-\frac{3}{4}\right)^2$$

$$=1-2\times\frac{9}{16}=-\frac{1}{8}$$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$$=(3-4\sin^2 2\theta).\sin 2\theta$$

$$=\left[3-4\left(\frac{9}{16}\right)\right].\left(-\frac{3}{4}\right)$$

$$\Rightarrow\left[\frac{3}{4}\right]\times\left(-\frac{3}{4}\right)=-\frac{9}{16}$$

$$16[\sin 2\theta+\cos 4\theta+\sin 6\theta]$$

$$16\left(-\frac{3}{4}-\frac{1}{8}-\frac{9}{16}\right)=-23$$

4. Official Ans. by NTA (1)

Sol. $\frac{\cos x}{1+\sin x}=|\tan 2x|$

$$\Rightarrow\frac{\cos^2 x/2-\sin^2 x/2}{(\cos x/2+\sin x/2)}=|\tan 2x|$$

$$\Rightarrow\tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\tan^2 2x$$

$$\Rightarrow 2x=n\pi\pm\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\Rightarrow x=\frac{-3\pi}{10},\frac{-\pi}{6},\frac{\pi}{10}$$

or sum $= \frac{-11\pi}{6}$.

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5. Official Ans. by NTA (56)

Sol. Given equation

$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 1 - \sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 2 - (\sin 2\theta)^2 - \sin 2\theta = 0$

$\Rightarrow (\sin 2\theta)^2 + (\sin 2\theta) - 2 = 0$

$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$

$\Rightarrow \sin 2\theta = 1$ or $\underset{\text{(not possible)}}{\boxed{\sin 2\theta = -2}}$

$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$\Rightarrow S = \frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4} = 7\pi$

$\Rightarrow \frac{8S}{\pi} = \frac{8\times 7\pi}{\pi} = 56.00$

6. Official Ans. by NTA (2)

Sol. $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$

In interval $\left[0, \frac{\pi}{4}\right]$ only one solution

7. Official Ans. by NTA (4)

Sol. $\sin 2\theta + \tan 2\theta > 0$

$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$

$\Rightarrow \sin 2\theta \frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0 \Rightarrow \tan 2\theta(2\cos^2\theta) > 0$

Note : $\cos 2\theta \neq 0$

$\Rightarrow 1 - 2\sin^2\theta \neq 0 \Rightarrow \sin\theta \neq \pm\frac{1}{\sqrt{2}}$

Now, $\tan 2\theta\,(1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ (as $\cos 2\theta + 1 > 0$)

$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

As $\sin\theta \neq \pm\frac{1}{\sqrt{2}}$; which has been already considered

8. Official Ans. by NTA (2)

Sol. $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$

$\cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)$

$+\frac{1}{4}.\cos^2\left(\frac{x-y}{2}\right) + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2}\cos\left(\frac{x-y}{2}\right)\right)^2 + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0$ and $\cos\left(\frac{x+y}{2}\right) = \frac{1}{2}\cos\left(\frac{x-y}{2}\right)$

$\Rightarrow x = y$ and $\cos x = \frac{1}{2} = \cos y$

$\therefore \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin x + \cos y = \frac{1+\sqrt{3}}{2}$

option (2)

9. Official Ans. by NTA (11)

Sol. $3\sin x + 4\cos x = k + 1$

$\Rightarrow k + 1 \in \left[-\sqrt{3^2+4^2}, \sqrt{3^2+4^2}\right]$

$\Rightarrow k + 1 \in [-5, 5]$

$\Rightarrow k \in [-6, 4]$

No. of integral values of k = 11

10. Official Ans. by NTA (1)

Sol. $\sqrt{3}(\cos x)^2 - \sqrt{3}\cos x + \cos x - 1 = 0$

$\Rightarrow \left(\sqrt{3}\cos x + 1\right)(\cos x - 1) = 0$

$\Rightarrow \cos x = 1$ or $\cos x = -\frac{1}{\sqrt{3}}$ (reject)

$\Rightarrow x = 0$ only

11. Official Ans. by NTA (2)

Sol. $x \in \left(0, \frac{\pi}{2}\right)$

$\log_{10}\sin x + \log_{10}\cos x = -1$

$\Rightarrow \quad \log_{10}\sin x.\cos x = -1$

$\Rightarrow \quad \sin x.\cos x = \frac{1}{10} \qquad .....(1)$

$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \quad \sin x + \cos x = 10^{\left(\log_{10}\sqrt{n} - \frac{1}{2}\right)} = \sqrt{\frac{n}{10}}$

by squaring

$1 + 2\sin x.\cos x = \frac{n}{10}$

$\Rightarrow \quad 1 + \frac{1}{5} = \frac{n}{10} \qquad \Rightarrow \quad n = 12$

12. Official Ans. by NTA (2)

Sol. $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$

$(81)^{\sin^2 x} = t$

$t + \frac{81}{t} = 30$

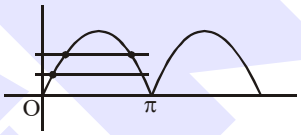
$t^2 - 30t + 81 = 0$

$(t - 3)(t - 27) = 0$

$(81)^{\sin^2 x} = 3^1 \quad$ or $\quad (81)^{\sin^2 x} = 3^3$

$3^{4\sin^2 x} = 3^1 \quad$ or $\quad 3^{4\sin^2 x} = 3^3$

$\sin^2 x = \frac{1}{4} \quad$ or $\quad \sin^2 x = \frac{3}{4}$



Total sol. = 4

13. Official Ans. by NTA (1)

Sol. If $\cot x > 0 \Rightarrow \frac{1}{\sin x} = 0$ (Not possible)

If $\cot x < 0 \Rightarrow 2\cot x + \frac{1}{\sin x} = 0$

$\Rightarrow 2\cos x = -1$

$\Rightarrow x = \frac{2\pi}{3} \text{or} \frac{4\pi}{3}$ (reject)

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