

QUADRATIC EQUATION

- यदि α तथा β , समीकरण $x^2 + (3)^{1/4}x + 3^{1/2} = 0$ के दो भिन्न मूल हैं, तो $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ का मान बराबर है :
 (1) 56×3^{25} (2) 56×3^{24}
 (3) 52×3^{24} (4) 28×3^{25}
- समीकरण, $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ के वास्तविक मूलों की संख्या है :
 (1) 2 (2) 4 (3) 6 (4) 1
- यदि समीकरण, $x^2 + 5(\sqrt{2})x + 10 = 0$, के α तथा β , $\alpha > \beta$ दो मूल हैं तथा $P_n = \alpha^n - \beta^n$, (प्रत्येक धन पूर्णांक n के लिए) है, तो $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right)$ का मान है _____।
- समीकरण, $x^2 - |x| - 12 = 0$ के वास्तविक हलों की संख्या है :
 (1) 2 (2) 3 (3) 1 (4) 4
- यदि $a + b + c = 1$, $ab + bc + ca = 2$ तथा $abc = 3$ हैं, तो $a^4 + b^4 + c^4$ बराबर है _____।
- मान α, β समीकरण $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$ के दो मूल हैं। तो $\alpha^8 + \beta^8$ बराबर है :
 (1) 10 (2) 100 (3) 50 (4) 160
- समीकरण $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ के वास्तविक मूलों की संख्या है _____।
- k ($k \neq 0$) के सभी पूर्णांक मानों, जिनके लिए x में समीकरण $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ का कोई वास्तविक मूल नहीं है, का योग है _____।
- माना $\lambda \neq 0, \mathbf{R}$ में है। यदि समीकरण $x^2 - x + 2\lambda = 0$ के मूल α तथा β हैं और समीकरण $3x^2 - 10x + 27\lambda = 0$ के मूल α तथा γ हैं, तो $\frac{\beta\gamma}{\lambda}$ बराबर है _____।

- यदि $x^2 + 9y^2 - 4x + 3 = 0$, $x, y \in \mathbf{R}$ हैं, तो x तथा y क्रमशः निम्न में से किस अंतराल में हैं?
 (1) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ तथा $\left[-\frac{1}{3}, \frac{1}{3}\right]$
 (2) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ तथा $[1, 3]$
 (3) $[1, 3]$ तथा $[1, 3]$
 (4) $[1, 3]$ तथा $\left[-\frac{1}{3}, \frac{1}{3}\right]$
- समीकरण $3x^4 + 4x^3 - 12x^2 + 4 = 0$ के भिन्न वास्तविक मूलों की संख्या है _____।
- $k > -1$ के सभी मानों, जिनके लिए समीकरण $(3x^2 + 4x + 3)^2 - (k+1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ के वास्तविक मूल हैं, का समुच्चय है:
 (1) $\left(1, \frac{5}{2}\right]$ (2) $[2, 3)$
 (3) $\left[-\frac{1}{2}, 1\right)$ (4) $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$
- $\operatorname{cosec} 18^\circ$ निम्न में से किस समीकरण का एक मूल है?
 (1) $x^2 + 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$
 (3) $x^2 - 2x + 4 = 0$ (4) $x^2 - 2x - 4 = 0$
- जब-जब α समीकरण $x^2 + ax + b = 0$, का एक मूल है, $\alpha^2 - 2$ भी इस समीकरण का एक मूल है। इसके लिए वास्तविक संख्याओं के युग्मों (a, b) की संख्या है :
 (1) 6 (2) 2 (3) 4 (4) 8
- समीकरण $(x+1)^2 + |x-5| = \frac{27}{4}$ के वास्तविक मूलों की संख्या है _____।
- यदि p तथा q दो धनात्मक संख्याएँ हैं, जिनके लिए $p + q = 2$ तथा $p^4 + q^4 = 272$ हैं, तो p तथा q जिस समीकरण के मूल हैं, वह है:
 (1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$
 (3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

17. पूर्णांक 'k', जिसके लिए असमिका $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$, R में प्रत्येक x के लिए, मान्य है, है :
- (1) 3 (2) 2 (3) 0 (4) 4
18. यदि $\alpha, \beta \in R$ है, जिसके लिए $z^2 + \alpha z + \beta = 0$, का एक मूल $1 - 2i$ (यहाँ $i^2 = -1$) है, तो $(\alpha - \beta)$ बराबर है :
- (1) -3 (2) -7 (3) 7 (4) 3
19. माना $x^2 - 6x - 2 = 0$ के मूल α तथा β हैं। यदि $n \geq 1$, के लिए $a_n = \alpha^n - \beta^n$ है, तो $\frac{a_{10} - 2a_8}{3a_9}$ का मान है :
- (1) 2 (2) 1 (3) 4 (4) 3
20. माना α तथा β दो वास्तविक संख्याएँ हैं जिनके लिए $\alpha + \beta = 1$ तथा $\alpha\beta = -1$ हैं। माना किसी पूर्णांक $n \geq 1$ के लिए $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ तथा $p_{n+1} = 29$ हैं। तो p_n^2 का मान है _____

21. समीकरण $\log_4(x - 1) = \log_2(x - 3)$ के हलों की संख्या है _____।
22. माना $f : [-1, 1] \rightarrow R$, $f(x) = ax^2 + bx + c \forall x \in [-1, 1]$, $a, b, c \in R$ द्वारा परिभाषित है, जबकि $f(-1) = 2$, $f'(-1) = 1$ हैं तथा $x \in (-1, 1)$ के लिए $f''(x)$ का अधिकतम मान $\frac{1}{2}$ है। यदि $f(x) \leq \alpha$, $x \in [-1, 1]$ है, तो α का निम्नतम मान है _____।
23. $3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$ का मान बराबर है :
- (1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$
 (3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

SOLUTION

$$\begin{aligned}
 1. \quad & \text{As, } (\alpha^2 + \sqrt{3}) = -(3)^{1/4} \cdot \alpha \\
 & \Rightarrow (\alpha^4 + 2\sqrt{3}\alpha^2 + 3) = \sqrt{3}\alpha^2 \quad (\text{On squaring}) \\
 & \therefore (\alpha^4 + 3) = (-)\sqrt{3}\alpha^2 \\
 & \Rightarrow \alpha^8 + 6\alpha^4 + 9 = 3\alpha^4 \quad (\text{Again squaring}) \\
 & \therefore \alpha^8 + 3\alpha^4 + 9 = 0 \\
 & \Rightarrow \boxed{\alpha^8 = -9 - 3\alpha^4}
 \end{aligned}$$

(Multiply by α^4)

$$\text{So, } \alpha^{12} = -9\alpha^4 - 3\alpha^8$$

$$\therefore \alpha^{12} = -9\alpha^4 - 3(-9 - 3\alpha^4)$$

$$\Rightarrow \alpha^{12} = \cancel{-9\alpha^4} + 27 + \cancel{9\alpha^4}$$

$$\text{Hence, } \boxed{\alpha^{12} = (27)^2}$$

$$\Rightarrow (\alpha^{12})^8 = (27)^8$$

$$\Rightarrow \alpha^{96} = (3)^{24}$$

$$\text{Similarly } \beta^{96} = (3)^{24}$$

$$\therefore \alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1) = (3)^{24} \times 52$$

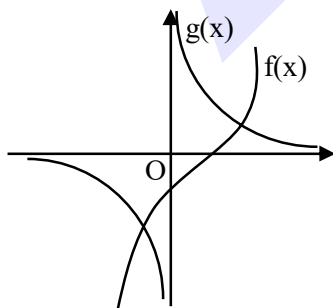
$\Rightarrow$ Option (3) is correct.

$$2. \quad e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$$

$$\Rightarrow (e^{3x} - 1)^2 - e^x(e^{3x} - 1) = 12e^{2x}$$

$$(e^{3x} - 1)^2 (e^x - e^{-x} - e^{-2x}) = 12$$

$$\Rightarrow \underbrace{e^x - e^{-x} - e^{-2x}}_{\text{increasing (let } f(x))} = \frac{12}{\underbrace{e^{3x} - 1}_{\text{decreasing (let } g(x))}}$$



$\Rightarrow$ No. of real roots = 2

$$3. \quad x^2 + 5\sqrt{2}x + 10 = 0$$

$$\& p_n = \alpha^n - \beta^n \quad (\text{Given})$$

$$\text{Now } \frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} = \frac{P_{17}(P_{20} + 5\sqrt{2}P_{19})}{P_{18}(P_{19} + 5\sqrt{2}P_{18})}$$

$$\frac{P_{17}(\alpha^{20} - \beta^{20} + 5\sqrt{2}(\alpha^{19} - \beta^{19}))}{P_{18}(\alpha^{19} - \beta^{19} + 5\sqrt{2}(\alpha^{18} - \beta^{18}))}$$

$$\frac{P_{17}(\alpha^{19}(\alpha + 5\sqrt{2}) - \beta^{19}(\beta + 5\sqrt{2}))}{P_{18}(\alpha^{18}(\alpha + 5\sqrt{2}) - \beta^{18}(\beta + 5\sqrt{2}))}$$

$$\text{Since } \alpha + 5\sqrt{2} = -10/\alpha \quad \dots(1)$$

$$\& \beta + 5\sqrt{2} = -10/\beta \quad \dots(2)$$

Now put there values in above expression

$$= -\frac{10P_{17}P_{18}}{-10P_{18}P_{17}} = 1$$

$$4. \quad |x|^2 - |x| - 12 = 0$$

$$(|x| + 3)(|x| - 4) = 0$$

$$|x| = 4 \Rightarrow x = \pm 2$$

$$5. \quad a^2 + b^2 + c^2 = (a + b + c)^2 - 2\Sigma ab = -3$$

$$(ab + bc + ca)^2 = \Sigma(ab)^2 + 2abc\Sigma a$$

$$\Rightarrow \Sigma(ab)^2 = -2$$

$$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - 2\Sigma(ab)^2$$

$$= 9 - 2(-2) = 13$$

$$6. \quad (x^2 + \sqrt{5})^2 = \sqrt{20}x^2$$

$$x^4 = -5 \Rightarrow x^8 = 25$$

$$\alpha^8 + \beta^8 = 50$$

$$7. \quad t^4 - t^3 - 4t^2 - t + 1 = 0, e^x = t > 0$$

$$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\Rightarrow \alpha^2 - \alpha - 6 = 0, \alpha = t + \frac{1}{t} \geq 2$$

$$\Rightarrow \alpha = 3, -2 \quad (\text{reject})$$

$$\Rightarrow t + \frac{1}{t} = 3$$

$\Rightarrow$ The number of real roots = 2

8. $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$
 $x \in \mathbb{R} - \{1, 2\}$
 $\Rightarrow k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$
 $\Rightarrow k(x - 3) = 2(x^2 - 3x + 2)$
 for $x \neq 3$, $k = 2\left(x - 3 + \frac{2}{x-3} + 3\right)$
 $x - 3 + \frac{2}{x-3} \geq 2\sqrt{2}, \forall x > 3$
 & $x - 3 + \frac{2}{x-3} \leq -2\sqrt{2}, \forall x < -3$
 $\Rightarrow 2\left(x - 3 + \frac{2}{x-3} + 3\right) \in (-\infty, 6 - 4\sqrt{2}] \cup [6 + 4\sqrt{2}, \infty)$
 for no real roots
 $k \in (6 - 4\sqrt{2}, 6 + 4\sqrt{2}) - \{0\}$
 Integral $k \in \{1, 2, \dots, 11\}$
 Sum of $k = 66$
9. $3\alpha^2 - 10\alpha + 27\lambda = 0$ _____(1)
 $\alpha^2 - \alpha + 2\lambda = 0$ _____(2)
 (1) - 3(2) gives
 $-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$
 Put $\alpha = 3\lambda$ in equation (1) we get
 $9\lambda^2 - 3\lambda + 2\lambda - 0$
 $9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \neq 0$
 Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$
 $\alpha + \beta = 1 \Rightarrow \beta = 2/3$
 $\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$
 $\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times 3}{\frac{1}{9}} = 18$

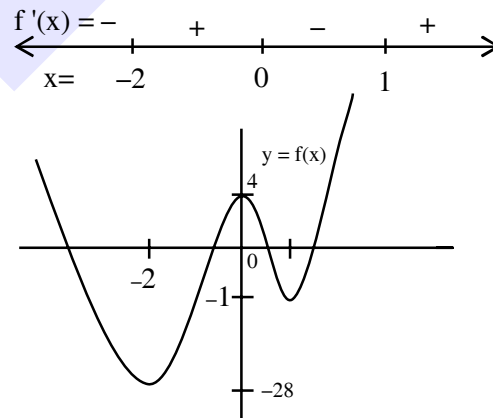
10. $x^2 + 9y^2 - 4x + 3 = 0$
 $(x^2 - 4x) + (9y^2) + 3 = 0$
 $(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$
 $(x - 2)^2 + (3y)^2 = 1$
 $\frac{(x-2)^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$ (equation of an ellipse).

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x - 2 \in [-1, 1]$ and $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

$x \in [1, 3]$ $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

11. $3x^4 + 4x^3 - 12x^2 + 4 = 0$
 So, Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$
 $\therefore f'(x) = 12x(x^2 + x - 2)$
 $= 12x(x + 2)(x - 1)$



12. $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$

Let $3x^2 + 4x + 3 = a$

and $3x^2 + 4x + 2 = b \Rightarrow b = a - 1$

Given equation becomes

$$\Rightarrow a^2 - (k + 1)ab + kb^2 = 0$$

$$\Rightarrow a(a - kb) - b(a - kb) = 0$$

$$\Rightarrow (a - kb)(a - b) = 0 \Rightarrow a = kb \text{ or } a = b$$

(reject)

$$\therefore a = kb$$

$$\Rightarrow 3x^2 + 4x + 3 = k(3x^2 + 4x + 2)$$

$$\Rightarrow 3(k - 1)x^2 + 4(k - 1)x + (2k - 3) = 0$$

for real roots $D \geq 0$

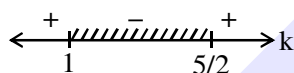
$$\Rightarrow 16(k - 1)^2 - 4(3(k - 1))(2k - 3) \geq 0$$

$$\Rightarrow 4(k - 1)\{4(k - 1) - 3(2k - 3)\} \geq 0$$

$$\Rightarrow 4(k - 1)\{-2k + 5\} \geq 0$$

$$\Rightarrow -4(k - 1)\{2k - 5\} \geq 0$$

$$\Rightarrow (k - 1)(2k - 5) \leq 0$$



$$\therefore k \in \left[1, \frac{5}{2}\right]$$

$$\therefore k \neq 1$$

$$\therefore k \in \left(1, \frac{5}{2}\right] \text{ Ans.}$$

13. $\operatorname{cosec} 18^\circ = \frac{1}{\sin 18^\circ} = \frac{4}{\sqrt{5} - 1} = \sqrt{5} + 1$

Let $\operatorname{cosec} 18^\circ = x = \sqrt{5} + 1$

$$\Rightarrow x - 1 = \sqrt{5}$$

Squaring both sides, we get

$$x^2 - 2x + 1 = 5$$

$$\Rightarrow x^2 - 2x - 4 = 0$$

14. Consider the equation $x^2 + ax + b = 0$

If has two roots (not necessarily real α & β)

Either $\alpha = \beta$ or $\alpha \neq \beta$

Case (1) If $\alpha = \beta$, then it is repeated root. Given

that $\alpha^2 - 2$ is also a root

$$\text{So, } \alpha = \alpha^2 - 2 \Rightarrow (\alpha + 1)(\alpha - 2) = 0$$

$$\Rightarrow \alpha = -1 \text{ or } \alpha = 2$$

$$\text{When } \alpha = -1 \text{ then } (a, b) = (2, 1)$$

$$\alpha = 2 \text{ then } (a, b) = (-4, 4)$$

Case (2) If $\alpha \neq \beta$ Then

$$\text{(I) } \alpha = \alpha^2 - 2 \text{ and } \beta = \beta^2 - 2$$

$$\text{Here } (\alpha, \beta) = (2, -1) \text{ or } (-1, 2)$$

$$\text{Hence } (a, b) = (-(\alpha + \beta), \alpha\beta)$$

$$= (-1, -2)$$

$$\text{(II) } \alpha = \beta^2 - 2 \text{ and } \beta = \alpha^2 - 2$$

$$\text{Then } \alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha)$$

$$\text{Since } \alpha \neq \beta \text{ we get } \alpha + \beta = \beta^2 + \alpha^2 - 4$$

$$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$$

$$\text{Thus } -1 = 1 - 2\alpha\beta - 4 \text{ which implies}$$

$$\alpha\beta = -1 \text{ Therefore } (a, b) = (-(\alpha + \beta), \alpha\beta)$$

$$= (1, -1)$$

$$\text{(III) } \alpha = \alpha^2 - 2 = \beta^2 - 2 \text{ and } \alpha \neq \beta$$

$$\Rightarrow \alpha = -\beta$$

$$\text{Thus } \alpha = 2, \beta = -2$$

$$\alpha = -1, \beta = 1$$

$$\text{Therefore } (a, b) = (0, -4) \text{ \& } (0, -1)$$

$$\text{(IV) } \beta = \alpha^2 - 2 = \beta^2 - 2 \text{ and } \alpha \neq \beta \text{ is same as (III)}$$

Therefore we get 6 pairs of (a, b)

Which are $(2, 1), (-4, 4), (-1, -2), (1, -1), (0, -4)$

Option (1)

15. Case-I

$$x \leq 5$$

$$(x+1)^2 - (x-5) = \frac{27}{4}$$

$$(x+1)^2 - (x+1) - \frac{3}{4} = 0$$

$$x+1 = \frac{3}{2}, -\frac{1}{2}$$

$$x = \frac{1}{2}, -\frac{3}{2}$$

Case-II

$$x > 5$$

$$(x+1) + (x-5) = \frac{27}{4}$$

$$(x+1)^2 + (x+1) - \frac{51}{4} = 0$$

$$x = \frac{-1 \pm \sqrt{52}}{2} \text{ (rejected as } x > 5 \text{)}$$

So, the equation have two real root.

16. Consider $(p^2 + q^2)^2 - 2p^2q^2 = 272$
 $((p+q)^2 - 2pq)^2 - 2p^2q^2 = 272$
 $16 - 16pq + 2p^2q^2 = 272$
 $(pq)^2 - 8pq - 128 = 0$

$$(pq)^2 - 8pq - 128 = 0$$

$$pq = \frac{8 \pm 24}{2} = 16, -8$$

$$\therefore pq = 16$$

$$\therefore \text{Required equation : } x^2 - (2)x + 16 = 0$$

17. $x^2 - 2(3K-1)x + 8K^2 - 7 > 0$

$$\text{Now, } D < 0$$

$$\Rightarrow 4(3K-1)^2 - 4 \times 1 \times (8K^2 - 7) < 0$$

$$\Rightarrow 9K^2 - 6K + 1 - 8K^2 + 7 < 0$$

$$\Rightarrow K^2 - 6K + 8 < 0$$

$$\Rightarrow (K-4)(K-2) < 0$$

$$\Rightarrow \boxed{K \in (2, 4)}$$

18. $\therefore \alpha, \beta \in \mathbb{R} \Rightarrow$ other root is $1 + 2i$

$$\alpha = -(\text{sum of roots}) = -(1 - 2i + 1 + 2i) = -2$$

$$\beta = \text{product of roots} = (1 - 2i)(1 + 2i) = 5$$

$$\therefore \alpha - \beta = -7$$

option (2)

19. $\alpha^2 - 6\alpha - 2 = 0$

$$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$$

Similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$

$$(\alpha^{10} - \beta^{10}) - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$$

$$\Rightarrow a_{10} - 6a_9 - 2a_8 = 0$$

$$\Rightarrow \frac{a_{10} - 2a_8}{3a_9} = 2$$

20. $x^2 - x - 1 = 0$ roots = α, β

$$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^{n+1} = \alpha^n + \alpha^{n-1}$$

$$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^{n+1} = \beta^n + \beta^{n-1}$$

+

$$P_{n+1} = P_n + P_{n-1}$$

$$29 = P_n + 11$$

$$P_n = 18$$

$$P_n^2 = 324$$

21. $\log_4(x-1) = \log_2(x-3)$

$$\Rightarrow \frac{1}{2} \log_2(x-1) = \log_2(x-3)$$

$$\Rightarrow \log_2(x-1)^{1/2} = \log_2(x-3)$$

$$\Rightarrow (x-1)^{1/2} = x-3$$

$$\Rightarrow x-1 = x^2 + 9 - 6x$$

$$\Rightarrow x^2 - 7x + 10 = 0$$

$$\Rightarrow (x-2)(x-5) = 0$$

$$\Rightarrow x = 2, 5$$

But $x \neq 2$ because it is not satisfying the domain of given equation i.e $\log_2(x-3) \rightarrow$ its domain $x > 3$

finally x is 5

$\therefore$ No. of solutions = 1.

22. $f : [-1, 1] \rightarrow \mathbb{R}$

$$f(x) = ax^2 + bx + c$$

$$f(-1) = a - b + c = 2 \quad \dots(1)$$

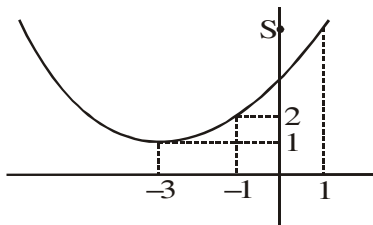
$$f'(-1) = -2a + b = 1 \quad \dots(2)$$

$$f''(x) = 2a$$

$$\Rightarrow \text{Max. value of } f''(x) = 2a = \frac{1}{2}$$

$$\Rightarrow a = \frac{1}{4}; b = \frac{3}{2}; c = \frac{13}{4}$$

$$\therefore f(x) = \frac{x^2}{4} + \frac{3}{2}x + \frac{13}{4}$$



$$\text{For, } x \in [-1, 1] \Rightarrow 2 \leq f(x) \leq 5$$

$\therefore$ Least value of α is 5

$$24. \text{ Let } x = 3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$$

$$\text{So, } x = 3 + \frac{1}{4 + \frac{1}{x}} = 3 + \frac{1}{\frac{4x+1}{x}}$$

$$\Rightarrow (x-3) = \frac{x}{(4x+1)}$$

$$\Rightarrow (4x+1)(x-3) = x$$

$$\Rightarrow 4x^2 - 12x + x - 3 = x$$

$$\Rightarrow 4x^2 - 12x - 3 = 0$$

$$x = \frac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \frac{12 \pm \sqrt{12(16)}}{8}$$

$$= \frac{12 \pm 4 \times 2\sqrt{3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$$

$$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$$

But only positive value is accepted

$$\text{So, } x = 1.5 + \sqrt{3}$$