

INVERSE TRIGONOMETRY FUNCTION

1. समीकरण $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{4}$

के वास्तविक मूलों की संख्या है :

- (1) 1 (2) 2 (3) 4 (4) 0

2. $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$ का मान बराबर है:

- (1) $\frac{-181}{69}$ (2) $\frac{220}{21}$ (3) $\frac{-291}{76}$ (4) $\frac{151}{63}$

3. यदि फलन $f(x) = \frac{\cos^{-1} \sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ का प्रान्त,

अन्तराल $(\alpha, \beta]$ है, तो $\alpha + \beta$ बराबर है -

- (1) $\frac{3}{2}$ (2) 2 (3) $\frac{1}{2}$ (4) 1

4. यदि $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$ है, तो $\tan p$ का मान है -

- (1) $\frac{101}{102}$ (2) $\frac{50}{51}$ (3) 100 (4) $\frac{51}{50}$

5. यदि $(\sin^{-1} x)^2 - (\cos^{-1} x)^2 = a$; $0 < x < 1$, $a \neq 0$, है, तो $2x^2 - 1$ का मान है:

- (1) $\cos\left(\frac{4a}{\pi}\right)$ (2) $\sin\left(\frac{2a}{\pi}\right)$
(3) $\cos\left(\frac{2a}{\pi}\right)$ (4) $\sin\left(\frac{4a}{\pi}\right)$

6. माना $\left[0, \frac{\pi}{2}\right]$ में फलन $f(x) = \tan^{-1}(\sin x + \cos x)$ के अधिकतम तथा न्यूनतम मान क्रमशः M तथा m हैं। तो $\tan(M-m)$ का मान बराबर है :

- (1) $2+\sqrt{3}$ (2) $2-\sqrt{3}$
(3) $3+2\sqrt{2}$ (4) $3-2\sqrt{2}$

7. फलन $f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$ का प्रांत है :

- (1) $\left[0, \frac{1}{4}\right]$ (2) $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$
(3) $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$ (4) $\left[0, \frac{1}{2}\right]$

8. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ बराबर है :

(प्रतिलोम त्रिकोणमितीय फलन मुख्य मान लेते हैं।)

- (1) $3\pi - 11$ (2) $4\pi - 9$
(3) $4\pi - 11$ (4) $3\pi + 1$

9. $\operatorname{cosec}\left[2\cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right]$ बराबर है :

- (1) $\frac{56}{33}$ (2) $\frac{65}{56}$ (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

10. यदि $0 < a, b < 1$, तथा $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, हैं, तो

$$(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \dots$$

का मान है :

- (1) $\log_e 2$ (2) $e^2 - 1$
(3) e (4) $\log_e\left(\frac{e}{2}\right)$

11. यदि $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$; $0 < x < 1$, तो

$$\cos\left(\frac{\pi c}{a+b}\right) \text{ का मान है :}$$

- (1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1-y^2$
(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

12. यह दिया गया है कि प्रतिलोम त्रिकोणमितीय फलन केवल मुख्य मान लेते हैं। तो

$$\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x \text{ को सन्तुष्ट करने}$$

वाले x के वास्तविक मानों की संख्या है :

- (1) 2 (2) 1 (3) 3 (4) 0

13. माना $S_k = \sum_{r=1}^k \tan^{-1} \left(\frac{6^r}{2^{2r+1} + 3^{2r+1}} \right)$ है। तो $\lim_{k \rightarrow \infty} S_k$

बराबर है :

- (1) $\tan^{-1} \left(\frac{3}{2} \right)$ (2) $\frac{\pi}{2}$
 (3) $\cot^{-1} \left(\frac{3}{2} \right)$ (4) $\tan^{-1}(3)$

14. समीकरण $\sin^{-1} \left[x^2 + \frac{1}{3} \right] + \cos^{-1} \left[x^2 - \frac{2}{3} \right] = x^2$,

$x \in [-1, 1]$, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है, के हलों की संख्या है :

- (1) 2 (2) 0
 (3) 4 (4) असंख्य

15. यदि $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots + 100$ पदों तक, तो α बराबर है :

- (1) 1.01 (2) 1.00 (3) 1.02 (4) 1.03

16. The sum of possible values of x for $\tan^{-1}(x+1) + \cot^{-1} \left(\frac{1}{x-1} \right) = \tan^{-1} \left(\frac{8}{31} \right)$ is :

- (1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$ (3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

16. $\tan^{-1}(x+1) + \cot^{-1} \left(\frac{1}{x-1} \right) = \tan^{-1} \left(\frac{8}{31} \right)$ के लिए

x के संभव मानों का योगफल है :

- (1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$ (3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

17. वास्तविक मान फलन $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x - [x]}}$, जहाँ $[x]$

महत्तम पूर्णांक $\leq x$ है, का प्रांत है :

- (1) पूर्णाकों के अतिरिक्त सभी वास्तविक संख्याएँ
 (2) अंतराल $[-1, 1]$ के अतिरिक्त सभी अपूर्णाक
 (3) 0, -1, 1 के अतिरिक्त सभी पूर्णाक
 (4) अंतराल $[-1, 1]$ के अतिरिक्त सभी वास्तविक संख्याएँ

SOLUTION

1. Official Ans. by NTA (4)

Sol. $\tan^{-1} \sqrt{x^2 + x} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{4}$

For equation to be defined,

$$x^2 + x \geq 0$$

$$\Rightarrow x^2 + x + 1 \geq 1$$

$\therefore$ only possibility that the equation is defined

$$x^2 + x = 0 \Rightarrow x = 0; x = -1$$

None of these values satisfy

$\therefore$ No of roots = 0

2. Official Ans. by NTA (2)

Sol. $\underbrace{\tan^{-1} \frac{3}{5} + \tan^{-1} \frac{3}{5}}_{x>0, y>0, xy<1} + \tan^{-1} \frac{5}{12}$

$$\tan^{-1} \frac{6}{1 - \frac{9}{25}} = \tan^{-1} \frac{15}{8} + \tan^{-1} \frac{5}{12}$$

$$\tan^{-1} \frac{\frac{15}{8} + \frac{5}{12}}{1 - \frac{15}{8} \cdot \frac{5}{12}} = \tan^{-1} \frac{220}{21}$$

$$\tan \left(\tan^{-1} \frac{220}{21} \right) = \frac{220}{21}$$

3. Official Ans. by NTA (1)

Sol. $0 \leq x^2 - x + 1 \leq 1$

$$\Rightarrow x^2 - x \leq 0$$

$$\Rightarrow x \in [0, 1]$$

Also, $0 < \sin^{-1} \left(\frac{2x-1}{2} \right) \leq \frac{\pi}{2}$

$$\Rightarrow 0 < \frac{2x-1}{2} \leq 1$$

$$\Rightarrow 0 < 2x - 1 \leq 2$$

$$1 < 2x \leq 3$$

$$\frac{1}{2} < x \leq \frac{3}{2}$$

Taking intersection

$$x \in \left(\frac{1}{2}, 1 \right]$$

$$\Rightarrow \alpha = \frac{1}{2}, \beta = 1$$

$$\Rightarrow \alpha + \beta = \frac{3}{2}$$

4. Official Ans. by NTA (2)

Sol. $\sum_{r=1}^{50} \tan^{-1} \left(\frac{2}{4r^2} \right) = \sum_{r=1}^{50} \tan^{-1} \left(\frac{(2r+1) - (2r-1)}{1 + (2r+1)(2r-1)} \right)$

$$\sum_{r=1}^{50} \tan^{-1} (2r+1) - \tan^{-1} (2r-1)$$

$$\tan^{-1} (101) - \tan^{-1} 1 \Rightarrow \tan^{-1} \frac{50}{51}$$

5. Official Ans. by NTA (2)

Sol. Given $a = (\sin^{-1} x)^2 - (\cos^{-1} x)^2$
 $= (\sin^{-1} x + \cos^{-1} x) (\sin^{-1} x - \cos^{-1} x)$

$$= \frac{\pi}{2} \left(\frac{\pi}{2} - 2 \cos^{-1} x \right)$$

$$\Rightarrow 2 \cos^{-1} x = \frac{\pi}{2} - \frac{2a}{\pi}$$

$$\Rightarrow \cos^{-1} (2x^2 - 1) = \frac{\pi}{2} - \frac{2a}{\pi}$$

$$\Rightarrow 2x^2 - 1 = \cos \left(\frac{\pi}{2} - \frac{2a}{\pi} \right) \text{ option (2)}$$

6. Official Ans. by NTA (4)

Sol. Let $g(x) = \sin x + \cos x = \sqrt{2} \sin \left(x + \frac{\pi}{4} \right)$

$$g(x) \in [1, \sqrt{2}] \text{ for } x \in [0, \pi/2]$$

$$f(x) = \tan^{-1}(\sin x + \cos x) \in \left[\frac{\pi}{4}, \tan^{-1} \sqrt{2} \right]$$

$$\tan \left(\tan^{-1} \sqrt{2} - \frac{\pi}{4} \right) = \frac{\sqrt{2}-1}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 3-2\sqrt{2}$$

7. Official Ans. by NTA (3)

Sol. $f(x) = \sin^{-1} \left(\frac{3x^2 + x - 1}{(x-1)^2} \right) + \cos^{-1} \left(\frac{x-1}{x+1} \right)$

$$-1 \leq \frac{x-1}{x+1} \leq 1 \Rightarrow 0 \leq x < \infty \quad \dots(1)$$

$$-1 \leq \frac{3x^2 + x - 1}{(x-1)^2} \leq 1 \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2} \right] \cup \{0\} \quad \dots(2)$$

(1) & (2)

$$\Rightarrow \text{Domain} = \left[\frac{1}{4}, \frac{1}{2} \right] \cup \{0\}$$

8. Official Ans. by NTA (3)

Sol. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$

$$\Rightarrow (2\pi - 5) + (6 - 2\pi) - (12 - 4\pi)$$

$$\Rightarrow 4\pi - 11.$$

9. Official Ans. by NTA (2)

Sol. $\operatorname{cosec} \left[2 \tan^{-1} \left(\frac{1}{5} \right) + \tan^{-1} \left(\frac{3}{4} \right) \right]$

$$\operatorname{cosec} \left[\tan^{-1} \left(\frac{5}{12} \right) + \tan^{-1} \left(\frac{3}{4} \right) \right]$$

$$= \operatorname{cosec} \left[\tan^{-1} \left(\frac{56}{33} \right) \right] = \frac{65}{56} \text{ option (2)}$$

10. Official Ans. by NTA (1)

Sol. $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4} \quad 0 < a, b < 1$

$$\Rightarrow \frac{a+b}{1-ab} = 1$$

$$a+b = 1-ab$$

$$(a+1)(b+1) = 2$$

$$\text{Now } \left[a - \frac{a^2}{2} + \frac{a^3}{3} - \dots \right] + \left[b - \frac{b^2}{2} + \frac{b^3}{3} - \dots \right]$$

$$= \log_e(1+a) + \log_e(1+b)$$

$$(\because \text{expansion of } \log_e(1+x))$$

$$= \log_e[(1+a)(1+b)]$$

$$= \log_e 2$$

11. Official Ans. by NTA (3)

Sol. $\frac{\sin^{-1} x}{r} = a, \frac{\cos^{-1} x}{r} = b, \frac{\tan^{-1} y}{r} = c$

$$\text{So, } a+b = \frac{\pi}{2r}$$

$$\cos \left(\frac{\pi c}{a+b} \right) = \cos \left(\frac{\pi \tan^{-1} y}{\frac{\pi}{2r}} \right)$$

$$= \cos(2 \tan^{-1} y), \text{ let } \tan^{-1} y = \theta$$

$$= \cos(2\theta)$$

$$= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - y^2}{1 + y^2}$$

12. Official Ans by NTA (3)

Sol. $\sin^{-1} \frac{3x}{5} + \sin^{-1} \frac{4x}{5} = \sin^{-1} x$

$$\sin^{-1} \left(\frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}} \right) = \sin^{-1} x$$

$$\frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}} = x$$

$$x = 0, 3\sqrt{25 - 16x^2} + 4\sqrt{25 - 9x^2} = 25$$

$$4\sqrt{25 - 9x^2} = 25 - 3\sqrt{25 - 16x^2} \text{ squaring we get}$$

$$16(25 - 9x^2) = 625 + 9(25 - 16x^2) - 150$$

$$\sqrt{25 - 16x^2}$$

$$400 = 625 + 225 - 150\sqrt{25 - 16x^2}$$

$$\sqrt{25 - 16x^2} = 3 \Rightarrow 25 - 16x^2 = 9$$

$$\Rightarrow x^2 = 1$$

Put $x = 0, 1, -1$ in the original equation

We see that all values satisfy the original equation.

Number of solution = 3

13. Official Ans. by NTA (3)

Sol. $S_k = \sum_{r=1}^k \tan^{-1} \left(\frac{6^r}{2^{2r+1} + 3^{2r+1}} \right)$

Divide by 3^{2r}

$$\sum_{r=1}^k \tan^{-1} \left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r} \cdot 2 + 3} \right)$$

$$\sum_{r=1}^k \tan^{-1} \left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1} + 1\right)} \right)$$

Let $\left(\frac{2}{3}\right)^r = t$

$$\sum_{r=1}^k \tan^{-1} \left(\frac{\frac{t}{3}}{1 + \frac{2}{3}t^2} \right)$$

$$\sum_{r=1}^k \tan^{-1} \left(\frac{t - \frac{2t}{3}}{1 + t \cdot \frac{2t}{3}} \right)$$

$$\sum_{r=1}^k \left(\tan^{-1}(t) - \tan^{-1}\left(\frac{2t}{3}\right) \right)$$

$$\sum_{r=1}^k \left(\tan^{-1}\left(\frac{2}{3}\right)^r - \tan^{-1}\left(\frac{2}{3}\right)^{r+1} \right)$$

$$S_k = \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right)^{k+1}$$

$$S_\infty = \lim_{k \rightarrow \infty} \left(\tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right)^{k+1} \right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}(0)$$

$$\therefore S_\infty = \tan^{-1}\left(\frac{2}{3}\right) = \cot^{-1}\left(\frac{3}{2}\right)$$

14. Official Ans. by NTA (2)**Sol.** Given equation

$$\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$$

Now, $\sin^{-1}\left[x^2 + \frac{1}{3}\right]$ is defined if

$$-1 \leq x^2 + \frac{1}{3} < 2 \Rightarrow \frac{-4}{3} \leq x^2 < \frac{5}{3}$$

$$\Rightarrow \boxed{0 \leq x^2 < \frac{5}{3}} \quad \dots(1)$$

and $\cos^{-1}\left[x^2 - \frac{2}{3}\right]$ is defined if

$$-1 \leq x^2 - \frac{2}{3} < 2 \Rightarrow \frac{-1}{3} \leq x^2 < \frac{8}{3}$$

$$\Rightarrow \boxed{0 \leq x^2 < \frac{8}{3}} \quad \dots(2)$$

So, from (1) and (2) we can conclude

$$\boxed{0 \leq x^2 < \frac{5}{3}}$$

Case - I if $0 \leq x^2 < \frac{2}{3}$

$$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$$

$$\Rightarrow x + \pi = x^2$$

$$\Rightarrow x^2 = \pi$$

$$\text{but } \pi \notin \left[0, \frac{2}{3}\right)$$

 $\Rightarrow$ No value of 'x'**Case - II** if $\frac{2}{3} \leq x^2 < \frac{5}{3}$

$$\sin^{-1}(1) + \cos^{-1}(0) = x^2$$

$$\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = x^2$$

$$\Rightarrow x^2 = \pi$$

$$\text{but } \pi \notin \left[\frac{2}{3}, \frac{5}{3}\right)$$

 $\Rightarrow$ No value of 'x'

So, number of solutions of the equation is zero.

Ans.(2)**15. Official Ans. by NTA (1)****Sol.** $\cot^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18) + \dots$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$$= \tan^{-1} 201 - \tan^{-1} 1$$

$$= \tan^{-1}\left(\frac{200}{202}\right)$$

$$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$$

$$\alpha = 1.01$$

16. Official Ans. by NTA (1)**Sol.** $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1} \frac{8}{31}$

Taking tangent both sides :-

$$\frac{(x+1) + (x-1)}{1 - (x^2 - 1)} = \frac{8}{31}$$

$$\Rightarrow \frac{2x}{2 - x^2} = \frac{8}{31}$$

$$\Rightarrow 4x^2 + 31x - 8 = 0$$

$$\Rightarrow x = -8, \frac{1}{4}$$

$$\text{But, if } x = \frac{1}{4}$$

$$\tan^{-1}(x+1) \in \left(0, \frac{\pi}{2}\right)$$

$$\& \cot^{-1}\left(\frac{1}{x-1}\right) \in \left(\frac{\pi}{2}, \pi\right)$$

$$\Rightarrow \text{LHS} > \frac{\pi}{2} \& \text{RHS} < \frac{\pi}{2}$$

(Not possible)

Hence, $x = -8$ **17. Official Ans. by NTA (2)****Sol.** $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{\{x\}}}$

$$\text{Domain} \in (-\infty, -1] \cup [1, \infty)$$

$$\{x\} \neq 0 \text{ so } x \neq \text{integers}$$