

AOD (TANGENT & NORMAL)

1. वक्रों $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ तथा $x^2 + y^2 = ab$, $a > b$, का

एक प्रतिच्छेदन कोण है :

(1) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (2) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

(3) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$ (4) $\tan^{-1}(2\sqrt{ab})$

2. यदि वक्र $y = ax^2 + bx + c$, $x \in \mathbb{R}$, बिन्दु $(1, 2)$ से होकर जाता है तथा मूल बिन्दु पर इसकी स्पर्श रेखा, $y = x$ है, तो a, b, c के संभावित मान है :

(1) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

(2) $a = 1, b = 0, c = 1$

(3) $a = 1, b = 1, c = 0$

(4) $a = -1, b = 1, c = 1$

3. यदि वक्र $y = x^3$ के बिन्दु $P(t, t^3)$ पर खींची गई स्पर्श रेखा वक्र को फिर से बिन्दु Q पर मिलती है, तो उस बिन्दु की कोटि जो रेखा-खण्ड PQ को आंतरिक अनुपात $1 : 2$ में काटता है, है:

(1) $-2t^3$ (2) 0 (3) $-t^3$ (4) $2t^3$

4. यदि वक्र $x = y^4$ तथा $xy = k$ एक दुसरे को समकोण पर काटते हैं, तो $(4k)^6$ बराबर है _____.

5. यदि वक्र $y(x) = \int_0^x (2t^2 - 15t + 10) dt$ के बिंदु (a, b) , $a > 1$, पर अभिलम्ब, रेखा $x + 3y = -5$ के समान्तर है, तो $|a + 6b|$ का मान बराबर है _____ ।

SOLUTION

1. Official Ans. by NTA (3)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $x^2 + y^2 = ab$

$$\frac{2x_1}{a^2} + \frac{2y_1 y'}{b^2} = 0$$

$$\Rightarrow y_1' = \frac{-x_1}{a^2} \frac{b^2}{y_1} \quad \dots(1)$$

$$\therefore 2x_1 + 2y_1 y' = 0$$

$$\Rightarrow y_2' = \frac{-x_1}{y_1} \quad \dots(2)$$

Here (x_1, y_1) is point of intersection of both curves

$$\therefore x_1^2 = \frac{a^2 b}{a+b}, \quad y_1^2 = \frac{ab^2}{a+b}$$

$$\therefore \tan \theta = \frac{|y_1' - y_2'|}{|1 + y_1' y_2'|} = \frac{\left| \frac{-x_1 b^2}{a^2 y_1} + \frac{x_1}{y_1} \right|}{\left| 1 + \frac{x_1^2 b^2}{a^2 y_1^2} \right|}$$

$$\tan \theta = \frac{|-b^2 x_1 y_1 + a^2 x_1 y_1|}{|a^2 y_1^2 + b^2 x_1^2|}$$

$$\tan \theta = \frac{|a-b|}{\sqrt{ab}}$$

2. Official Ans. by NTA (3)

Sol. $a + b + c = 2 \quad \dots(1)$

and $\frac{dy}{dx} \Big|_{(0,0)} = 1$

$$2ax + b \Big|_{(0,0)} = 1$$

$$b = 1$$

Curve passes through origin

So, $c = 0$

and $a = 1$

3. Official Ans. by NTA (1)

Sol. Slope of tangent at $P(t, t^3) = \frac{dy}{dx} \Big|_{(t, t^3)}$

$$= (3x^2)_{x=t} = 3t^2$$

So equation tangent at $P(t, t^3)$:

$$y - t^3 = 3t^2(x - t)$$

for point of intersection with $y = x^3$

$$x^3 - t^3 = 3t^2x - 3t^3$$

$$\Rightarrow (x - t)(x^2 + xt + t^2) = 3t^2(x - t)$$

for $x \neq t$

$$x^2 + xt + t^2 = 3t^2$$

$$\Rightarrow x^2 + xt - 2t^2 = 0 \Rightarrow (x - t)(x + 2t) = 0$$

So for Q : $x = -2t$, $Q(-2t, -8t^3)$

ordinate of required point : $\frac{2t^3 - 8t^3}{2+1} = -2t^3$

4. Official Ans. by NTA (4)

Sol. $x = y^4$, $xy = k$

for intersection $y^5 = k \quad \dots(1)$

Also $x = y^4$

$$\Rightarrow 1 = 4y^3 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{4y^3}$$

for $xy = k \Rightarrow x = \frac{k}{y}$

$$\Rightarrow 1 = -\frac{k}{y^2} \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{k}$$

\therefore Curve cut orthogonally

$$\Rightarrow \frac{1}{4y^3} \times \left(\frac{-y^2}{k} \right) = -1$$

$$\Rightarrow y = \frac{1}{4k}$$

\therefore from (1) $y^5 = k$

$$\Rightarrow \frac{1}{(4k)^5} = k$$

$$\Rightarrow 4 = (4k)^6$$

5. Official Ans. by NTA (406)

$$\text{Sol. } y(x) = \int_0^x (2t^2 - 15t + 10) dt$$

$$y'(x) \Big|_{x=a} = [2x^2 - 15x + 10]_a = 2a^2 - 15a + 10$$

$$\text{Slope of normal} = -\frac{1}{3}$$

$$\Rightarrow 2a^2 - 15a + 10 = 3 \Rightarrow a = 7$$

$$\& \quad a = \frac{1}{2} \text{ (rejected)}$$

$$b = y(7) = \int_0^7 (2t^2 - 15t + 10) dt$$

$$= \left[\frac{2t^3}{3} - \frac{15t^2}{2} + 10t \right]_0^7$$

$$\Rightarrow 6b = 4 \times 7^3 - 45 \times 49 + 60 \times 7$$

$$|a + 6b| = 406$$