

SOLUTION OF TRIANGLE

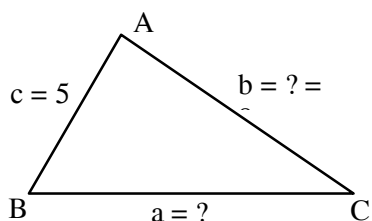
1. If in a triangle ABC, $AB = 5$ units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :
 (1) $10 + 6\sqrt{2}$ (2) $8 + 2\sqrt{2}$
 (3) $6 + 8\sqrt{3}$ (4) $4 + 2\sqrt{3}$
2. Let in a right angled triangle, the smallest angle be θ . If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :
 (1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{5}-1}{2}$
 (3) $\frac{\sqrt{2}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

3. In ΔABC , the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC , then the value of $2R + r$ (in cm) is equal to _____.

SOLUTION

1. Official Ans. by NTA (3)

Sol.



$$\text{As, } \cos B = \frac{3}{5} \Rightarrow \boxed{B = 53^\circ}$$

$$A_s, R = 5 \Rightarrow \frac{c}{\sin c} = 2R$$

$$\Rightarrow \frac{5}{10} = \sin c \Rightarrow \boxed{C = 30^\circ}$$

$$\text{Now, } \frac{b}{\sin B} = 2R \Rightarrow \boxed{b = 2(5)\left(\frac{4}{5}\right) = 8}$$

Now, by cosine formula

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$$

$$\Rightarrow a^2 - 6a - 3g = 0$$

$$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$$

$$\Rightarrow \boxed{3+4\sqrt{3}} \text{ (Reject } a = 3-4\sqrt{3}\text{)}$$

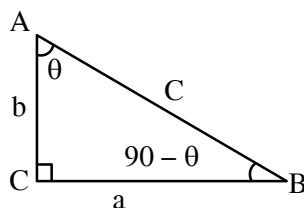
$$\text{Now, } \Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(3+4\sqrt{3})$$

$$\Rightarrow \Delta = (6 + 8\sqrt{3})$$

⇒ Option (3) is correct.

2. Official Ans. by NTA (2)

Sol.



$\angle A = \theta$

$$\angle B = 90 - \theta$$

a = smallest side

$$c^2 = a^2 + b^2$$

$$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$$

$$\frac{b^2c^2}{a^2} = b^2 + c^2$$

Use $a = 2R \sin A = 2R \sin \theta$

$$b = 2R \sin B = 2R \sin (90 - \theta) = 2R \cos \theta$$

$$c = 2R \sin C = 2 \sin 90^\circ = 2R$$

$$\frac{4R^2 \cos^2 \theta}{4R^2 \sin^2 \theta} = 4R^2 \cos^2 \theta + 4R^2$$

$$\cos^2\theta = \sin^2\theta \cos^2\theta + \sin^2\theta$$

$$1 - \sin^2\theta = \sin^2\theta (1 - \sin^2\theta) + \sin^2\theta$$

$$\sin^2 \theta = \frac{3 - \sqrt{5}}{2}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{5}-1}{2}$$

3. Official Ans by NTA (15)

Sol. $\Delta = \frac{1}{2} \cdot 5 \cdot 12 \cdot \sin A = 30$

$$\sin A = 1$$

$$A = 90^\circ \Rightarrow BC = 13$$

$$\text{BC} = 2\text{R} = 13$$

$$r = \frac{\Delta}{S} = \frac{30}{15} = 2$$

$$2R + r = 15$$

