PERMUTATION & COMBINATION

1. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is__________.

2. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _______.

3. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _______.

4. If ${}^{n}P_{r} = {}^{n}P_{r+1}$ and ${}^{n}C_{r} = {}^{n}C_{r-1}$, then the value of r is equal to:

(1) 1 (2) 4 (3) 2 (4) 3

5. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10}.(11)^{11}.(13)^{13}$ is equal to_____ .

6. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ________.

7. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _______.

8. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _________.

9. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set T = {A ⊆ S : A ≠ ϕ and the sum of all the elements of A is not a multiple of 3} is _________.

10. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _________.

11. Let $P_1, P_2, \ldots, P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is :

(1) 12 (2) 419 (3) 443 (4) 455

12. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is ______.

13. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

14. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

(1) 1625 (2) 575 (3) 560 (4) 1050

15. The total number of positive integral solutions (x, y, z) such that xyz = 24 is :

(1) 36 (2) 24 (3) 45 (4) 30

16. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ______.

17. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then :

(1) $y = 273x$ (2) $2y = 91x$

(3) $y = 91x$ (4) $2y = 273x$

18. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ______.

19. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 42 (2) 82 (3) 77 (4) 35

20. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :

(1) 11 (2) 6 (3) 6x (4) 12

21. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

(1) 795 (2) 1173 (3) 1890 (4) 717

22. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :

(1) 364 (2) 240 (3) 333 (4) 360

23. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

(1) 5 (2) 2 (3) 4 (4) 6

24. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

(1) 26664 (2) 122664

(3) 122234 (4) 22264

25. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

26. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ______.

SOLUTION

1. Official Ans. by NTA (777)

Sol. 15 : Players

6 : Bowlers

7 : Batsman

2 : Wicket keepers

Total number of ways for :

at least 4 bowlers, 5 batsman & 1 wicket keeper

$= {}^6C_4({}^7C_6 \times {}^2C_1 + {}^7C_5 \times {}^2C_2) + {}^6C_5 \times {}^7C_5 \times {}^2C_1$

$= \boxed{777}$

2. Official Ans. by NTA (96)

Sol.

2,4,6,8				
4	4	3	2	1

$= 4 \times 4 \times 3 \times 2 = 96$

3. Official Ans. by NTA (238)

Sol.

Class	10^{th}	11^{th}	12^{th}
Total student	5	6	8
	2	3	5

$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5$

Number of selection	2	2	6

$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6$

	3	2	5

$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5$

$\Rightarrow$ Total number of ways = 23800

According to question

$100\,K = 23800$

$\Rightarrow K = 238$

4. Official Ans. by NTA (3)

Sol. ${}^nP_r = {}^nP_{r+1} \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow (n-r) = 1$ ...(1)

${}^nC_r = {}^nC_{r-1}$

$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!}$

$\Rightarrow \frac{1}{r(n-r)!} = \frac{1}{(n-r+1)(n-r)!}$

$\Rightarrow n - r + 1 = r$

$\Rightarrow n + 1 = 2r$ ...(2)

$(1) \Rightarrow 2r - 1 - r = 1 \Rightarrow r = 2$

5. Official Ans. by NTA (924)

Sol. $N = 2^{10} \times 5^{10} \times 11^{11} \times 13^{13}$

Now, power of 2 must be zero,

power of 5 can be anything,

power of 13 can be anything.

But, power of 11 should be even.

So, required number of divisors is

$1 \times 11 \times 14 \times 6 = 924$

6. Official Ans. by NTA (52)

Sol. (i) When '0' is at unit place

		0
5	4	

Number of numbers = 20

(ii) When 4 or 6 are at unit place

OX		4,6
4	× 4	2

Number of numbers = 32

So number of numbers = 52

7. Official Ans. by NTA (7744)

Sol. 209, 220, 231, ..., 495

Sum = $\frac{27}{2}(209+495)=9504$

Number containing 1 at unit place $\underline{2}\ \underline{3}\ \underline{1}$, $\underline{3}\ \underline{4}\ \underline{1}$, $\underline{4}\ \underline{5}\ \underline{1}$

Number containing 1 at 10^{th} place $\underline{3}\ \underline{1}\ \underline{9}$, $\underline{4}\ \underline{1}\ \underline{8}$

Required = 9501 – (231 + 341 + 451 + 319 + 418)

7744

8. Official Ans. by NTA (100)

Sol.

5	a	b	b	a	5

It is always divisible by 5 and 11.

So, required number = 10 × 10 = 100

9. Official Ans. by NTA (80)

Sol. 3n type $\rightarrow 3, 6, 9 = P$

3n – 1 type $\rightarrow 2, 5 = Q$

3n – 2 type $\rightarrow 1,4 = R$

number of subset of S containing one element which are not divisible by 3 = $^2C_1 + {}^2C_1 = 4$

number of subset of S containing two numbers whose some is not divisible by 3

$= {}^3C_1\times{}^2C_1+{}^3C_1\times{}^2C_1+{}^2C_2+{}^2C_2=14$

number of subsets containing 3 elements whose sum is not divisible by 3

$={}^3C_2\times{}^4C_1+\left({}^2C_2\times{}^2C_1\right)2+{}^3C_1\left({}^2C_2+{}^2C_2\right)=22$

number of subsets containing 4 elements whose sum is not divisible by 3

$={}^3C_3\times{}^4C_1+{}^3C_2\left({}^2C_2+{}^2C_2\right)+\left({}^3C_1{}^2C_1\times{}^2C_2\right)2$

= 4 + 6 + 12 = 22.

number of subsets of S containing 5 elements whose sum is not divisible by 3.

$={}^3C_3\left({}^2C_2+{}^2C_2\right)+\left({}^3C_2{}^2C_1\times{}^2C_2\right)\times 2=2+12=14$

number of subsets of S containing 6 elements whose sum is not divisible by 3 = 4

$\Rightarrow$ Total subsets of Set A whose sum of digits is not divisible by 3 = 4 + 14 + 22 + 22 + 14 + 4 = 80.

10. Official Ans. by NTA (576)

Sol. VOWELS $\rightarrow$ 2 Vowels, 4 Consonants

All Consonants should not be together

= Total – All consonants together,

= 6! – 3! 4! = 576

11. Official Ans. by NTA (3)

Sol. Total Number of Triangles = $^{15}C_3$

i + j + k = 15 (Given)

5 Cases			4 Cases			3 Cases			1 Cases		
i	j	k	i	j	k	i	j	k	i	j	k
1	2	12	2	3	10	3	4	8	4	5	6
1	3	11	2	4	9	3	5	7			
1	4	10	2	5	8						
1	5	9	2	6	7						
1	6	8									

Number of Possible triangles using the vertices P_i, P_j, P_k such that i + j + k ≠ 15 is equal to $^{15}C_3 - 12 = 443$ Option (3)

12. Official Ans. by NTA (77)

Sol. FARMER (6)

A, E, F, M, R, R

A					
E					
F	A	E			
F	A	M			
F	A	R	E		
F	A	R	M	E	R

$\frac{\lfloor 5}{\lfloor 2}-\lfloor 4=60-24=36$

$\frac{\lfloor 3}{\lfloor 2}-\lfloor 2=3-2=1$

= 1

= 2

= 1

77

13. Official Ans. by NTA (31650)

Sol. If group C has one student then number of groups

$^{10}C_1[2^9 - 2] = 5100$

If group C has two students then number of groups

$^{10}C_2[2^8 - 2] = 11430$

If group C has three students then number of groups

$= {}^{10}C_3 \times [2^7 - 2] = 15120$

So total groups = 31650

14. Official Ans. by NTA (1)

Sol.

Indians	Foreigners	Number of ways
2	4	$^6C_2 \times {}^8C_4 = 1050$
3	6	$^6C_3 \times {}^8C_6 = 560$
4	8	$^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

15. Official Ans. by NTA (4)

Sol. $xyz = 2^3 \times 3^1$

Let $x = 2^{\alpha_1} \times 3^{\beta_1}$

$y = 2^{\alpha_2} \times 3^{\beta_2}$

$z = 2^{\alpha_3} \times 3^{\beta_2}$

Now $\alpha_1 + \alpha_2 + \alpha_3 = 3.$

No. of non-negative intergal sol = $^5C_2 = 10$

& $\beta_1 + \beta_2 + \beta_3 = 1$

No. of non-negative intergal sol^n = $^3C_2 = 3$

Total ways = 10 × 3 = 30.

16. Official Ans. by NTA (32)

Sol. We need three digits numbers.

Since 1 + 2 + 3 + 4 + 5 = 15

So, number of possible triplets for multiple of 15 is 1 × 2 × 2

so Ans. = $4 \times \underline{|3} + 4 \times 3 - 1 \times 2 \times \underline{|2} = 32$

17. Official Ans. by NTA (2)

Sol. $x = {}^5C_3 \times 3! = 60$

$y = {}^{15}C_3 \times 3! = 15 \times 14 \times 13 = 30 \times 91$

$\therefore 2y = 91x$

18. Official Ans. by NTA (45)

Sol. for $3^n + 7^n$ to be divisible by 10

n can be any odd number

∴ Number of odd two digit numbers = 45

19. Official Ans. by NTA (3)

Sol. (I) First possiblity is 1, 1, 1, 1, 1, 2, 3

required number = $\frac{7!}{5!} = 7 \times 6 = 42$

(II) Second possiblity is 1, 1, 1, 1, 2, 2, 2

required number = $\frac{7!}{4!\ 3!} = \frac{7 \times 6 \times 5}{6} = 35$

Total = 42 + 35 = 77

20. Official Ans. by NTA (4)

Sol. $y + z = 5$

$\frac{1}{y} + \frac{1}{z} = \frac{5}{6} \quad y > z$

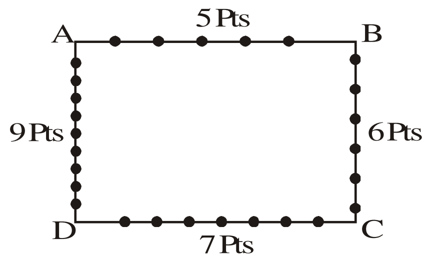
$\Rightarrow y = 3, z = 2$

$\Rightarrow n = 2^x.3^3.5^2 = (2.2.2 \ldots)(3.3.3)(5.5)$

Number of odd divisors = 4 × 3 = 12

21. Official Ans by NTA (4)

Sol.



α = Number of triangles

$\alpha = 5\cdot6\cdot7 + 5\cdot7\cdot9 + 5\cdot6\cdot9 + 6\cdot7\cdot9$

$= 210 + 315 + 270 + 378$

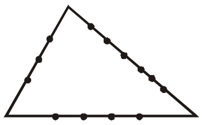
$= 1173$

β = Number of Quadrilateral

$\beta = 5\cdot6\cdot7\cdot9 = 1890$

$\beta - \alpha = 1890 - 1173 = 717$

22. Official Ans. by NTA (3)

Sol. 

Total Number of triangles formed

$= {}^{14}C_3 - {}^{3}C_3 - {}^{5}C_3 - {}^{6}C_3$

$= 333$ **Option (3)**

23. Official Ans. by NTA (3)

Sol. Total matches between boys of both team

$= {}^{7}C_1 \times {}^{4}C_1 = 28$

Total matches between girls of both

team $= {}^{n}C_1\ {}^{6}C_1 = 6n$

Now, $28 + 6n = 52$

$\Rightarrow n = 4$

24. Official Ans. by NTA (1)

Sol. Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12$.

total numbers when 1 at unit place is 3.

2 at unit place is 6

3 at unit place is 3.

So, sum $= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

$= (1111) \times 24$

$= 26664$

25. Official Ans. by NTA (300)

Sol. 3_ _ = $10 \times 10 = 100$

_3 _ = $10 \times 10 = 100$

_ _ 3 = $10 \times 10 = \underline{100}$

300

26. Official Ans. by NTA (1000)

Sol. Let N be the four digit number

$\gcd(N,18) = 3$

Hence N is an odd integer which is divisible by 3 but not by 9.

4 digit odd multiples of 3

1005, 1011,......., 9999 $\rightarrow$ 1500

4 digit odd multiples of 9

1017, 1035,......., 9999 $\rightarrow$ 500

Hence number of such N = 1000