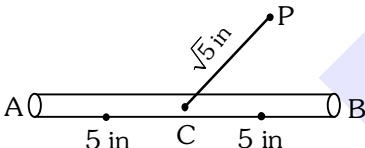


HEIGHT & DISTANCE

1. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75° . Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

- (1) $8(2+2\sqrt{3}+\sqrt{2})$ (2) $8(\sqrt{6}+\sqrt{2}+2)$
(3) $8(\sqrt{2}+2+\sqrt{3})$ (4) $8(\sqrt{6}-\sqrt{2}+2)$

2. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :

- 
- (1) $\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}(1)$
(3) $\tan^{-1}\left(\frac{4}{3}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

3. Two poles, AB of length a metres and CD of length a + b ($b \neq a$) metres are erected at the same horizontal level with bases at B and D. If

$$BD = x \text{ and } \tan \angle ACB = \frac{1}{2}, \text{ then:}$$

- (1) $x^2 + 2(a + 2b)x - b(a + b) = 0$
(2) $x^2 + 2(a + 2b)x + a(a + b) = 0$
(3) $x^2 - 2ax + b(a + b) = 0$
(4) $x^2 - 2ax + a(a + b) = 0$

4. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

- (1) $12\sqrt{15}$ (2) $12\sqrt{10}$
(3) $8\sqrt{10}$ (4) $6\sqrt{10}$

5. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45° . Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

- (1) 10 (2) $10\sqrt{3}$
(3) $10(\sqrt{3}+1)$ (4) $10(\sqrt{3}-1)$

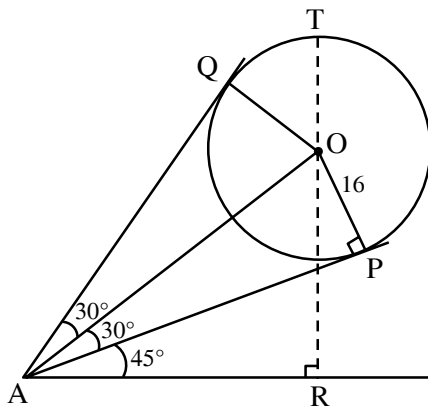
6. The angle of elevation of a jet plane from a point A on the ground is 60° . After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30° . If the jet plane is flying at a constant height, then its height is :

- (1) $1800\sqrt{3}$ m (2) $3600\sqrt{3}$ m
(3) $2400\sqrt{3}$ m (4) $1200\sqrt{3}$ m

7. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :
- (1) $20\sqrt{3}$ (2) $25\sqrt{3}$
(3) 30 (4) 25
8. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle to $\triangle ABC$ is 2, then the height of the pole is equal to :
- (1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

SOLUTION

1.



O → centre of sphere

P, Q → point of contact of tangents from A

Let T be top most point of balloon & R be foot of perpendicular from O to ground.

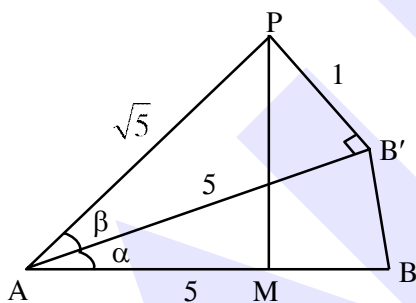
From triangle OAP, $OA = 16 \operatorname{cosec} 30^\circ = 32$

From triangle ABO, $OR = OA \sin 75^\circ = 32 \frac{(\sqrt{3}+1)}{2\sqrt{2}}$

So level of top most point = $OR + OT$

$$= 8(\sqrt{6} + \sqrt{2} + 2)$$

2.



From figure.

$$\sin \beta = \frac{1}{\sqrt{5}}$$

$$\therefore \tan \beta = \frac{1}{2}$$

$$\tan (\alpha + \beta) = 2$$

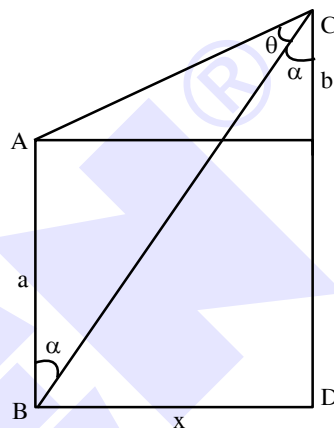
$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = 2$$

$$\frac{\tan \alpha + \frac{1}{2}}{1 - \tan \alpha \left(\frac{1}{2} \right)} = 2$$

$$\tan \alpha = \frac{3}{4}$$

$$\alpha = \tan^{-1} \left(\frac{3}{4} \right)$$

3.



$$\tan \theta = \frac{1}{2}$$

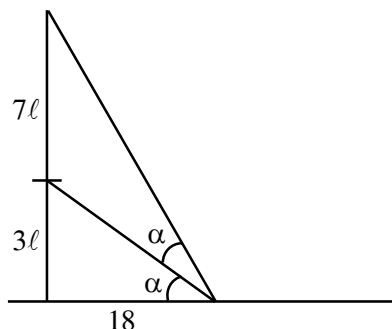
$$\tan (\theta + \alpha) = \frac{x}{b}, \tan \alpha = \frac{x}{a+b}$$

$$\Rightarrow \frac{1}{2} + \frac{x}{a+b}$$

$$\Rightarrow \frac{\frac{1}{2} + \frac{x}{a+b}}{1 - \frac{1}{2} \times \frac{x}{a+b}} = \frac{x}{b}$$

$$\Rightarrow x^2 - 2ax + ab + b^2 = 0$$

4.



Let height of pole = 10ℓ

$$\tan \alpha = \frac{3\ell}{18} = \frac{\ell}{6}$$

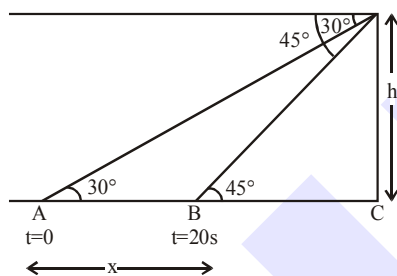
$$\tan 2\alpha = \frac{10\ell}{18}$$

$$\frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{10\ell}{18}$$

$$\text{use } \tan \alpha = \frac{\ell}{6} \Rightarrow \ell = \sqrt{\frac{72}{5}}$$

$$\text{height of pole} = 10\ell = 12\sqrt{10}$$

5.



Let speed of boat is u m/s and height of tower is h meter & distance $AB = x$ metre

$$\therefore x = h \cot 30^\circ - h \cot 45^\circ$$

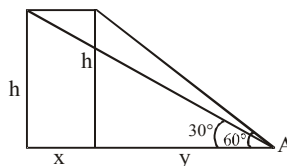
$$\Rightarrow x = h(\sqrt{3} - 1)$$

$$\therefore u = \frac{x}{20} = \frac{h(\sqrt{3} - 1)}{20} \text{ m/s}$$

\therefore Time taken to travel from B to C (Distance = h meter)

$$= \frac{h}{u} = \frac{h}{\frac{h(\sqrt{3} - 1)}{20}} = \frac{20}{\sqrt{3} - 1} = 10(\sqrt{3} + 1) \text{ sec.}$$

6.



$$\tan 60^\circ = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y} \Rightarrow h = \sqrt{3}y \dots (1)$$

$$\tan 30^\circ = \frac{h}{x + y}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x + y} \Rightarrow \sqrt{3}h = x + y \dots (2)$$

$$\text{Speed } 432 \text{ km/h} \Rightarrow \frac{432 \times 20}{60 \times 60} \Rightarrow \frac{12}{5} \text{ km}$$

$$\sqrt{3}h = \frac{12}{5} + y$$

$$\sqrt{3}h - \frac{12}{5} = y$$

from (1)

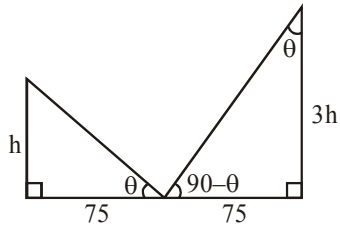
$$h = \sqrt{3} \left[\sqrt{3}h - \frac{12}{5} \right]$$

$$h = 3h - \frac{12\sqrt{3}}{5}$$

$$h = \frac{6\sqrt{3}}{5} \text{ km}$$

$$h = 1200\sqrt{3} \text{ m}$$

7.



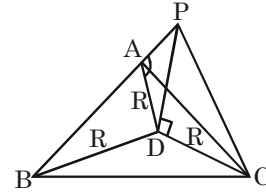
$$\tan \theta = \frac{h}{75} = \frac{75}{3h}$$

$$\Rightarrow h^2 = \frac{(75)^2}{3}$$

$$h = 25\sqrt{3} \text{ m}$$

8. Let $PD = h$, $R = 2$

As angle of elevation of top of pole from A, B, C are equal So D must be circumcentre of ΔABC



$$\tan\left(\frac{\pi}{3}\right) = \frac{PD}{R} = \frac{h}{R}$$

$$h = R \tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$