

COMPOUND ANGLE

1. The value of $\cot \frac{\pi}{24}$ is :
 - (1) $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$
 - (2) $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$
 - (3) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$
 - (4) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$
2. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to :
 - (1) 4
 - (2) 3
 - (3) 0
 - (4) 1
3. Let $\alpha = \max_{x \in \mathbb{R}} \{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$ and $\beta = \min_{x \in \mathbb{R}} \{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c - b$ is equal to :
 - (1) 42
 - (2) 47
 - (3) 43
 - (4) 50
4. The value of $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ is :
 - (1) $\frac{1}{4\sqrt{2}}$
 - (2) $\frac{1}{4}$
 - (3) $\frac{1}{8}$
 - (4) $\frac{1}{8\sqrt{2}}$
5. Let $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :
 - (1) $b^2 - a^2 = a^2 + c^2$
 - (2) b^2, c^2, a^2 are in A.P.
 - (3) c^2, a^2, b^2 are in A.P.
 - (4) a^2, b^2, c^2 are in A.P.

6. If n is the number of solutions of the equation $2\cos x \left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1$, $x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is :
 - (1) (3, $13\pi/9$)
 - (2) (2, $2\pi/3$)
 - (3) (2, $8\pi/9$)
 - (4) (3, $5\pi/3$)
7. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:
 - (1) $\frac{1}{\sqrt{7}}$
 - (2) $2\sqrt{2} - 1$
 - (3) $\sqrt{7} - 1$
 - (4) $\frac{1}{2\sqrt{2}}$
8. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \dots \infty) \log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x} \left(0 < x < \frac{\pi}{2}\right)$ is
 - (1) $2\sqrt{3}$
 - (2) $\frac{3}{2}$
 - (3) $\sqrt{3}$
 - (4) $\frac{1}{2}$
9. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to :
 - (1) 350
 - (2) 500
 - (3) 400
 - (4) 250

SOLUTION

$$1. \quad \cot \theta = \frac{1 + \cos 2\theta}{\sin 2\theta} = \frac{1 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)} \quad \theta = \frac{\pi}{24}$$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{(2\sqrt{2} + \sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$$

$$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2$$

$$2. \quad x = \frac{1}{2} \left(\tan \frac{\pi}{9} + \tan \frac{7\pi}{18} \right)$$

$$\text{and } 2y = \tan \frac{\pi}{9} + \tan \frac{5\pi}{18}$$

$$\text{so, } x - 2y = \frac{1}{2} \left(\tan \frac{\pi}{9} + \tan \frac{7\pi}{18} \right) - \left(\tan \frac{\pi}{9} + \tan \frac{5\pi}{18} \right)$$

$$\Rightarrow |x - 2y| = \left| \frac{\cot \frac{\pi}{9} - \tan \frac{\pi}{9}}{2} - \tan \frac{5\pi}{18} \right|$$

$$= \left| \cot \frac{2\pi}{9} - \cot \frac{2\pi}{9} \right| = 0$$

$$\left(\text{as } \tan \frac{5\pi}{18} = \cot \frac{2\pi}{9}; \tan \frac{7\pi}{18} = \cot \frac{\pi}{9} \right)$$

$$3. \quad \alpha = \max \{ 8^{2\sin 3x} \cdot 4^{4\cos 3x} \}$$

$$= \max \{ 2^{6\sin 3x} \cdot 2^{8\cos 3x} \}$$

$$= \max \{ 2^{6\sin 3x + 8\cos 3x} \}$$

$$\text{and } \beta = \min \{ 8^{2\sin 3x} \cdot 4^{4\cos 3x} \} = \min \{ 2^{6\sin 3x + 8\cos 3x} \}$$

Now range of $6\sin 3x + 8\cos 3x$

$$= [-\sqrt{6^2 + 8^2}, +\sqrt{6^2 + 8^2}] = [-10, 10]$$

$$\alpha = 2^{10} \text{ \& } \beta = 2^{-10}$$

$$\text{So, } \alpha^{1/5} = 2^2 = 4$$

$$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$$

quadratic $8x^2 + bx + c = 0$, $c - b$

$$= 8 \times [(\text{product of roots}) + (\text{sum of roots})]$$

$$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4} \right] = 8 \times \left[\frac{21}{4} \right] = 42$$

$$4. \quad 2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

$$2\sin^2 \frac{\pi}{8} \sin^2 \frac{2\pi}{8} \sin^2 \frac{3\pi}{8}$$

$$\sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8}$$

$$\sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8}$$

$$\frac{1}{4} \sin^2 \left(\frac{\pi}{4} \right) = \frac{1}{8}$$

$$5. \quad \frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$$

As A, B, C are angles of triangle

$$A + B + C = \pi$$

$$A = \pi - (B + C)$$

$$\text{So, } \sin A = \sin(B + C) \dots (1)$$

$$\text{Similarly } \sin B = \sin(A + C) \dots (2)$$

From (1) and (2)

$$\frac{\sin(B + C)}{\sin(A + C)} = \frac{\sin(A - C)}{\sin(C - B)}$$

$$\sin(C + B) \cdot \sin(C - B) = \sin(A - C) \sin(A + C)$$

$$\sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$$

$$\{ \because \sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y \}$$

$$2\sin^2 C = \sin^2 A + \sin^2 B$$

By sine rule

$$2c^2 = a^2 + b^2$$

$$\Rightarrow b^2, c^2 \text{ and } a^2 \text{ are in A.P.}$$

6. $2 \cos x \left(4 \sin \left(\frac{\pi}{4} + x \right) \sin \left(\frac{\pi}{4} - x \right) - 1 \right) = 1$

$$2 \cos x \left(4 \left(\sin^2 \frac{\pi}{4} - \sin^2 x \right) - 1 \right) = 1$$

$$2 \cos x \left(4 \left(\frac{1}{2} - \sin^2 x \right) - 1 \right) = 1$$

$$2 \cos x (2 - 4 \sin^2 x - 1) = 1$$

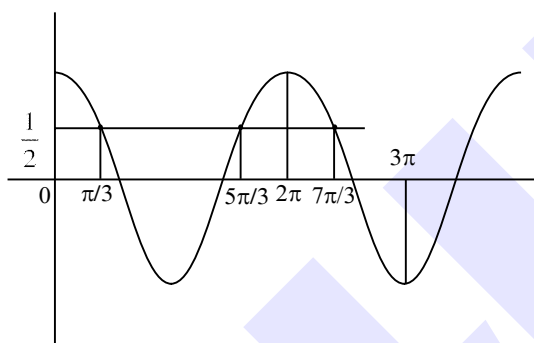
$$2 \cos x (1 - 4 \sin^2 x) = 1$$

$$2 \cos x (4 \cos^2 x - 3) = 1$$

$$4 \cos^3 x - 3 \cos x = \frac{1}{2}$$

$$\cos 3x = \frac{1}{2}$$

$$x \in [0, \pi] \therefore 3x \in [0, 3\pi]$$



7. Let $\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} = \theta$

$$\sin 4\theta = \frac{\sqrt{63}}{8}$$

$$\cos 4\theta = \frac{1}{8}$$

$$2 \cos^2 2\theta - 1 = \frac{1}{8}$$

$$\cos^2 2\theta = \frac{9}{16}$$

$$\cos 2\theta = \frac{3}{4}$$

$$2 \cos^2 \theta - 1 = \frac{3}{4}$$

$$\cos^2 \theta = \frac{7}{8}$$

$$\cos \theta = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\tan \theta = \frac{1}{\sqrt{7}}$$

8. $e^{(\cos^2 \theta + \cos^4 \theta + \dots \infty) \ln^2} = 2^{\cos^2 \theta + \cos^4 \theta + \dots \infty}$

$$= 2^{\cot^2 \theta}$$

$$\text{Now } t^2 - 9t + 9 = 0 \Rightarrow t = 1, 8$$

$$\Rightarrow 2^{\cot^2 \theta} = 1, 8 \Rightarrow \cot^2 \theta = 0, 3$$

$$0 < \theta < \frac{\pi}{2} \Rightarrow \cot \theta = \sqrt{3}$$

$$\Rightarrow \frac{2 \sin \theta}{\sin \theta + \sqrt{3} \cos \theta} = \frac{2}{1 + \sqrt{3} \cot \theta} = \frac{2}{4} = \frac{1}{2}$$

9. $15 \sin^4 \alpha + 10 \cos^4 \alpha = 6$

$$15 \sin^4 \alpha + 10 \cos^4 \alpha = 6(\sin^2 \alpha + \cos^2 \alpha)^2$$

$$(3 \sin^2 \alpha - 2 \cos^2 \alpha)^2 = 0$$

$$\tan^2 \alpha = \frac{2}{3} \therefore \cot^2 \alpha = \frac{3}{2}$$

$$\Rightarrow 27 \sec^6 \alpha + 8 \operatorname{cosec}^6 \alpha$$

$$= 27(\sec^6 \alpha)^3 + 8(\operatorname{cosec}^6 \alpha)^3$$

$$= 27(1 + \tan^2 \alpha)^3 + 8(1 + \cot^2 \alpha)^3$$

$$= 250$$