

**AOD (MONOTONICITY)**

1. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be defined as

$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x & , x > 0 \\ 3xe^x & , x \leq 0 \end{cases}. \text{ Then } f \text{ is}$$

increasing function in the interval

- (1)  $\left(-\frac{1}{2}, 2\right)$       (2)  $(0, 2)$   
 (3)  $\left(-1, \frac{3}{2}\right)$       (4)  $(-3, -1)$

2. Let  $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$ ,  
 $x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$ . Then,  $f$  is :

- (1) increasing in  $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$   
 (2) decreasing in  $\left(0, \frac{\pi}{2}\right)$   
 (3) increasing in  $\left(-\frac{\pi}{6}, 0\right)$   
 (4) decreasing in  $\left(-\frac{\pi}{6}, 0\right)$

3. The number of real roots of the equation  $e^{4x} + 2e^{3x} - e^x - 6 = 0$  is :  
 (1) 2      (2) 4      (3) 1      (4) 0

4. If 'R' is the least value of 'a' such that the function  $f(x) = x^2 + ax + 1$  is increasing on  $[1, 2]$  and 'S' is the greatest value of 'a' such that the function  $f(x) = x^2 + ax + 1$  is decreasing on  $[1, 2]$ , then the value of  $|R - S|$  is \_\_\_\_\_.

5. Let  $f$  be any continuous function on  $[0, 2]$  and twice differentiable on  $(0, 2)$ . If  $f(0) = 0$ ,  $f(1) = 1$  and  $f(2) = 2$ , then

- (1)  $f''(x) = 0$  for all  $x \in (0, 2)$   
 (2)  $f''(x) = 0$  for some  $x \in (0, 2)$   
 (3)  $f'(x) = 0$  for some  $x \in [0, 2]$   
 (4)  $f''(x) > 0$  for all  $x \in (0, 2)$

6. The function  $f(x) = x^3 - 6x^2 + ax + b$  is such that  $f(2) = f(4) = 0$ . Consider two statements.

(S1) there exists  $x_1, x_2 \in (2, 4)$ ,  $x_1 < x_2$ , such that  $f'(x_1) = -1$  and  $f'(x_2) = 0$ .

(S2) there exists  $x_3, x_4 \in (2, 4)$ ,  $x_3 < x_4$ , such that  $f$  is decreasing in  $(2, x_4)$ , increasing in  $(x_4, 4)$

and  $2f'(x_3) = \sqrt{3}f(x_4)$ . Then

- (1) both (S1) and (S2) are true  
 (2) (S1) is false and (S2) is true  
 (3) both (S1) and (S2) are false  
 (4) (S1) is true and (S2) is false

7. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be defined as,

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let  $A = \{x \in \mathbf{R} : f \text{ is increasing}\}$ . Then  $A$  is equal to :

- (1)  $(-\infty, -5) \cup (4, \infty)$   
 (2)  $(-5, \infty)$   
 (3)  $(-\infty, -5) \cup (-4, \infty)$   
 (4)  $(-5, -4) \cup (4, \infty)$

8. The function

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x :$$

- (1) increases in  $\left[\frac{1}{2}, \infty\right)$

- (2) increases in  $\left(-\infty, \frac{1}{2}\right]$

- (3) decreases in  $\left[\frac{1}{2}, \infty\right)$

- (4) decreases in  $\left(-\infty, \frac{1}{2}\right]$

- 11.** Let  $f$  be a real valued function, defined on  $R - \{-1, 1\}$  and given by

$$f(x) = 3 \log_e \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function  $f(x)$  is increasing?

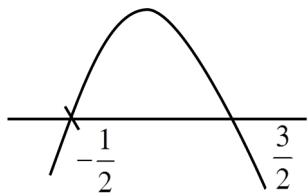
  - (1)  $(-\infty, -1) \cup \left( \left[ \frac{1}{2}, \infty \right) - \{1\} \right)$
  - (2)  $(-\infty, \infty) - \{-1, 1\}$
  - (3)  $\left[ -1, \frac{1}{2} \right]$
  - (4)  $\left( -\infty, \frac{1}{2} \right] - \{-1\}$

**12.** Consider the function  $f : R \rightarrow R$  defined by

$$f(x) = \begin{cases} \left( 2 - \sin \left( \frac{1}{x} \right) \right) |x|, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ Then } f \text{ is :}$$
  - (1) monotonic on  $(-\infty, 0) \cup (0, \infty)$
  - (2) not monotonic on  $(-\infty, 0)$  and  $(0, \infty)$
  - (3) monotonic on  $(0, \infty)$  only
  - (4) monotonic on  $(-\infty, 0)$  only

**SOLUTION****1. Official Ans. by NTA (3)**

**Sol.**  $f'(x) \begin{cases} -4x^2 + 4x + 3 & x > 0 \\ 3e^x(1+x) & x \leq 0 \end{cases}$



For  $x > 0$ ,  $f'(x) = -4x^2 + 4x + 3$

$f(x)$  is increasing in  $\left(-\frac{1}{2}, \frac{3}{2}\right)$

For  $x \leq 0$ ,  $f'(x) = 3e^x(1+x)$

$f'(x) > 0 \quad \forall x \in (-1, 0)$

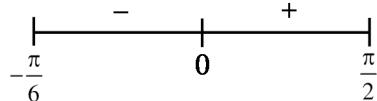
$\Rightarrow f(x)$  is increasing in  $(-1, 0)$

So, in complete domain,  $f(x)$  is increasing in  $\left(-1, \frac{3}{2}\right)$

**2. Official Ans. by NTA (4)**

**Sol.**  $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3, x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$

$$\begin{aligned} f'(x) &= 12\sin^3 x \cos x + 30\sin^2 x \cos x + 12\sin x \cos x \\ &= 6\sin x \cos x (2\sin^2 x + 5\sin x + 2) \\ &= 6\sin x \cos x (2\sin x + 1)(\sin x + 2) \end{aligned}$$



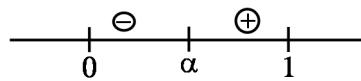
Decreasing in  $\left(-\frac{\pi}{6}, 0\right)$

**3. Official Ans. by NTA (3)**

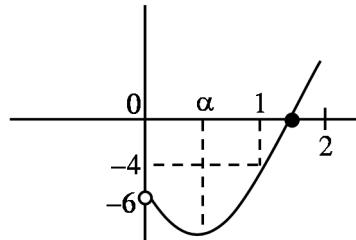
**Sol.** Let  $e^x = t > 0$

$$f(t) = t^4 + 2t^3 - t - 6 = 0$$

$$f'(t) = 4t^3 + 6t^2 - 1$$



$$f''(t) = 12t^2 + 12t > 0$$



$$f(0) = -6, f(1) = -4, f(2) = 24$$

$\Rightarrow$  Number of real roots = 1

**4. Official Ans. by NTA (2)**

**Sol.**  $f(x) = x^2 + ax + 1$

$$f'(x) = 2x + a$$

when  $f(x)$  is increasing on  $[1, 2]$

$$2x + a \geq 0 \quad \forall x \in [1, 2]$$

$$a \geq -2x \quad \forall x \in [1, 2]$$

$$R = -4$$

when  $f(x)$  is decreasing on  $[1, 2]$

$$2x + a \leq 0 \quad \forall x \in [1, 2]$$

$$a \leq -2x \quad \forall x \in [1, 2]$$

$$S = -2$$

$$|R - S| = |-4 + 2| = 2$$

**5. Official Ans. by NTA (2)**

**Sol.**  $f(0) = 0 \quad f(1) = 1$  and  $f(2) = 2$

Let  $h(x) = f(x) - x$  has three roots

By Rolle's theorem  $h'(x) = f'(x) - 1$  has at least two roots

$h''(x) = f''(x) = 0$  has at least one roots

## 6. Official Ans. by NTA (1)

Sol.  $f(x) = x^3 - 6x^2 + ax + b$

$$f(2) = 8 - 24 + 2a + b = 0$$

$$2a + b = 16 \quad \dots(1)$$

$$f(4) = 64 - 96 + 4a + b = 0$$

$$4a + b = 32 \quad \dots(2)$$

Solving (1) and (2)

$$a = 8, b = 0$$

$$\boxed{f(x) = x^3 - 6x^2 + 8x}$$

$$f(x) = x^3 - 6x^2 + 8x$$

$$f'(x) = 3x^2 - 12x + 8$$

$$f''(x) = 6x - 12$$

$\Rightarrow f(x)$  is  $\uparrow$  for  $x > 2$ , and  $f(x)$  is  $\downarrow$  for  $x < 2$

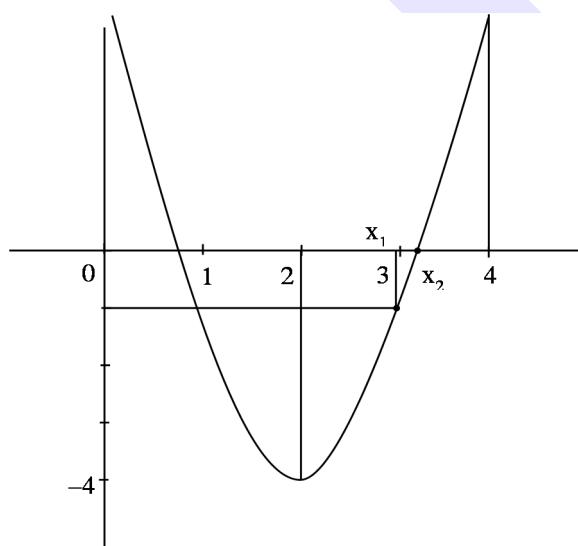
$$f(2) = 12 - 24 + 8 = -4$$

$$f(4) = 48 - 48 + 8 = 8$$

$$f(x) = 3x^2 - 12x + 8$$

vertex  $(2, -4)$

$$f(2) = -4, f(4) = 8, f(3) = 27 - 36 + 8$$



$$f(x_1) = -1, \text{ then } x_1 = 3$$

$$f(x_2) = 0$$

Again  $f(x) < 0$  for  $x \in (2, x_4)$

$f(x) > 0$  for  $x \in (x_4, 4)$

$$x_4 \in (3, 4)$$

$$f(x) = x^3 - 6x^2 + 8x$$

$$f(3) = 27 - 54 + 24 = -3$$

$$f(4) = 64 - 96 + 32 = 0$$

For  $x_4(3, 4)$

$$f(x_4) < -3\sqrt{3}$$

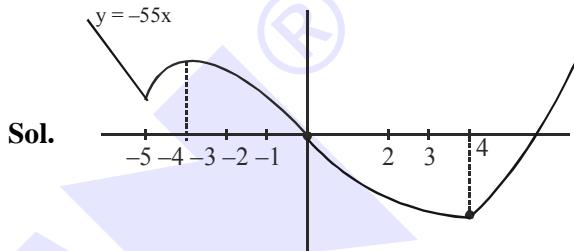
and  $f(x_3) > -4$

$$2f(x_3) > -8$$

$$\text{So, } 2f(x_3) = \sqrt{3} \ f(x_4)$$

Correct Ans. (1)

## 7. Official Ans. by NTA (4)



$$f'(x) = \begin{cases} -55; & x < -5 \\ 6(x-5)(x+4); & -5 < x < 4 \\ 6(x-3)(x+2); & x > 4 \end{cases}$$

$f(x)$  is increasing in

$$x \in (-5, -4) \cup (4, \infty)$$

## 8. Official Ans. by NTA (1)

$$\text{Sol. } f(x) = \frac{4x^3 - 3x^2}{6} - 2 \sin x + (2x-1) \cos x$$

$$f'(x) = (2x^2 - x) - 2 \cos x + 2 \cos x - \sin x(2x-1)$$

$$= (2x-1)(x - \sin x)$$

for  $x > 0, x - \sin x > 0$

$x < 0, x - \sin x < 0$

$$\text{for } x \in (-\infty, 0] \cup \left[ \frac{1}{2}, \infty \right), f'(x) \geq 0$$

$$\text{for } x \in \left[ 0, \frac{1}{2} \right], f'(x) \leq 0$$

$$\Rightarrow f(x) \text{ increases in } \left[ \frac{1}{2}, \infty \right).$$

**9. Official Ans. by NTA (1)**

**Sol.**  $f(1) = f(2)$

$$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$$

$$\Rightarrow 3a - b = 7 \quad \dots\dots(1)$$

Also  $f'(x) = 0$  (given)

$$\Rightarrow (3x^2 - 2ax + b)_{x=\frac{4}{3}} = 0$$

$$\Rightarrow \frac{16}{3} - \frac{8a}{3} + b = 0$$

$$\Rightarrow 8a - 3b - 16 = 0 \quad \dots\dots(2)$$

Solving (1) and (2)

$$a = 5, b = 8$$

**10. Official Ans. by NTA (2)**

**Sol.** Let  $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10 = f(x)$

Now  $f(-2) = -34$  and  $f(-1) = 3$

Hence  $f(x)$  has a root in  $(-2, -1)$

Further  $f'(x) = 10x^4 + 20x^3 + 20x^2 + 20x + 10$

$$= 10x^2 \left[ \left( x^2 + \frac{1}{x^2} \right) + 2 \left( x + \frac{1}{x} \right) + 20 \right]$$

$$= 10x^2 \left[ \left( x + \frac{1}{x} + 1 \right)^2 + 17 \right] > 0$$

Hence  $f(x)$  has only one real root, so  $|a| = 2$

**11. Official Ans by NTA (1)**

**Sol.**  $f(x) = 3\ln(x-1) - 3\ln(x+1) - \frac{2}{x-1}$

$$f'(x) = \frac{3}{x-1} - \frac{3}{x+1} + \frac{2}{(x-1)^2}$$

$$f'(x) = \frac{4(2x-1)}{(x-1)^2(x+1)}$$

$$f'(x) \geq 0$$

$$\Rightarrow x \in (-\infty, -1) \cup \left[ \frac{1}{2}, 1 \right) \cup (1, \infty)$$

**12. Official Ans. by NTA (2)**

**Sol.**  $f(x) = \begin{cases} -x \left( 2 - \sin \left( \frac{1}{x} \right) \right) & x < 0 \\ 0 & x = 0 \\ x \left( 2 - \sin \left( \frac{1}{x} \right) \right) & x > 0 \end{cases}$

$$f'(x) = \begin{cases} -\left( 2 - \sin \frac{1}{x} \right) - x \left( -\cos \frac{1}{x} \cdot \left( -\frac{1}{x^2} \right) \right) & x < 0 \\ \left( 2 - \sin \frac{1}{x} \right) + x \left( -\cos \frac{1}{x} \left( -\frac{1}{x^2} \right) \right) & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} -2 + \sin \frac{1}{x} - \frac{1}{x} \cos \frac{1}{x} & x < 0 \\ 2 - \sin \frac{1}{x} + \frac{1}{x} \cos \frac{1}{x} & x > 0 \end{cases}$$

$f'(x)$  is an oscillating function which is non-monotonic in  $(-\infty, 0) \cup (0, \infty)$ .

**Option (2)**