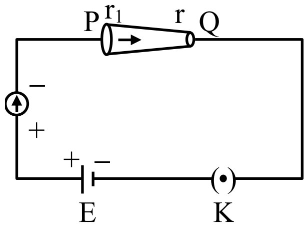
FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 27th July, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. In the given figure, a battery of emf E is connected across a conductor PQ of length 'l' and different area of cross-sections having radii r_1 and r_2 ($r_2 < r_1$).



Choose the correct option as one moves from P to Q :

(1) Drift velocity of electron increases.

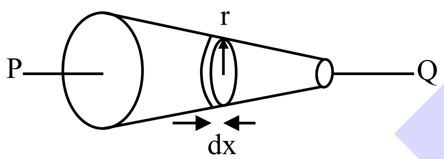
(2) Electric field decreases.

(3) Electron current decreases.

(4) All of these

Official Ans. by NTA (1)

Sol.



Current is constant in conductor

i = constant

Resistance of element $dR = \frac{\rho dx}{\pi r^2}$

$dV = idR = \frac{i\rho dx}{\pi r^2}$

$E = \frac{dV}{dx} = \frac{i\rho}{\pi r^2}$

$\& \ V_d = \frac{eE\tau}{m}$

$\therefore V_d \propto E$

$\rightarrow \quad E \propto \frac{1}{r^2}$

if r decreases , E will increase $\therefore$ V_d will increase

2. The number of molecules in one litre of an ideal gas at 300 K and 2 atmospheric pressure with mean kinetic energy 2×10^{-9} J per molecules is :

(1) 0.75×10^{11} (2) 3×10^{11}

(3) 1.5×10^{11} (4) 6×10^{11}

Official Ans. by NTA (3)

Sol. $KE = \frac{3}{2}kT$

$PV = \frac{N}{N_A}RT$

$N = \frac{PV}{kT}$

$= N = 1.5 \times 10^{11}$

3. The relative permittivity of distilled water is 81. The velocity of light in it will be :

(Given $\mu_r = 1$)

(1) 4.33×10^7 m/s (2) 2.33×10^7 m/s

(3) 3.33×10^7 m/s (4) 5.33×10^7 m/s

Official Ans. by NTA (3)

Sol. $V = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$= 3.33 \times 10^7$ m/sec

4.

List-I	List-II
(a) MI of the rod (length L, Mass M, about an axis ⊥ to the rod passing through the midpoint)	(i) $8 ML^2/3$
(b) MI of the rod (length L, Mass 2M, about an axis ⊥ to the rod passing through one of its end)	(ii) $ML^2/3$
(c) MI of the rod (length 2L, Mass M, about an axis ⊥ to the rod passing through its midpoint)	(iii) $ML^2/12$
(d) MI of the rod (Length 2L, Mass 2M, about an axis ⊥ to the rod passing through one of its end)	(iv) $2 ML^2/3$

Choose the **correct** answer from the options given below :

(1) (a)–(ii), (b)–(iii), (c)– (i), (d)–(iv)

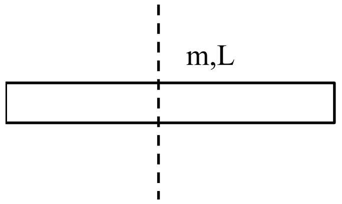
(2) (a)–(ii), (b)–(i), (c)– (iii), (d)–(iv)

(3) (a)–(iii), (b)–(iv), (c)– (ii), (d)–(i)

(4) (a)–(iii), (b)–(iv), (c)– (i), (d)–(ii)

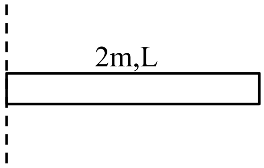
Official Ans. by NTA (3)

Sol. (a)



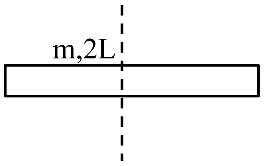
$I = \frac{mL^2}{12}$

(b)



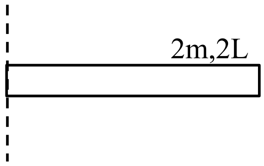
$I = \frac{(2m)(L^2)}{3}$

(c)



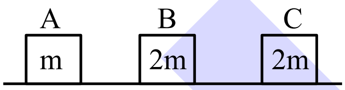
$I = \frac{m(2L)^2}{12} = \frac{mL^2}{3}$

(d)



$I = \frac{2m(2L)^2}{3} = \frac{8}{3}mL^2$

5. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, 2 m and 2 m respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is :

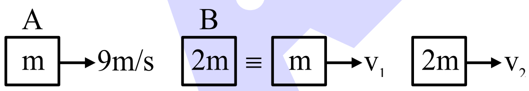


(1) 6 m/s (2) 9 m/s

(3) 4 m/s (4) 3 m/s

Official Ans. by NTA (4)

Sol. Collision between A and B

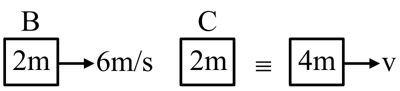


$m \times 9 = mv_1 + 2m\, v_2$ (from momentum conservation)

$e = 1 = \frac{v_2 - v_1}{9}$

$\Rightarrow v_2 = 6$ m/sec. , $v_1 = -3$ m/sec.

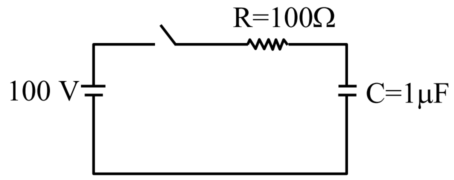
collision between B and C



$2m \times 6 = 4mv$ (from momentum conservation)

$v = 3$ m/s

6.



A capacitor of capacitance C=1 μF is suddenly connected to a battery of 100 volt through a resistance R = 100 Ω. The time taken for the capacitor to be charged to get 50 V is :

[Take ln 2 = 0.69]

(1) 1.44×10^{-4} s (2) 3.33×10^{-4} s

(3) 0.69×10^{-4} s (4) 0.30×10^{-4} s

Official Ans. by NTA (3)

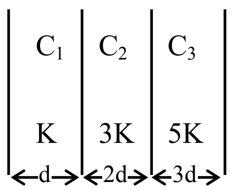
Sol. $V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$

$50 = 100\left(1 - e^{-\frac{t}{RC}}\right)$

$t = 0.69 \times 10^{-4}$ sec.

7. In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be :

(Given area of plate = A)



(1) $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$ (2) $\frac{15}{6}\frac{K\varepsilon_0 A}{d}$

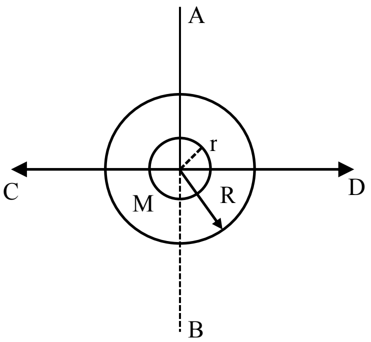
(3) $\frac{25}{6}\frac{K\varepsilon_0 A}{d}$ (4) $\frac{9}{6}\frac{K\varepsilon_0 A}{d}$

Official Ans. by NTA (1)

Sol. $\frac{1}{C_{eff}} = \frac{d}{K\in_0 A} + \frac{2d}{3K\in_0 A} + \frac{3d}{5K\in_0 A}$

$C_{eff} = \frac{15K\in_0 A}{34d}$

8. The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (Which is $\perp$ to the plane of the disc and passing through its centre) of MI of smaller disc around one of its diameters lying on its plane? Given 'M' is the mass of the larger disc. (MI stands for moment of inertia)



(1) $R^2 : r^2$ (2) $2r^4 : R^4$

(3) $2R^2 : r^2$ (4) $2R^4 : r^4$

Official Ans. by NTA (4)

Sol. Ratio of moment of inertia = $\dfrac{\frac{1}{2}MR^2}{\frac{1}{4}mr^2}$

$= \dfrac{2\sigma\pi R^2 R^2}{\sigma\pi r^2 r^2} = \dfrac{2R^4}{r^4}$

9. In Young's double slit experiment, if the source of light changes from orange to blue then :

(1) the central bright fringe will become a dark fringe.

(2) the distance between consecutive fringes will decrease.

(3) the distance between consecutive fringes will increase.

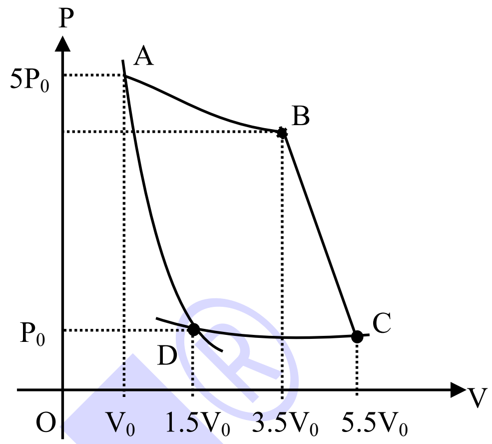
(4) the intensity of the minima will increase.

Official Ans. by NTA (2)

Sol. Fringe width = $\lambda D/d$

as λ decreases , fringe width also decreases

10. In the reported figure, there is a cyclic process ABCDA on a sample of 1 mol of a diatomic gas. The temperature of the gas during the process $A \rightarrow B$ and $C \rightarrow D$ are T_1 and T_2 ($T_1 > T_2$) respectively.



Choose the correct option out of the following for work done if processes BC and DA are adiabatic.

(1) $W_{AB} = W_{DC}$ (2) $W_{AD} = W_{BC}$

(3) $W_{BC} + W_{DA} > 0$ (4) $W_{AB} < W_{CD}$

Official Ans. by NTA (2)

Sol. Work done in adiabatic process = $\dfrac{-nR}{\gamma - 1}(T_f - T_i)$

$\therefore W_{AD} = \dfrac{-nR}{\gamma - 1}(T_2 - T_1)$

and $W_{BC} = \dfrac{-nR}{\gamma - 1}(T_2 - T_1)$

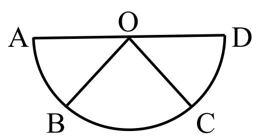
$\therefore W_{AD} = W_{BC}$

11. **Assertion A :** If A, B, C, D are four points on a semi-circular arc with centre at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\,\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

Reason R : Polygon law of vector addition yields

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AD} = 2\,\overrightarrow{AO}$



In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **A** is correct but **R** is not correct.

(2) **A** is not correct but **R** is correct.

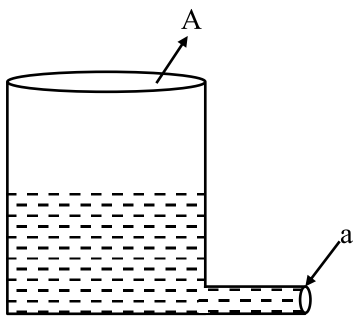
(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

Official Ans. by NTA (4)

Sol. Polygon law is applicable in both but the equation given in the reason is not useful in explaining the assertion.

12. A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a < < A) :



(1) $\frac{A}{2a}$ (2) None of these

(3) $\frac{2a}{A}$ (4) $\frac{a}{A}$

Official Ans. by NTA (3)

Sol. For no sliding

$f \geq \rho a v^2$

$\mu m g \geq \rho a v^2$

$\mu \rho A h g \geq \rho a 2 g h$

$\boxed{\mu \geq \frac{2a}{A}}$

Option (3)

13. A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by :

(1) $y = a$

(2) $y = \frac{a}{\sqrt{2}}$

(3) $y = \frac{a\sqrt{3}}{2}$

(4) $y = \frac{a}{2}$

Official Ans. by NTA (4)

Sol. $E = \frac{1}{2}Ka^2$

$\frac{3E}{4} = \frac{1}{2}K(a^2 - y^2)$

$\frac{3}{4} \times \frac{1}{2}Ka^2 = \frac{1}{2}K(a^2 - y^2)$

$y^2 = a^2 - \frac{3a^2}{4}$

$y = \frac{a}{2}$

14. If '*f*' denotes the ratio of the number of nuclei decayed (N_d) to the number of nuclei at t = 0 (N_0) then for a collection of radioactive nuclei, the rate of change of '*f*' with respect to time is given as :

[λ is the radioactive decay constant]

(1) $-\lambda (1 - e^{-\lambda t})$ (2) $\lambda (1 - e^{-\lambda t})$

(3) $\lambda e^{-\lambda t}$ (4) $-\lambda e^{-\lambda t}$

Official Ans. by NTA (3)

Sol. $N = N_0 e^{-\lambda t}$

$N_d = N_0 - N$

$N_d = N_0 (1 - e^{-\lambda t})$

$\frac{N_d}{N_0} = f = 1 - e^{-\lambda t}$

$\frac{df}{dt} = \lambda e^{-\lambda t}$

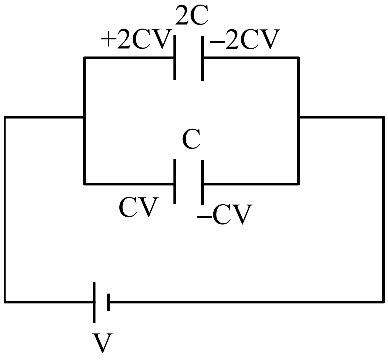
15. Two capacitors of capacities 2C and C are joined in parallel and charged up to potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. The potential difference across the capacitors will now be :

(1) $\frac{V}{K+2}$ (2) $\frac{V}{K}$

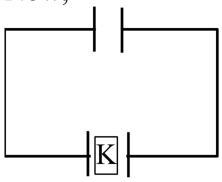
(3) $\frac{3V}{K+2}$ (4) $\frac{3V}{K}$

Official Ans. by NTA (3)

Sol.



Now,



$$V_C = \frac{2CV + CV}{KC + 2C}$$

$$= \frac{3V}{K+2}$$

16. A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

(1) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (2) $\frac{1}{3}$

(3) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Official Ans. by NTA (3)

Sol. $u = \sqrt{2gh}$

Now,

$S = \frac{h}{3}$ $a = -g$

$$S = ut + \frac{1}{2}at^2$$

$$\frac{h}{3} = \sqrt{2gh}\,t + \frac{1}{2}(-g)t^2$$

$$t^2\left(\frac{g}{2}\right) - \sqrt{2gh}\,t + \frac{h}{3} = 0$$

From quadratic equation

$$t_1, t_2 = \frac{\sqrt{2gh} \pm \sqrt{2gh - \frac{4g}{2}\frac{h}{3}}}{g}$$

$$\frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

17. A 0.07 H inductor and a 12 Ω resistor are connected in series to a 220 V, 50 Hz ac source. The approximate current in the circuit and the phase angle between current and source voltage are respectively. [Take π as $\frac{22}{7}$]

(1) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(2) 88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(3) 0.88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(4) 8.8 A and $\tan^{-1}\left(\frac{6}{11}\right)$

Official Ans. by NTA (1)

Sol. $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

$X_L = \omega L$

$X_L = 2\times\frac{22}{7}\times 50\times 0.07 = 22\,\Omega$

$\phi = \tan^{-1}\left(\frac{22}{12}\right)$

$R = 12\,\Omega$

$$\phi = \tan^{-1}\left(\frac{11}{6}\right)$$

$$Z = \sqrt{X_L^2 + R^2} = 25.059$$

$$I = \frac{V}{Z} = \frac{220}{25.059} = 8.77\,A$$

18. Two identical tennis balls each having mass 'm' and charge 'q' are suspended from a fixed point by threads of length 'l'. What is the equilibrium separation when each thread makes a small angle 'θ' with the vertical ?

(1) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{2}}$

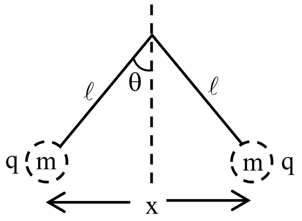
(2) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

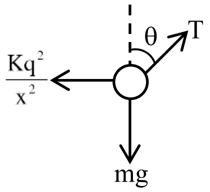
(3) $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g}\right)^{\frac{1}{3}}$

(4) $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{\frac{1}{3}}$

Official Ans. by NTA (2)

Sol.





$T\cos\theta = mg$

$T\sin\theta = \frac{kq^2}{x^2}$

$\tan\theta = \frac{kq^2}{x^2 mg}$

as $\tan\theta \approx \sin\theta \approx \frac{x}{2L}$

$\frac{x}{2L} = \frac{Kq^2}{x^2 mg}$

$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}$

19. **Assertion A :** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R :

Least Count $= \frac{\text{Pitch}}{\text{Total divisions on circular scale}}$

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) **A** is not correct but **R** is correct.

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(3) **A** is correct but **R** is not correct.

(4) Both **A** and **R** are correct and **R** is NOT the correct explanation of **A**.

Official Ans. by NTA (1)

Sol. Least count $= \frac{\text{Pitch}}{\text{total division on circular scale}}$

In 5 revolution, distance travel, 5 mm

In 1 revolution, it will travel 1 mm.

So least count $= \frac{1}{50} = 0.02$

20. A body takes 4 min. to cool from 61° C to 59°C. If the temperature of the surroundings is 30°C, the time taken by the body to cool from 51°C to 49° C is :

(1) 4 min. (2) 3 min.

(3) 8 min. (4) 6 min.

Official Ans. by NTA (4)

Sol. $\frac{\Delta T}{\Delta t} = K(T_t - T_S)$ T_t = average temp.

T_S = surrounding temp.

$\frac{61-59}{4} = K\left(\frac{61+59}{2} - 30\right)$ ....(1)

$\frac{51-49}{t} = K\left(\frac{51+49}{2} - 30\right)$ ....(2)

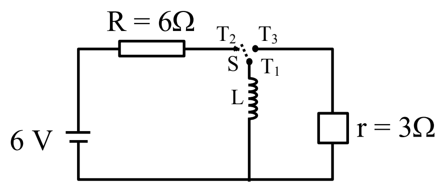
Divide (1) & (2)

$\frac{t}{4} = \frac{60-30}{50-30} = \frac{30}{20}$

so, $t = 6$ minutes

SECTION-B

1. Consider an electrical circuit containing a two way switch 'S'. Initially S is open and then T_1 is connected to T_2. As the current in R = 6 Ω attains a maximum value of steady state level, T_1 is disconnected from T_2 and immediately connected to T_3. Potential drop across r = 3 Ω resistor immediately after T_1 is connected to T_3 is______V. (Round off to the Nearest Integer)



Official Ans. by NTA (3)

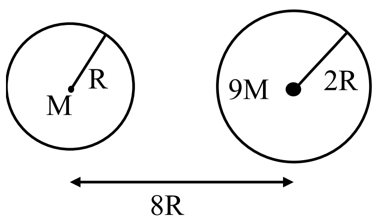
Sol. When T_1 and T_2 are connected, then the steady state current in the inductor $I = \frac{6}{6} = 1A$

When T_1 and T_3 are connected then current through inductor remains same. So potential difference across 3Ω

$V = Ir = 1 \times 3 = 3$ volt

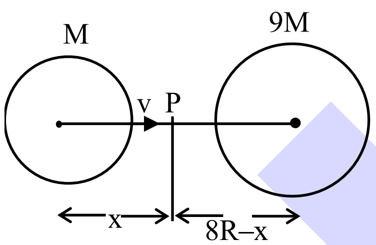
2. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is ________.

[Given : The two planets are fixed in their position]



Official Ans. by NTA (4)

Sol.



Acceleration due to gravity will be zero at P therefore,

$$\frac{GM}{x^2}=\frac{G9M}{(8R-x)^2}$$

$$8R-x=3x$$

$$x=2R$$

Apply conservation of energy and consider velocity at P is zero.

$$\frac{1}{2}mv^2-\frac{GMm}{R}-\frac{G9Mm}{7R}=0-\frac{GMm}{2R}-\frac{G9Mm}{6R}$$

$$\therefore V=\sqrt{\frac{4}{7}\frac{GM}{R}}$$

3. In Bohr's atomic model, the electron is assumed to revolve in a circular orbit of radius 0.5 Å. If the speed of electron is 2.2×16^{6} m/s, then the current associated with the electron will be ___________ $\times 10^{-2}$ mA. [Take π as $\frac{22}{7}$]

Official Ans. by NTA (112)

Sol. $I=\frac{e}{T}=\frac{e\omega}{2\pi}=\frac{eV}{2\pi r}$

$$I=\frac{1.6\times10^{-19}\times2.2\times10^{6}\times7}{2\times22\times0.5\times10^{-10}}$$

$$=1.12\text{ mA}$$

112×10^{-2} mA

4. A radioactive sample has an average life of 30 ms and is decaying. A capacitor of capacitance 200 μF is first charged and later connected with resistor 'R'. If the ratio of charge on capacitor to the activity of radioactive sample is fixed with respect to time then the value of 'R' should be ______Ω.

Official Ans. by NTA (150)

Sol. $T_m=30$ ms

$C=200\ \mu F$

$$\frac{q}{N}=\frac{Q_0e^{-t/RC}}{N_0e^{-\lambda t}}=\frac{Q_0}{N_0}e^{t\left(\lambda-\frac{1}{RC}\right)}$$

Since q/N is constant hence

$$\lambda=\frac{1}{RC}$$

$$R=\frac{1}{\lambda C}=\frac{T_m}{C}=\frac{30\times10^{-3}}{200\times10^{-6}}=150\ \Omega$$

5. A particle of mass 9.1×10^{-31} kg travels in a medium with a speed of 10^{6} m/s and a photon of a radiation of linear momentum 10^{-27} kg m/s travels in vacuum. The wavelength of photon is _______ times the wavelength of the particle.

Official Ans. by NTA (910)

Sol. For photon $\lambda_1 = \dfrac{h}{P} = \dfrac{6.6\times10^{-34}}{10^{-27}}$

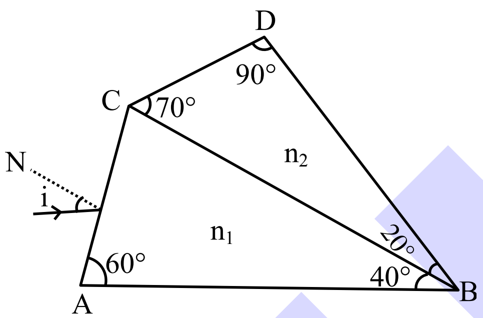
For particle $\lambda_2 = \dfrac{h}{mv} = \dfrac{6.6\times10^{-34}}{9.1\times10^{-31}\times10^{6}}$

$\therefore \dfrac{\lambda_1}{\lambda_2} = 910$

6. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together (as shown in the figure). n_1 and n_2 depend on λ, the wavelength of light, according to the relation

$$n_1 = 1.2 + \frac{10.8\times10^{-14}}{\lambda^2} \text{ and } n_2 = 1.45 + \frac{1.8\times10^{-14}}{\lambda^2}$$

The wavelength for which rays incident at any angle on the interface BC pass through without bending at that interface will be _________ nm.



Official Ans. by NTA (600)

Sol. For no bending, $n_1 = n_2$

$$1.2 + \frac{10.8\times10^{-14}}{\lambda^2} = 1.45 + \frac{1.8\times10^{-4}}{\lambda^2}$$

On solving,

$9 \times 10^{-14} = 25\,\lambda^2$

$\lambda = 6 \times 10^{-7}$

$\lambda = 600$ nm

7. A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is __________ m/s.

(Young's modulus of rubber = 0.5×10^9 N/m^2)

Official Ans. by NTA (20)

Sol. By energy conservation

$$\frac{1}{2}.\frac{YA}{L}.x^2 = \frac{1}{2}mv^2$$

$$\frac{0.5\times10^9\times10^{-6}\times(0.04)^2}{0.1} = \frac{20}{1000}v^2$$

$\therefore\ v^2 = 400$

$\boxed{v = 20\ m/s}$

8. A transistor is connected in common emitter circuit configuration, the collector supply voltage is 10 V and the voltage drop across a resistor of 1000 Ω in the collector circuit is 0.6 V. If the current gain factor (β) is 24, then the base current is _______ μA. (Round off to the Nearest Integer)

Official Ans. by NTA (25)

Sol. $\beta = \dfrac{I_C}{I_B} = 24$; $R_C = 1000$

$\Delta V = 0.6$

$I_C = \dfrac{0.6}{1000}$

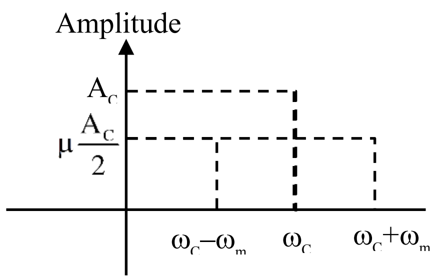
$I_C = 6 \times 10^{-4}$

$$I_B = \frac{I_C}{\beta} = \frac{6\times10^{-4}}{24} = 25\mu A$$

9. The amplitude of upper and lower side bands of A.M. wave where a carrier signal with frequency 11.21 MHz, peak voltage 15 V is amplitude modulated by a 7.7 kHz sine wave of 5V amplitude are $\dfrac{a}{10}$ V and $\dfrac{b}{10}$ V respectively. Then the value of $\dfrac{a}{b}$ is ___________.

Official Ans. by NTA (1)

Sol.



$$\frac{a}{10} = \frac{b}{10} = \frac{\mu A_C}{2}$$

$$\Rightarrow \frac{a}{b} = 1$$

10. In a uniform magnetic field, the magnetic needle has a magnetic moment 9.85×10^{-2} A/m^2 and moment of inertia 5×10^{-6} kg m^2. If it performs 10 complete oscillations in 5 seconds then the magnitude of the magnetic field is ________ mT. [Take π^2 as 9.85]

Official Ans. by NTA (8)

Sol. $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$B = 80 \times 10^{-4} = 8\text{mT}$

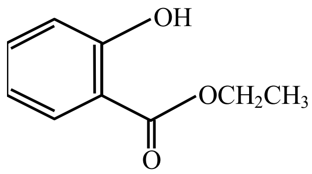
FINAL JEE–MAIN EXAMINATION – JULY, 2021

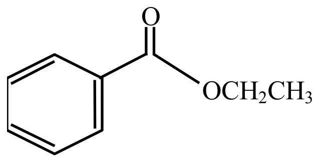
(Held On Tuesday 27th July, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

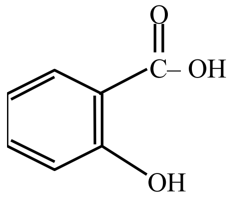
CHEMISTRY **TEST PAPER WITH SOLUTION**

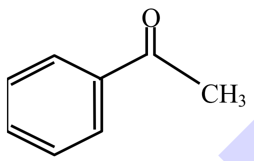
SECTION-A

1. Which one of the following compounds will give orange precipitate when treated with 2,4-dinitrophenyl hydrazine ?

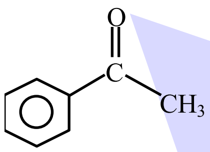
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. 

Explanation $\Rightarrow$ 2-4-D.N.P test is used for carbonyl compound (aldehyde & ketone)

2. The product obtained from the electrolytic oxidation of acidified sulphate solutions, is :

(1) HSO_4^-

(2) HO_3SOOSO_3H

(3) HO_2SOSO_2H

(4) HO_3SOSO_3H

Official Ans. by NTA (2)

Sol. Electrolysis of concentrated solution of acidified sulphate solution yields $H_2S_2O_8$.

3. The parameters of the unit cell of a substance are a = 2.5, b = 3.0, c = 4.0, $\alpha = 90°$, $\beta = 120°$ $\gamma = 90°$. The crystal system of the substance is :

(1) Hexagonal (2) Orthorhombic

(3) Monoclinic (4) Triclinic

Official Ans. by NTA (3)

Sol. $a \neq b \neq c$ and $\alpha = \gamma = 90^o \neq \beta$

are parameters of monoclinic unit cell.

4. The oxidation states of 'P' in $H_4P_2O_7$, $H_4P_2O_5$ and $H_4P_2O_6$, respectively, are :

(1) 7, 5 and 6 (2) 5, 4 and 3

(3) 5, 3 and 4 (4) 6, 4 and 5

Official Ans. by NTA (3)

Sol. Oxidation state of P in $H_4P_2O_7$, $H_4P_2O_5$ and $H_4P_2O_6$ is 5, 3 & 4 respectively

$H_4P_2O_7$

$2x + 4(+1) + 7(-2) = 0$

$x = +5$

$H_4\underline{P_2}O_5$

$2x + 4(+1) + 5(-2) = 0$

$x = +3$

$H_4\underline{P_2}O_6$

$2x + 4(+1) + 6(-2) = 0$

$x = +4$

5. For a reaction of order n, the unit of the rate constant is :

(1) $mol^{1-n}\,L^{1-n}\,s$ (2) $mol^{1-n}\,L^{2n}\,s^{-1}$

(3) $mol^{1-n}\,L^{n-1}s^{-1}$ (4) $mol^{1-n}\,L^{1-n}\,s^{-1}$

Official Ans. by NTA (3)

Sol. Rate = $k[A]^n$

comparing units

$$\frac{(mol/\ell)}{sec} = k\left(\frac{mol}{\ell}\right)^n$$

$\Rightarrow k = mol^{(1-n)}\,\ell^{(n-1)}\,s^{-1}$

6. Given below are two statements :

Statement I : Aniline is less basic than acetamide.

Statement II : In aniline, the lone pair of electrons on nitrogen atom is delocalised over benzene ring due to resonance and hence less available to a proton.

Choose the **most appropriate** option ;

(1) Statement I is true but statement II is false.

(2) Statement I is false but statement II is true.

(3) Both statement I and statement II are true.

(4) Both statement I and statement II are false.

Official Ans. by NTA (2)

Sol. Explanation :- aniline is more basic than acetamide because in acetamide, lone pair of nitrogen is delocalised to more electronegative element oxygen.

In Aniline lone pair of nitrogen delocalised over benzene ring.

7. The type of hybridisation and magnetic property of the complex $[MnCl_6]^{3-}$, respectively, are :

(1) sp^3d^2 and diamagnetic

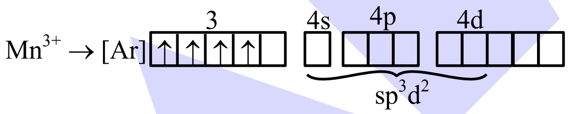
(2) d^2sp^3 and diamagnetic

(3) d^2sp^3 and paramagnetic

(4) sp^3d^2 and paramagnetic

Official Ans. by NTA (4)

Sol. $[MnCl_6]^{3-}$



Paramagnetic and having 4 unpaired electrons.

8. The number of geometrical isomers found in the metal complexes $[PtCl_2(NH_3)_2]$, $[Ni(CO)_4]$, $[Ru(H_2O)_3Cl_3]$ and $[CoCl_2(NH_3)_4]^+$ respectively, are :

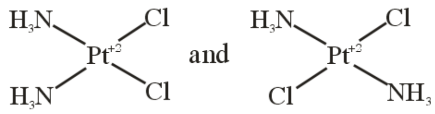
(1) 1, 1, 1, 1 (2) 2, 1, 2, 2

(3) 2, 0, 2, 2 (4) 2, 1, 2, 1

Official Ans. by NTA (3)

Sol.

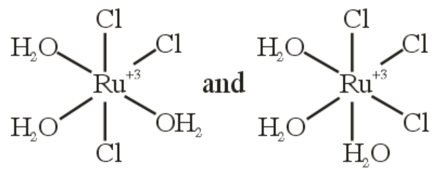
$[PtCl_2(NH_3)_2]$



2 Geometrical isomers

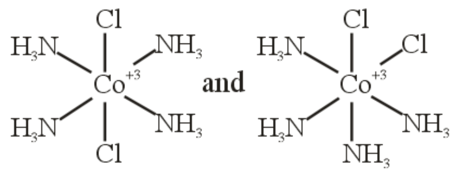
$[Ni(CO)_4] \rightarrow$ All ligands are same Zero Geometrical isomers

$[Ru(H_2O)_3Cl_3]$



2 Geometrical isomers

$[CoCl_2(NH_3)_4]^+$



2 Geometrical isomers

9. Which one of the following statements is **NOT** correct ?

(1) Eutrophication indicates that water body is polluted ?

(2) The dissolved oxygen concentration below 6 ppm inhibits fish growth

(3) Eutrophication leads to increase in the oxygen level in water

(4) Eutrophication leads to anaerobic conditions

Official Ans. by NTA (3)

Sol. Eutrophication leads to decrease in oxygen level of water.

3rd statement is incorrect

10. Given below are two statements :

Statement I : Rutherford's gold foil experiment cannot explain the line spectrum of hydrogen atom.

Statement II : Bohr's model of hydrogen atom contradicts Heisenberg's uncertainty principle.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is false but **statement II** is true.

(2) **Statement I** is true but **statement II** is false.

(3) Both **statement I** and **statement II** are false.

(4) Both **statement I** and **statement II** are true.

Official Ans. by NTA (4)

Sol. Rutherford's gold foil experiment only proved that electrons are held towards nucleus by electrostatic forces of attraction and move in circular orbits with very high speeds.

Bohr's model gave exact formula for simultaneous calculation of speed & distance of electron from the nucleus, something which was deemed impossible according to Heisenberg.

11. Presence of which reagent will affect the reversibility of the following reaction, and change it to a irreversible reaction :

$$CH_4 + I_2 \underset{\text{Reversible}}{\overset{hv}{\rightleftharpoons}} CH_3 - I + HI$$

(1) HOCl

(2) dilute HNO_2

(3) Liquid NH_3

(4) Concentrated HIO_3

Official Ans. by NTA (4)

Sol. Iodination of alkane is reversible reaction.

It can be irreversible in the presence of strong oxidising agent like conc. HNO_3 or conc. HIO_3

12. Which one among the following chemical tests is used to distinguish monosaccharide from disaccharide ?

(1) Seliwanoff's test

(2) Iodine test

(3) Barfoed test

(4) Tollen's test

Official Ans. by NTA (3)

Sol. Barford test is used for distinguish mono-saccharide from disaccharide

13. Match **List-I** with **List-II** :

List-I (Drug)	**List-II (Class of Drug)**
(a) Furacin	(i) Antibiotic
(b) Arsphenamine	(ii) Tranquilizers
(c) Dimetone	(iii) Antiseptic
(d) Valium	(iv) Synthetic antihistamines

Choose the **most appropriate** match :

(1) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(2) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(3) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Official Ans. by NTA (4)

Sol. → furacine acts as Antiseptic

→ Arsphenamine also known as salvarsan acts as antibiotic

→ Dimetone is synthetic histamine

→ valium is a Tranqulizer

14. The statement that is INCORRECT about Ellingham diagram is

(1) provides idea about the reaction rate.

(2) provides idea about free energy change.

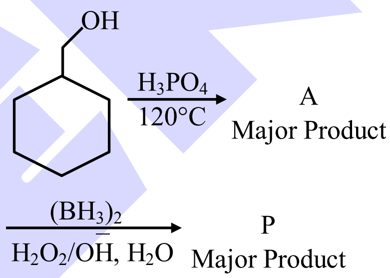
(3) provides idea about changes in the phases during the reaction.

(4) provides idea about reduction of metal oxide.

Official Ans. by NTA (1)

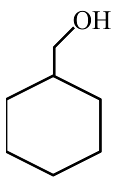
Sol. Ellingham diagram is a plot between ΔG° and T and does not give any information regarding rate of reaction

15.

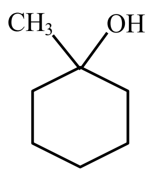


Consider the above reaction and identify the Product P :

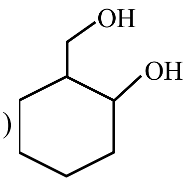
(1)



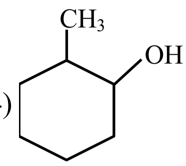
(2)



(3)

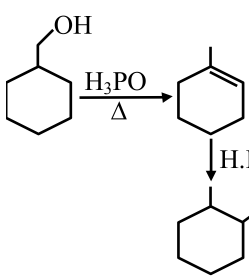


(4)



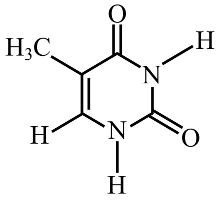
Official Ans. by NTA (4)

Sol.



dehedration of alcohol by E^1 Rxn

H-attached at more hindard site while OH attached at less hindard site in H.BO. Reaction

16. 

(A)

The compound 'A' is a complementary base of __________ in DNA stands.

(1) Uracil (2) Guanine

(3) Adenine (4) Cytosine

Official Ans. by NTA (3)

Sol. Given structure is Thymine and Thymine being paired with adenine

17. Staggered and eclipsed conformers of ethane are :

(1) Polymers (2) Rotamers

(3) Enantiomers (4) Mirror images

Official Ans. by NTA (2)

Sol. Staggered and eclipsed conformers of ethane also known as rotamers

18. Match **List - I** with **List - II** :

List - I	List - II
(a) NaOH	(i) Acidic
(b) $Be(OH)_2$	(ii) Basic
(c) $Ca(OH)_2$	(iii) Amphoteric
(d) $B(OH)_3$	
(e) $Al(OH)_3$	

Choose the **most appropriate** answer from the options given below

(1) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(ii), (e)-(iii)

(2) (a)-(ii), (b)-(iii), (c)-(ii), (d)-(i), (e)-(iii)

(3) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(i), (e)-(iii)

(4) (a)-(ii), (b)-(i), (c)-(ii), (d)-(iii), (e)-(iii)

Official Ans. by NTA (2)

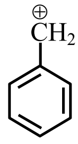
Sol. NaOH $\rightarrow$ Basic

$Be(OH)_2 \rightarrow$ Amphoteric

$Ca(OH)_2 \rightarrow$ Basic

$B(OH)_3 \rightarrow$ Acidic

$Al(OH)_3 \rightarrow$ Amphoteric

19.  $CH_2{=}\overset{\oplus}{C}H$ $CH_3{-}\overset{\oplus}{C}H_2$ $HC{\equiv}\overset{\oplus}{C}$

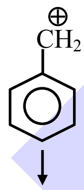
A B C D

The correct order of stability of given carbocation is :

(1) A > C > B > D (2) D >B > C > A

(3) D > B > A > C (4) C > A > D > B

Official Ans. by NTA (1)

Sol.  $> CH_3{-}\overset{\oplus}{C}H_2 > CH_2 = \overset{\oplus}{C}H > CH{\equiv}\overset{\oplus}{C}$

+ve charge stable on less electronegative elements

Stable due to Resonance

20. Given below are two statements : One is labelled as **Assertion A** and the other labelled as **Reason R.**

Assertion A : Lithium halides are some what covalent in nature.

Reason R : Lithium possess high polarisation capability.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) **A** is true but **R** is false

(2) **A** is false but **R** is true

(3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Official Ans. by NTA (4)

Sol. Lithium due to small size has very high polarization capability and thus increases covalent nature in Halides.

SECTION-B

1. The density of NaOH solution is 1.2 g cm^{-3}. The molality of this solution is ________ m.

(Round off to the Nearest Integer)

[Use : Atomic masses : Na : 23.0 u O : 16.0 u H : 1.0 u

Density of H_2O : 1.0 g cm^{-3}]

Official Ans. by NTA (5)

Sol. Consider 1ℓ solution

mass of solution = (1.2 × 1000)g

= 1200 gm

Neglecting volume of NaOH

Mass of water = 1000 gm

⇒ Mass of NaOH = (1200 – 1000)gm

= 200 gm

$$\Rightarrow \text{Moles of NaOH} = \frac{200\text{g}}{50\text{g}/\text{mol}} = 5 \text{ mol}$$

$$\Rightarrow \text{molality} = \frac{5\,\text{mol}}{1\text{kg}} = 5 \text{ m}$$

2. CO_2 gas adsorbs on charcoal following Freundlich adsorption isotherm. For a given amount of charcoal, the mass of CO_2 adsorbed becomes 64 times when the pressure of CO_2 is doubled.

The value of n in the Freundlich isotherm equation is _________ × 10^{-2}. (Round off to the Nearest Integer)

Official Ans. by NTA (17)

Sol. Freundlich isotherm. ;

$$\frac{x}{m} = k.p^{\frac{1}{n}}$$

Substituting values ;

$$\left(\frac{64}{1}\right) = (2)^{\frac{1}{n}} \Rightarrow n = \frac{1}{6} = 0.166$$

$$\simeq 17 \times 10^{-2}$$

3. The conductivity of a weak acid HA of concentration 0.001 mol L^{-1} is 2.0 × 10^{-5} S cm^{-1}. If $\Lambda_m^{o}(HA) = 190 \text{ S cm}^2 \text{ mol}^{-1}$, the ionization constant (K_a) of HA is equal to _________ × 10^{-6}.

(Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol.
$$\Lambda_m = 1000 \times \frac{\kappa}{M}$$

$$= 1000 \times \frac{2\times10^{-5}}{0.001} = 20 \text{ S cm}^2 \text{ mol}^{-1}$$

$$\Rightarrow \alpha = \frac{\Lambda_m}{\Lambda_m^{\infty}} = \frac{20}{190} = \left(\frac{2}{19}\right)$$

$$HA \rightleftharpoons H^+ + A^-$$

0.001 (1– α) 0.001α 0.001 α

$$\Rightarrow k_a = 0.001\left(\frac{\alpha^2}{1-\alpha}\right) = \frac{0.001\times\left(\frac{2}{19}\right)^2}{1-\left(\frac{2}{19}\right)}$$

$$= 12.3 \times 10^{-6}$$

4. 1.46 g of a biopolymer dissolved in a 100 mL water at 300 K exerted an osmotic pressure of 2.42 × 10^{-3} bar.

The molar mass of the biopolymer is _____ × 10^4 g mol^{-1}. (Round off to the Nearest Integer)

[Use : R = 0.083 L bar mol^{-1} K^{-1}]

Official Ans. by NTA (15)

Sol. π = CRT ; π = osmotic pressure

C = molarity

T = Temperature of solution

let the molar mass be M gm / mol

2.42 × 10^{-3} bar =

$$\frac{\left(\frac{1.46\text{g}}{\text{Mgm}/\text{mol}}\right)}{0.1\ell} \times \left(\frac{0.083\ell-\text{bar}}{\text{mol}-\text{K}}\right) \times (300\text{K})$$

⇒ M = 15.02 × 10^4 g/mol

5. An organic compound is subjected to chlorination to get compound A using 5.0 g of chlorine. When 0.5 g of compound A is reacted with $AgNO_3$ [Carius Method], the percentage of chlorine in compound A is _______ when it forms 0.3849 g of AgCl. (Round off to the Nearest Integer)

(Atomic masses of Ag and Cl are 107.87 and 35.5 respectively)

Official Ans. by NTA (19)

Sol. $n_{c\ell}$ in compound $= n_{AgCl} = \dfrac{0.3849g}{(107.87+35.5)}$ g/mol

$\Rightarrow$ mass of chlorine $= n_{Cl} \times 35.5 = 0.0953$ gm

$\Rightarrow$ % wt of chlorine $= \dfrac{0.0953}{0.5}\times 100$

$= 19.06\%$

OR

Sol. Mass of organic compound = 0.5 gm.

mass of formed AgCl = 0.3849 gm

$$\%\text{of Cl} = \frac{\text{atomic mass of Cl}\times\text{ mass formed AgCl}}{\text{molecular mass of AgCl}\times\text{mass of organic compound}}\times 100$$

$$= \frac{35.5\times 0.3849}{143.37\times 0.5}\times 100$$

$= 19.06$

$\boxed{\approx 19}$

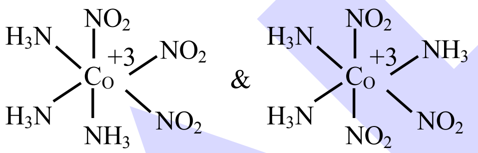
6. The number of geometrical isomers possible in triamminetrinitrocobalt (III) is X and in trioxalatochromate (III) is Y. Then the value of X + Y is _______.

Official Ans. by NTA (2)

Sol. Triamminetrinitrocobalt(III) $\rightarrow [Co(NO_2)_3(NH_3)_3]$

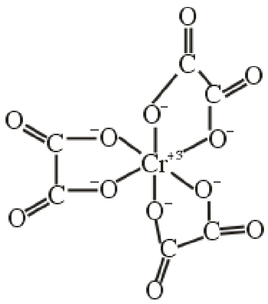
trioxalatochromate(III) ion $\rightarrow [Cr(C_2O_4)_3]^{3-}$

$[Co(NO_2)_3(NH_3)_3]$



Two geometrical isomers (X)

$[Cr(C_2O_4)_3]^{3-}$



Zero geometrical isomer (Y)

$X + Y = 2 + 0 = 2.0$

7. In gaseous triethyl amine the "–C–N–C–" bond angle is ________ degree.

Official Ans. by NTA (108)

Sol. In gaseous triethyl amine the "–C–N–C–" bond angle is 108 degree.

8. For water at 100°C and 1 bar,

$\Delta_{vap}H - \Delta_{vap}U =$ ________ $\times 10^2$ J mol^{-1}.

(Round off to the Nearest Integer)

[Use : R=8.31 J mol^{-1} K^{-1}]

[Assume volume of $H_2O(l)$ is much smaller than volume of $H_2O(g)$. Assume $H_2O(g)$ treated as an ideal gas]

Official Ans. by NTA (31)

Sol. $H_2O_{(\ell)} \rightleftharpoons H_2O_{(V)}$

$\Delta H = \Delta U + \Delta n_g RT$

for 1 mole waters ;

$\Delta n_g = 1$

$\therefore \Delta n_g RT = 1$ mol $\times$ 8.31 J/mol-k $\times$ 373 K

$= 3099.63$ J $\cong 31 \times 10^2$ J

9. $PCl_5 \rightleftharpoons PCl_3 + Cl_2$ $\quad K_c = 1.844$

3.0 moles of PCl_5 is introduced in a 1 L closed reaction vessel at 380 K. The number of moles of PCl_5 at equilibrium is _______ $\times 10^{-3}$.

(Round off to the Nearest Integer)

Official Ans. by NTA (1396)

Sol. $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$ $\quad K_2 = 1.844$

t = 0 3 moles

t = ∞ x x

$$\Rightarrow \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{x^2}{3-x} = 1.844$$

$\Rightarrow x^2 + 1.844 - 5.532 = 0$

$$\Rightarrow x = \frac{-1.844+\sqrt{(1.844)^2+4\times 5.532}}{2}$$

$\cong 1.604$

$\Rightarrow$ Moles of $PCl_5 = 3 - 1.604 \cong 1.396$

10. The difference between bond orders of CO and $NO^{\oplus}$ is $\dfrac{x}{2}$ where x = _______.

(Round off to the Nearest Integer)

Official Ans. by NTA (0)

Sol. Bond order of CO = 3

Bond order of $NO^+ = 3$

Difference $= 0 = \dfrac{x}{2}$

$x = 0$

FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 27th July, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. If the mean and variance of the following data:
6, 10, 7, 13, a, 12, b, 12
are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to:

(1) 24 (2) 12 (3) 32 (4) 16

Official Ans. by NTA (4)

Sol. Mean $=\frac{6+10+7+13+a+12+b+12}{8}=9$

$60 + a + b = 72$

$a + b = 12$ …(1)

variance $=\frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2=\frac{37}{4}$

$\Sigma x_i^2=6^2+10^2+7^2+13^2+a^2+b^2+12^2+12^2$

$=a^2+b^2+642$

$\frac{a^2+b^2+642}{8}-(9)^2=\frac{37}{4}$

$\frac{a^2+b^2}{8}+\frac{321}{4}-81=\frac{37}{4}$

$\frac{a^2+b^2}{8}=81+\frac{37}{4}-\frac{321}{4}$

$\frac{a^2+b^2}{8}=81-71$

$\therefore a^2+b^2=80$ …(2)

From (1) $a^2+b^2+2ab=144$

$80+2ab=144$ $\therefore 2ab=64$

$(a-b)^2=a^2+b^2-2ab=80-64=16$

2. The value of $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{j=1}^{n}\frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

(1) $5+\log_e\left(\frac{3}{2}\right)$ (2) $2-\log_e\left(\frac{2}{3}\right)$

(3) $3+2\log_e\left(\frac{2}{3}\right)$ (4) $1+2\log_e\left(\frac{3}{2}\right)$

Official Ans. by NTA (4)

Sol. $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{j=1}^{n}\frac{\left(\frac{2j}{n}-\frac{1}{n}+8\right)}{\left(\frac{2j}{n}-\frac{1}{n}+4\right)}$

$\int_0^1\frac{2x+8}{2x+4}dx=\int_0^1 dx+\int_0^1\frac{4}{2x+4}dx$

$=1+4\frac{1}{2}(\ell n|2x+4|)\Big|_0^1$

$=1+2\ell n\left(\frac{3}{2}\right)$

3. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product $\left(\vec{a}+\vec{b}\right)\times\left(\left(\vec{a}\times\left(\left(\vec{a}-\vec{b}\right)\times\vec{b}\right)\right)\times\vec{b}\right)$ is equal to :

(1) $5\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$ (2) $7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

(3) $7\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$ (4) $5\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$

Official Ans. by NTA (2)

Sol. $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

$\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}+\vec{b}=3\hat{j}+5\hat{k}$; $\vec{a}.\vec{b}=-1+2+6=7$

$\left(\left(\vec{a}\times\left(\left(\vec{a}-\vec{b}\right)\times\vec{b}\right)\right)\times\vec{b}\right)$

$\left(\left(\vec{a}\times\left(\vec{a}\times\vec{b}-\vec{b}\times\vec{b}\right)\right)\times\vec{b}\right)$

$\left(\vec{a}\times\left(\vec{a}\times\vec{b}-0\right)\right)\times\vec{b}$

$\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\times\vec{b}$

$\left((\vec{a}.\vec{b})\vec{a}-(\vec{a}.\vec{a})\vec{b}\right)\times\vec{b}$

$(\vec{a}.\vec{b})\vec{a}\times\vec{b}-(\vec{a}.\vec{a})(\vec{b}\times\vec{b})$

$(\vec{a}.\vec{b})(\vec{a}\times\vec{b})$

$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k\\ 1 & 1 & 2\\ -1 & 2 & 3\end{vmatrix}=-\hat{i}-5\hat{j}+3\hat{k}$

$\therefore 7\left(-\hat{i}-5\hat{j}+3\hat{k}\right)$

$\left(\vec{a}+\vec{b}\right)\times\left(7\left(-\hat{i}-5\hat{j}+3\hat{k}\right)\right)$

$7\left(0\hat{i}+3\hat{j}+5\hat{k}\right)\times\left(-\hat{i}-5\hat{j}+3\hat{k}\right)$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k}\\ 0 & 3 & 5\\ -1 & -5 & 3\end{vmatrix}$

$\Rightarrow 34\hat{i}-(5)\hat{j}+(3\hat{k})$

$\Rightarrow 34\hat{i}-5\hat{j}+3\hat{k}$

$\therefore 7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

4. The value of the definite integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$$

is equal to :

(1) $-\frac{\pi}{2}$ (2) $\frac{\pi}{2\sqrt{2}}$ (3) $-\frac{\pi}{4}$ (4) $\frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (2)

Sol. $I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$ ...(1)

Using $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$$I=\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{-x\cos x})(\sin^4 x+\cos^4 x)}$$

Add (1) and (2)

$$2I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\sin^4 x+\cos^4 x}$$

$$2I=2\int_0^{\frac{\pi}{4}} \frac{dx}{\sin^4 x+\cos^4 x}$$

$$I=\int_0^{\frac{\pi}{4}} \frac{(1+\tan^2 x)\sec^2 x}{\tan^4 x+1}dx$$

$$I=\int_0^{\frac{\pi}{4}} \frac{\left(1+\frac{1}{\tan^2 x}\right)\sec^2 x}{\left(\tan x-\frac{1}{\tan x}\right)^2+2}dx$$

$$\tan x-\frac{1}{\tan x}=t$$

$$\left(1+\frac{1}{\tan^2 x}\right)\sec^2 x\,dx=dt$$

$$I=\int_{-\infty}^{0} \frac{dt}{t^2+2}=\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_{-\infty}^{0}$$

$$I=0-\frac{1}{\sqrt{2}}\left(-\frac{\pi}{2}\right)=\frac{\pi}{2\sqrt{2}}$$

5. Let C be the set of all complex numbers. Let

$S_1=\{z\in C \mid |z-3-2i|^2=8\}$,

$S_2=\{z\in C \mid \text{Re}(z)\ge 5\}$ and

$S_3=\{z\in C \mid |z-\bar{z}|\ge 8\}$.

Then the number of elements in $S_1\cap S_2\cap S_3$ is equal to

(1) 1 (2) 0 (3) 2 (4) Infinite

Official Ans. by NTA (1)

Sol. $S_1: |z-3-2i|^2=8$

$|z-3-2i|=2\sqrt{2}$

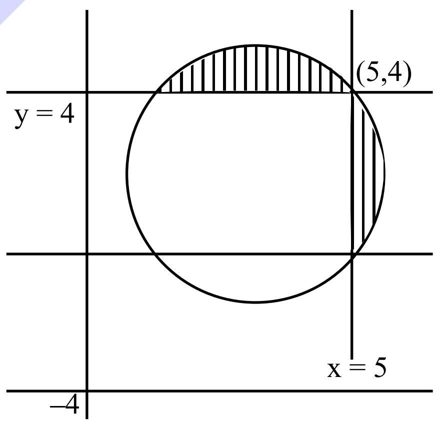
$(x-3)^2+(y-2)^2=(2\sqrt{2})^2$

$S_2: x\ge 5$

$S_3: |z-\bar{z}|\ge 8$

$|2iy|\ge 8$

$2|y|\ge 8 \therefore y\ge 4, y\le -4$



$n(S_1\cap S_2\cap S_3)=1$

6. If the area of the bounded region

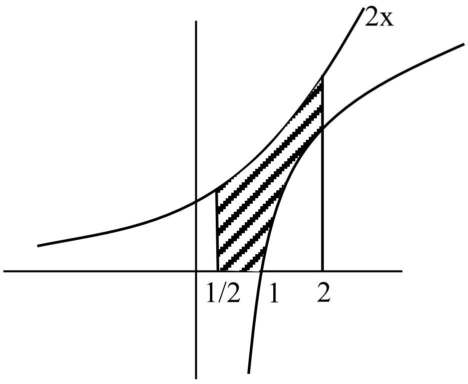
$$R=\left\{(x,y): \max\{0,\log_e x\}\le y\le 2^x, \frac{1}{2}\le x\le 2\right\}$$

is, $\alpha(\log_e 2)^{-1}+\beta(\log_e 2)+\gamma$, then the value of $(\alpha+\beta-2\gamma)^2$ is equal to :

(1) 8 (2) 2 (3) 4 (4) 1

Official Ans. by NTA (2)

Sol. $R=\left\{(x,y):\max\left(0,\log_e x\right)\le y\le 2^x,\frac{1}{2}\le x\le 2\right\}$



$\int_{\frac{1}{2}}^{2} 2^x dx-\int_{1}^{2}\ell nx\,dx$

$\Rightarrow \left[\frac{2^x}{\ln 2}\right]_{1/2}^{2}-[x\ln x-x]_1^2$

$\Rightarrow \frac{(2^2)-2^{1/2}}{\log_e 2}-(2\ln 2-1)$

$\Rightarrow \frac{\left(2^2-\sqrt{2}\right)}{\log_e 2}-2\ln 2+1$

$\therefore\ \alpha=2^2-\sqrt{2},\ \beta=-2,\gamma=1$

$\Rightarrow(\alpha+\beta+2\gamma)^2$

$\Rightarrow(2^2-\sqrt{2}-2-2)^2$

$\Rightarrow(\sqrt{2})^2=2$

7. A ray of light through (2,1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be:

(1) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$

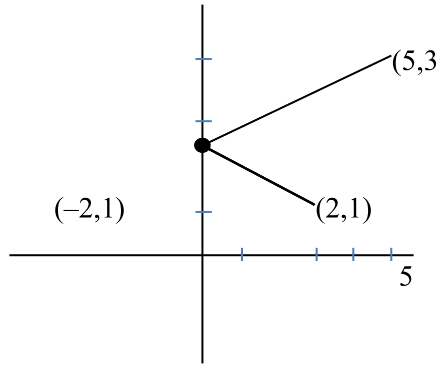
(2) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

(3) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

(4) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

Official Ans. by NTA (3)

Sol.



Equation of reflected Ray

$y-1=\frac{2}{7}(x+2)$

$7y-7=2x+4$

$2x-7y+11=0$

Let the equation of other directrix is

$2x-7y+\lambda$

Distance of directrix from Focub

$\frac{a}{e}-ae=\frac{8}{\sqrt{53}}$

$3a-\frac{a}{3}=\frac{8}{\sqrt{53}}$ or $a=\frac{3}{\sqrt{53}}$

Distance from other focus $\frac{a}{e}+ae$

$3a+\frac{a}{3}=\frac{10a}{3}=\frac{10}{3}\times\frac{3}{\sqrt{53}}=\frac{10}{\sqrt{53}}$

Distance between two directrix $=\frac{2a}{e}$

$=2\times 3\times\frac{3}{\sqrt{53}}=\frac{18}{\sqrt{53}}$

$\left|\frac{\lambda-11}{\sqrt{53}}\right|=\frac{18}{\sqrt{53}}$

$\lambda - 11 = 18$ or -18

$\lambda = 29$ or -7

$2x - 7y - 7 = 0$ or $2x - 7y + 29 = 0$

8. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b\ne 0$, are equal, then the value of b is equal to:

(1) 2 (2) –1 (3) 1 (4) –2

Official Ans. by NTA (3)

Sol. Coefficient of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$

$^{11}C_r(x^2)^{11-r}.\left(\frac{1}{bx}\right)^r$

$^{11}C_r x^{22-3r}.\frac{1}{b^r}$

$22-3r=7$

$r=5$

$\therefore\ ^{11}C_5.\frac{1}{b^5}.x^7$

Coefficient of x^{-7} in $\left(x-\frac{b}{bx^2}\right)^{11}$

$^{11}C_r(x)^{11-r}.\left(-\frac{1}{bx^2}\right)^r$

$^{11}C_r x^{11-3r}.\frac{(-1)^r}{b^r}$

$11-3r=-7\ \therefore\ r=6$

$^{11}C_6.\frac{1}{b^6}x^{-7}$

$^{11}C_5.\frac{1}{b^5}={}^{11}C_6.\frac{1}{b^6}$

Since $b\neq 0\ \therefore\ b=1$

9. The compound statement $(P\vee Q)\wedge(\sim P)\Rightarrow Q$ is equivalent to:

(1) $P\vee Q$ (2) $P\wedge\sim Q$

(3) $\sim(P\Rightarrow Q)$ (4) $\sim(P\Rightarrow Q)\Leftrightarrow P\wedge\sim Q$

Official Ans. by NTA (4)

Sol. Using Truth Table

P	Q	$P\vee Q$	$\sim P$	$(P\vee Q)\wedge P$	$(P\vee Q)\wedge\sim P\to Q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

P	Q	$\sim Q$	$P\wedge\sim Q$	$P\to Q$	$\sim(P\to Q)$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	F	F	T	F
F	F	T	F	T	F

$\sim(P\to Q)$	$P\wedge\sim Q$	$\sim(P\to Q)\Leftrightarrow P\wedge\sim Q$
F	F	T
T	T	T
F	F	T
F	F	T

10. If $\sin\theta+\cos\theta=\frac{1}{2}$, then $16(\sin(2\theta)+\cos(4\theta)+\sin(6\theta))$ is equal to:

(1) 23 (2) –27 (3) –23 (4) 27

Official Ans. by NTA (3)

Sol. $\sin\theta+\cos\theta=\frac{1}{2}$

$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{1}{4}$

$\sin 2\theta=-\frac{3}{4}$

Now :

$\cos 4\theta=1-2\sin^2 2\theta$

$=1-2\left(-\frac{3}{4}\right)^2$

$=1-2\times\frac{9}{16}=-\frac{1}{8}$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$=(3-4\sin^2 2\theta).\sin 2\theta$

$=\left[3-4\left(\frac{9}{16}\right)\right].\left(-\frac{3}{4}\right)$

$\Rightarrow\left[\frac{3}{4}\right]\times\left(-\frac{3}{4}\right)=-\frac{9}{16}$

$16[\sin 2\theta+\cos 4\theta+\sin 6\theta]$

$16\left(-\frac{3}{4}-\frac{1}{8}-\frac{9}{16}\right)=-23$

11. Let $A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$. If $A^{-1}=\alpha I+\beta A$, $\alpha,\beta\in\mathbf{R}$, I is a 2×2 identity matrix, then $4(\alpha-\beta)$ is equal to :

(1) 5 (2) $\frac{8}{3}$ (3) 2 (4) 4

Official Ans. by NTA (4)

Sol. $A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$, $|A|=6$

$A^{-1}=\frac{adjA}{|A|}=\frac{1}{6}\begin{bmatrix}4 & -2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}\frac{2}{3} & -\frac{1}{3}\\ \frac{1}{6} & \frac{1}{6}\end{bmatrix}$

$\begin{bmatrix}\frac{2}{3} & -\frac{1}{3}\\ \frac{1}{6} & \frac{1}{6}\end{bmatrix}=\begin{bmatrix}\alpha & 0\\ 0 & \alpha\end{bmatrix}+\begin{bmatrix}\beta & 2\beta\\ -\beta & 4\beta\end{bmatrix}$

$\left.\begin{aligned}\alpha+\beta&=\frac{2}{3}\\ \beta&=-\frac{1}{6}\end{aligned}\right\}\Rightarrow\alpha=\frac{2}{3}+\frac{1}{6}=\frac{5}{6}$

$4(\alpha-\beta)=4(1)=4$

12. Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}} & , \quad -\frac{\pi}{4}<x<0\\ b & , \quad x=0\\ e^{\cot 4x/\cot 2x} & , \quad 0<x<\frac{\pi}{4}\end{cases}$$

If f is continuous at x = 0, then the value of $6a + b^2$ is equal to:

(1) 1 – e (2) e –1 (3) 1 + e (4) e

Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to 0} f(x)=b$

$$\lim_{x\to 0^+} xe^{\frac{\cot 4x}{\cot 2x}}=e^{\frac{1}{2}}=b$$

$$\lim_{x\to 0^-}(1+|\sin x|)^{\frac{3a}{|\sin x|}}=e^{3a}=e^{\frac{1}{2}}$$

$$\lim_{x\to 0^-}(1+|\sin x|)^{\frac{3a}{|\sin x|}}=e^{3a}=e^{\frac{1}{2}}$$

$$a=\frac{1}{6}\Rightarrow 6a=1$$

$(6a + b^2) = (1 + e)$

13. Let y = y(x) be solution of the differential equation

$\log_e\left(\frac{dy}{dx}\right)=3x+4y$, with y(0) = 0.

If $y\left(-\frac{2}{3}\log_e 2\right)=\alpha\log_e 2$, then the value of α is equal to:

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) 2 (4) $-\frac{1}{2}$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}=e^{3x}.e^{4y}\Rightarrow\int e^{-4y}dy=\int e^{3x}dx$

$$\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+C\Rightarrow-\frac{1}{4}-\frac{1}{3}=C\Rightarrow C=-\frac{7}{12}$$

$$\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}-\frac{7}{12}\Rightarrow e^{-4y}=\frac{4e^{3x}-7}{-3}$$

$$e^{4y}=\frac{3}{7-4e^{3x}}\Rightarrow 4y=\ell n\left(\frac{3}{7-4e^{3x}}\right)$$

$$4y=\ell n\left(\frac{3}{6}\right)\ \text{when}\ x=-\frac{2}{3}\ell n2$$

$$y=\frac{1}{4}\ell n\left(\frac{1}{2}\right)=-\frac{1}{4}\ell n2$$

14. Let the plane passing through the point (–1, 0, –2) and perpendicular to each of the planes 2x + y – z = 2 and x – y – z = 3 be ax + by + cz + 8 = 0. Then the value of a + b + c is equal to:

(1) 3 (2) 8 (3) 5 (4) 4

Official Ans. by NTA (4)

Sol. Normal of req. plane $\left(2\hat{i}+\hat{j}-\hat{k}\right)\times\left(\hat{i}-\hat{j}-\hat{k}\right)$

$=-2\hat{i}+\hat{j}-3\hat{k}$

Equation of plane

–2(x + 1) + 1(y – 0) – 3(z + 2) = 0

–2x + y – 3z – 8 = 0

2x – y + 3z + 8 = 0

a + b + c = 4

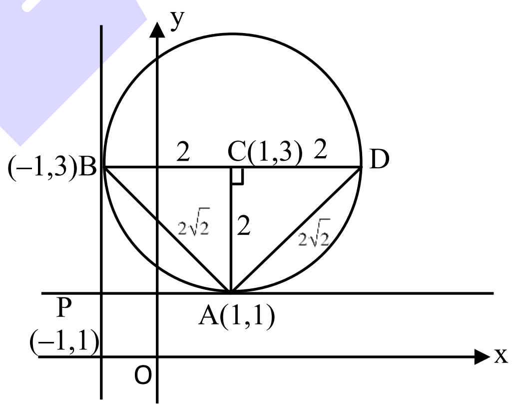
15. Two tangents are drawn from the point P(–1, 1) to the circle $x^2 + y^2 – 2x – 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to:

(1) 2 (2) $(3\sqrt{2}+2)$

(3) 4 (4) $3(\sqrt{2}-1)$

Official Ans. by NTA (3)

Sol.



$$\Delta ABD=\frac{1}{2}\times 2\times 4$$

$= 4$

16. Let f : **R** → **R** be a function such that f(2) = 4 and $f'(2)=1$. Then, the value of $\lim\limits_{x\to 2}\frac{x^2f(2)-4f(x)}{x-2}$ is equal to :

(1) 4 (2) 8 (3) 16 (4) 12

Official Ans. by NTA (4)

Sol. Apply L'Hopital Rule

$$\lim_{x\to 2}\left(\frac{2xf(2)-4f'(x)}{1}\right)$$

$$=\frac{4(4)-4}{1}=12$$

17. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P, Q} is equal to

(1) $\{(4,0),(0,6)\}$

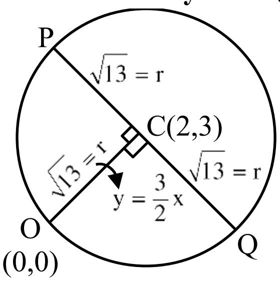
(2) $\left\{\left(2+2\sqrt{2},3-\sqrt{5}\right),\left(2-2\sqrt{2},3+\sqrt{5}\right)\right\}$

(3) $\left\{\left(2+2\sqrt{2},3+\sqrt{5}\right),\left(2-2\sqrt{2},3-\sqrt{5}\right)\right\}$

(4) $\{(-1,5),(5,1)\}$

Official Ans. by NTA (4)

Sol.



$\tan\theta = -\frac{2}{3}$

Using symmetric from of line

P,Q : $\left(2\pm\sqrt{13}\cos\theta, 3\pm\sqrt{13}\sin\theta\right)$

$\left(2\pm\sqrt{13}.\left(-\frac{3}{\sqrt{13}}\right), 3\pm\sqrt{13}\left(\frac{2}{\sqrt{13}}\right)\right)$

(–1, 5) & (5, 1)

18. Let α, β be two roots of the equation $x^2+(20)^{1/4}x+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to

(1) 10 (2) 100 (3) 50 (4) 160

Official Ans. by NTA (3)

Sol. $\left(x^2+\sqrt{5}\right)^2 = \sqrt{20}\,x^2$

$x^4 = -5 \Rightarrow x^8 = 25$

$\alpha^8 + \beta^8 = 50$

19. The probability that a randomly selected 2-digit number belongs to the set $\{n \in N : (2^n - 2)$ is a multiple of 3$\}$ is equal to

(1) $\frac{1}{6}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Official Ans. by NTA (3)

Sol. Total number of cases = ${}^{90}C_1 = 90$

Now, $2^n - 2 = (3-1)^n - 2$

${}^nC_0 3^n - {}^nC_1.3^{n-1} + ... + (-1)^{n-1}.{}^nC_{n-1}3 + (-1)^n.{}^nC_n - 2$

$3\left(3^{n-1} - n3^{n-2} + ... + (-1)^{n-1}.n\right) + (-1)^n - 2$

$(2^n - 2)$ is multiply of 3 only when n is odd

Req. Probability $= \frac{45}{90} = \frac{1}{2}$

20. Let

$A = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid 2x^2+2y^2-2x-2y=1\}$,

$B = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid 4x^2+4y^2-16y+7=0\}$ and

$C = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid x^2+y^2-4x-2y+5 \le r^2\}$.

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to

(1) $\frac{3+\sqrt{10}}{2}$ (2) $\frac{2+\sqrt{10}}{2}$

(3) $\frac{3+2\sqrt{5}}{2}$ (4) $1+\sqrt{5}$

Official Ans. by NTA (3)

Sol. $S_1 : x^2+y^2-x-y-\frac{1}{2}=0$; $C_1\left(\frac{1}{2},\frac{1}{2}\right)$

$r_1 = \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{2}} = 1$

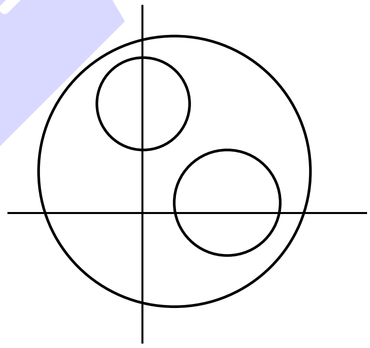
$S_2 : x^2+y^2-4y+\frac{7}{4}=0$; $C_2 : (0,2)$

$r_2 = \sqrt{4-\frac{7}{4}} = \frac{3}{2}$

$S_3 : x^2+y^2-4x-2y+5-r^2=0$

$C_3 : (2,1)$

$r_3 = \sqrt{4+1-5+r^2} = |r|$



$C_1C_3 = \sqrt{\frac{5}{2}}$

$\sqrt{\frac{5}{2}} \le |r-1| \Rightarrow \left.\begin{matrix} r \le 1+\sqrt{\frac{5}{2}} \\ r \ge \frac{3}{2}+\sqrt{5} \end{matrix}\right\}$

$C_2C_3 = \sqrt{5} \le \left|r-\frac{3}{2}\right|$

$\left.\begin{matrix} r-\frac{3}{2} \ge \sqrt{5} \\ r-\frac{3}{2} \le -\sqrt{5} \end{matrix}\right\}$

SECTION-B

1. For real numbers α and β, consider the following system of linear equations :

$x + y - z = 2,\ x + 2y + \alpha z = 1,\ 2x - y + z = \beta$.

If the system has infinite solutions, then $\alpha + \beta$ is equal to ________

Official Ans. by NTA (5)

Sol. For infinite solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$\Delta = 3(2+\alpha) = 0$

$\Rightarrow \alpha = -2$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$1(1+2\beta) - 2(1+4) - (\beta - 2) = 0$

$\beta - 7 = 0$

$\beta = 7$

$\therefore\ \alpha + \beta = 5$ Ans.

2. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to _______.

Official Ans. by NTA (2)

Sol. $\vec{a} \times \vec{b} = c$

Take Dot with $\vec{c}$

$(\vec{a} \times \vec{b}).\vec{c} = |\vec{c}|^2 = 2$

Projection of $\vec{b}$ or $\vec{a} \times \vec{c} = \ell$

$$\frac{|\vec{b}.(\vec{a} \times \vec{c})|}{|\vec{a} \times \vec{c}|} = \ell$$

$$\therefore \ell = \frac{2}{\sqrt{6}} \Rightarrow \ell^2 = \frac{4}{6}$$

$3\ell^2 = 2$

3. If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to __________.

Official Ans. by NTA (3)

Sol. $2\log_3(2^x - 5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$

Let $2^x = t$

$\log_3(t-5)^2 = \log_3 2\left(t - \frac{7}{2}\right)$

$(t-5)^2 = 2t - 7$

$t^2 - 12t + 32 = 0$

$(t-4)(t-8) = 0$

$\Rightarrow 2^x = 4$ or $2^x = 8$

X = 2 (Rejected)

Or x = 3

4. Let the domain of the function

$f(x) = \log_4\left(\log_5\left(\log_3\left(18x - x^2 - 77\right)\right)\right)$ be (a, b).

Then the value of the integral

$\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a+b-x))} dx$ is equal to ________.

Official Ans. by NTA (1)

Sol. For domain

$\log_5\left(\log_3\left(18x - x^2 - 77\right)\right) > 0$

$\log_3\left(18x - x^2 - 77\right) > 1$

$18x - x^2 - 77 > 3$

$x^2 - 18x + 80 < 0$

$x \in (8, 10)$

$\Rightarrow a = 8$ and $b = 10$

$$I = \int_a^b \frac{\sin^3 x}{\sin^3 x + \sin^3(a+b-x)} dx$$

$$I = \int_a^b \frac{\sin^3(a+b-x)}{\sin^3 x + \sin^3(a+b-x)}$$

$$2I = (b-a) \Rightarrow I = \frac{b-a}{2} \quad (\because a = 8 \text{ and } b = 10)$$

$$I = \frac{10-8}{2} = 1$$

5. Let

$$f(x)=\begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x\in[0,\pi]$$

Then the maximum value of f(x) is equal to ________.

Official Ans. by NTA (6)

Sol. $\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}\begin{pmatrix} R_1\to R_1-R_2 \\ \& \ R_2\to R_2-R_3 \end{pmatrix}$

$-2(\cos^2 x)+2(2+2\cos 2x+\sin^2 x)$

$4+4\cos 2x-2(\cos^2 x-\sin^2 x)$

$f(x)=4+\underbrace{2\cos 2x}_{\max=1}$

$f(x)_{\max}=4+2=6$

6. Let F : [3, 5] → **R** be a twice differentiable function on (3, 5) such that

$$F(x)=e^{-x}\int_3^x(3t^2+2t+4F'(t))\,dt.$$

If $F'(4)=\dfrac{\alpha e^{\beta}-224}{(e^{\beta}-4)^2}$, then $\alpha+\beta$ is equal to ________.

Official Ans. by NTA (16)

Sol. $F(3)=0$

$e^xF(x)=\int_3^x(3t^2+2t+4F'(t))dt$

$e^xF(x)+e^xF'(x)=3x^2+2x+4F'(x)$

$(e^x-4)\dfrac{dy}{dx}+e^xy=(3x^2+2x)$

$\dfrac{dy}{dx}+\dfrac{e^x}{(e^x-4)}y=\dfrac{(3x^2+2x)}{(e^x-4)}$

$ye^{\int\frac{e^x}{(e^x-4)}dx}=\int\dfrac{(3x^2+2x)}{(e^x-4)}e^{\int\frac{e^x}{e^x-4}dx}dx$

$y.(e^x-4)=\int(3x^2+2x)dx+c$

$y(e^x-4)=x^3+x^2+c$

Put x = 3 ⇒ c = –36

$F(x)=\dfrac{(x^3+x^2-36)}{(e^x-4)}$

$F'(x)=\dfrac{(3x^2+2x)(e^x-4)-(x^3+x^2-36)e^x}{(e^x-4)^2}$

Now put value of x = 4 we will get α = 12 & β = 4

7. Let a plane P pass through the point (3, 7, –7) and contain the line, $\dfrac{x-2}{-3}=\dfrac{y-3}{2}=\dfrac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to ________.

Official Ans. by NTA (3)

Sol. $\overrightarrow{BA}=(\hat{i}+4\hat{j}-5\hat{k})$

$\overrightarrow{BA}=(\hat{i}+4\hat{j}-5\hat{k})$

$\overrightarrow{BA}\times\vec{\ell}=\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix}$

$a\hat{i}+b\hat{j}+c\hat{k}=-14\hat{i}-\hat{j}(14)+\hat{k}(-14)$

a = 1, b = 1, c = 1

Plane is (x – 2) + (y – 3) + (z + 2) = 0

x + y + z –3 = 0

$d=\sqrt{3}\Rightarrow d^2=3$

8. Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions f : S → S such that f(m·n) = f(m)·f(n) for every m, n ∈ S and m · n ∈ S is equal to______.

Official Ans. by NTA (490)

Sol. F(mn) = f(m). f(n)

Put m = 1 **f(n) = f(1).** f(n) ⇒ f(1) = 1

Put m = n = 2

$f(4)=f(2).f(2)\begin{cases} f(2)=1\Rightarrow f(4)=1 \\ \text{or} \\ f(2)=2\Rightarrow f(4)=4 \end{cases}$

Put m = 2, n = 3

$f(6)=f(2).f(3)\begin{cases} \text{when } f(2)=1 \\ f(3)=1 \text{ to } 7 \\ \\ f(2)=2 \\ f(3)=1 \text{ or } 2 \text{ or } 3 \end{cases}$

f(5), f(7) can take any value

Total = (1 × 1 × 7 × 1 × 7 × 1 × 7)

+ (1 × 1 × 3 × 1 × 7 × 1 × 7)

= 490

9. If $y = y(x)$, $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$$\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0$$, with $y(0) = 0$,

then $5y'\left(\frac{\pi}{2}\right)$ is equal to ________.

Official Ans. by NTA (2)

Sol. $\sec y \frac{dy}{dx} = 2\sin x \cos y$

$\sec^2 y dy = 2\sin x dx$

$\tan y = -2\cos x + c$

$c = 2$

$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \frac{\pi}{2}$

$\tan y = 2$

$\sec^2 y \frac{dy}{dx} = 2\sin x$

$5\frac{dy}{dx} = 2$

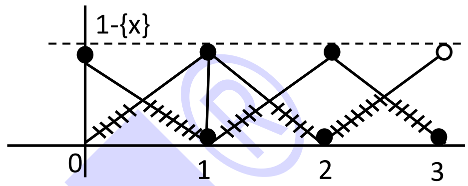
10. Let $f : [0, 3] \to \mathbf{R}$ be defined by

$f(x) = \min \{x - [x], 1 + [x] - x\}$

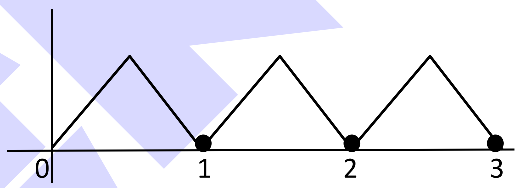
where $[x]$ is the greatest integer less than or equal to x. Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to ________.

Official Ans. by NTA (5)

Sol.



$1 - \{x\} = 1 - x; 0 \le x < 1$



Non differentiable at

$x = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$