FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 27th July, 2021) **TIME : 2 : 00 PM to 5 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. An electron and proton are separated by a large distance. The electron starts approaching the proton with energy 3 eV. The proton captures the electrons and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4000 Å. What is the maximum kinetic energy of the emitted photoelectron?

(1) 7.61 eV

(2) 1.41 eV

(3) 3.3 eV

(4) No photoelectron would be emitted

Official Ans. by NTA (2)

Sol. Initially, energy of electron = +3eV

finally, in 2^{nd} excited state,

energy of electron $= -\dfrac{(13.6\text{eV})}{3^2}$

$= -1.51\text{eV}$

Loss in energy is emitted as photon,

So, photon energy $\dfrac{hc}{\lambda} = 4.51$ eV

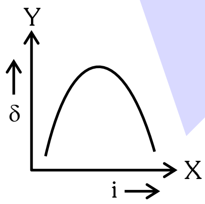
Now, photoelectric effect equation

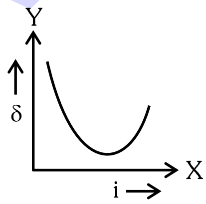
$$KE_{max} = \frac{hc}{\lambda} - \phi = 4.51 - \left(\frac{hc}{\lambda_{th}}\right)$$

$$= 4.51 \text{ eV} - \frac{12400 \text{ eVÅ}}{4000 \text{ Å}}$$

$$= 1.41 \text{ eV}$$

2. The expected graphical representation of the variation of angle of deviation 'δ' with angle of incidence 'i' in a prism is :

(1)


(2)


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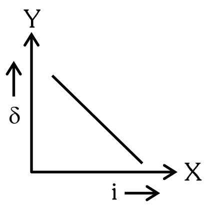

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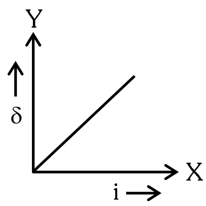


Official Ans. by NTA (2)

Sol. Standard graph between angle of deviation and incident angle.

3. A raindrop with radius R = 0.2 mm falls from a cloud at a height h = 2000 m above the ground. Assume that the drop is spherical throughout its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is :

[Density of water $f_w = 1000$ kg m^{-3} and Density of air $f_a = 1.2$ kg m^{-3}, g = 10 m/s^2

Coefficient of viscosity of air = 1.8×10^{-5} Nsm^{-2}]

(1) 250.6 ms^{-1}

(2) 43.56 ms^{-1}

(3) 4.94 ms^{-1}

(4) 14.4 ms^{-1}

Official Ans. by NTA (3)

Sol. At terminal speed

$a = 0$

$F_{net} = 0$

$mg = F_v = 6\pi \eta R v$

$$v = \frac{mg}{6\pi\eta R v}$$

$$v = \frac{\rho_w \frac{4\pi}{3} R^3 g}{6\pi\eta R}$$

$$= \frac{2\rho_w R^2 g}{9\eta}$$

$$= \frac{400}{81} \text{ m/s}$$

$= 4.94$ m/s

4. One mole of an ideal gas is taken through an adiabatic process where the temperature rises from 27°C to 37°C. If the ideal gas is composed of polyatomic molecule that has 4 vibrational modes, which of the following is true?
$[R = 8.314 \text{ J mol}^{-1} \text{ k}^{-1}]$
(1) work done by the gas is close to 332 J
(2) work done on the gas is close to 582 J
(3) work done by the gas is close to 582 J
(4) work done on the gas is close to 332 J
Official Ans. by NTA (2)

Sol. Since, each vibrational mode, corresponds to two degrees of freedom, hence, f = 3 (trans.) + 3(rot.) + 8 (vib.) = 14

& $\gamma = 1 + \frac{2}{f}$

$\gamma = 1 + \frac{2}{14} = \frac{8}{7}$

$W = \frac{nR\Delta T}{\gamma - 1} = -582$

As W < 0. work is done on the gas.

5. An object of mass 0.5 kg is executing simple harmonic motion. It amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}$ s starting from mean position. Assume that the initial phase of the oscillation is zero.
(1) 0.62 J (2) 6.2×10^{-3} J
(3) 1.2×10^{3} J (4) 6.2×10^{3} J
Official Ans. by NTA (1)

Sol. $T = 2\pi\sqrt{\frac{m}{k}}$

$0.2 = 2\pi\sqrt{\frac{0.5}{k}}$

$k = 50\pi^2$
≈ 500

$x = A \sin(\omega t + \phi)$

$= 5 \text{ cm} \sin\left(\frac{\omega T}{4} + 0\right)$

$= 5 \text{ cm} \sin\left(\frac{\pi}{2}\right)$

$= 5 \text{ cm}$

$PE = \frac{1}{2}kx^2$

$= \frac{1}{2}(500)\left(\frac{5}{100}\right)^2$

$= 0.6255$

6. Match **List I** with **List II**.

List-I	**List-II**
(a) Capacitance, C	(i) $M^1L^1T^{-3}A^{-1}$
(b) Permittivity of free space, ε_0	(ii) $M^{-1}L^{-3}T^4A^2$
(c) Permeability of free space, μ_0	(iii) $M^{-1}L^{-2}T^4A^2$
(d) Electric field, E	(iv) $M^1L^1T^{-2}A^{-2}$

Choose the correct answer from the options given below

(1) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)
(2) (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)
(3) (a) → (iv), (b) → (ii), (c) → (iii), (d) → (i)
(4) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

Official Ans. by NTA (1)

Sol. $q = CV$

$[C] = \left[\frac{q}{V}\right] = \frac{(A \times T)^2}{ML^2T^{-2}}$

$= M^{-1}L^{-2}T^4A^2$

$[E] = \left[\frac{F}{q}\right] = \frac{MLT^{-2}}{AT}$

$= MLT^{-3}A^{-1}$

$F = \frac{q_1q_2}{4\pi\epsilon_o r^2}$

$[\epsilon_o] = M^{-1}L^{-3}T^4A^2$

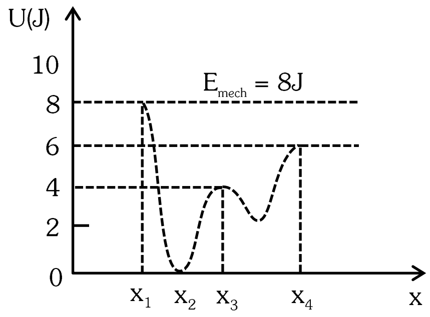
Speed of light $c = \frac{1}{\sqrt{\mu_o \epsilon_o}}$

$\mu_o = \frac{1}{\epsilon_o c^2}$

$[\mu_o] = \frac{1}{[M^{-1}L^{-3}T^4A^2][LT^{-1}]^2}$

$= [M^1L^1T^{-2}A^{-2}]$

7. Given below is the plot of a potential energy function U(x) for a system, in which a particle is in one dimensional motion, while a conservative force F(x) acts on it. Suppose that E_{mech} = 8 J, the incorrect statement for this system is :



[where K.E. = kinetic energy]

(1) at $x > x_4$, K.E. is constant throughout the region.

(2) at $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.

(3) at $x = x_2$, K.E. is greatest and the particle is moving at the fastest speed.

(4) at $x = x_3$, K.E. = 4 J.

Official Ans. by NTA (2)

Sol. $E_{mech.} = 8J$

(A) at $x > x_4$, U = constant = 6J

$K = E_{mech.} - U = 2J$ = constant

(B) at $x < x_1$, U = constant = 8J

$K = E_{mech.} - U = 8 - 8 = 0$ J

Particle is at rest.

(C) At $x = x_2$, $U = 0 \Rightarrow E_{mech.} = K = 8$ J

KE is greatest, and particle is moving at fastest speed.

(D) At $x = x_3$, $U = 4$ J

$U + K = 8$ J

$K = 4$ J

8. A 100 Ω resistance, a 0.1 μF capacitor and an inductor are connected in series across a 250 V supply at variable frequency. Calculate the value of inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 Hz.

(1) 0.70 H (2) 70.3 mH

(3) 7.03×10^{-5} H (4) 70.3 H

Official Ans. by NTA (4)

Sol. $C = 0.1\ \mu F = 10^{-7}$ F

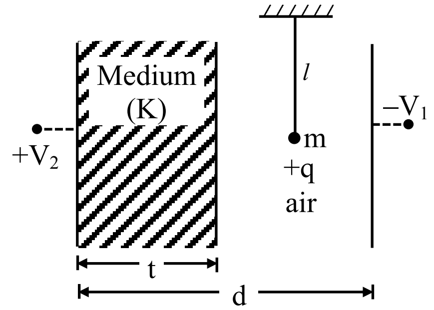
Resonant frequency = 60 Hz

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$2\pi f_o = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{4\pi^2 f_o^2 C}$$

by putting values $L \simeq 70.3$ Hz.

9. A simple pendulum of mass 'm', length 'l' and charge '+q' suspended in the electric field produced by two conducting parallel plates as shown. The value of deflection of pendulum in equilibrium position will be



(1) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

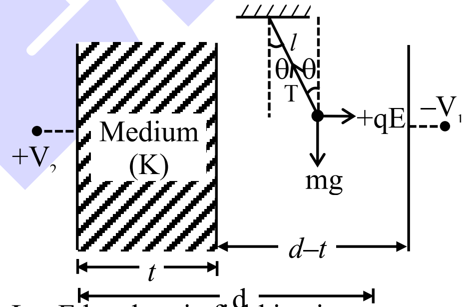
(2) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

(3) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

(4) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

Official Ans. by NTA (3)

Sol.

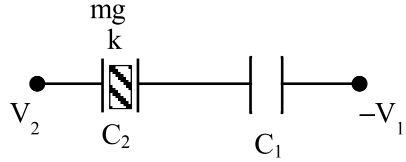


Let E be electric field in air

$T \sin\theta = qE$

$T \cos\theta = mg$

$$\tan\theta = \frac{qE}{mg}$$



$$Q = \left[\frac{C_1 C_2}{C_1 + C_2}\right][V_1 + V_2]$$

$$E = \frac{Q}{A\epsilon_o} = \left[\frac{C_1 C_2}{C_1 + C_2}\right]\frac{[V_1 + V_2]}{A\epsilon_o}$$

$$C_1 = \frac{\epsilon_o A}{d - t} \Rightarrow E = \frac{C_2[V_1 + V_2]}{(C_1 + C_2)(d - t)}$$

$$\text{Now } \theta = \tan^{-1}\left[\frac{q.E}{mg}\right]$$

$$\theta = \tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$$

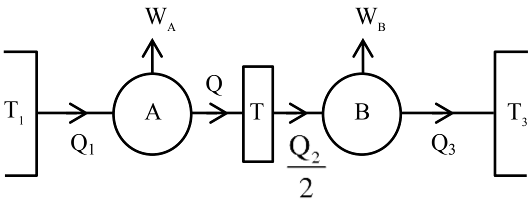
10. Two Carnot engines A and B operate in series such that engine A absorbs heat at T_1 and rejects heat to a sink at temperature T. Engine B absorbs half of the heat rejected by Engine A and rejects heat to the sink at T_3. When workdone in both the cases is equal, to value of T is :

(1) $\frac{2}{3}T_1+\frac{3}{2}T_3$ (2) $\frac{1}{3}T_1+\frac{2}{3}T_3$

(3) $\frac{3}{2}T_1+\frac{1}{3}T_3$ (4) $\frac{2}{3}T_1+\frac{1}{3}T_3$

Official Ans. by NTA (4)

Sol.



$$W_A = 1-\frac{Q_2}{Q_1} = 1-\frac{T}{T_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{T}{T_1}$$

$$W_B = 1-\frac{Q_3}{(Q_2/2)} = 1-\frac{T_3}{T} \Rightarrow \frac{2Q_3}{Q_2} = \frac{T_3}{T}$$

Now, $W_A = W_B$

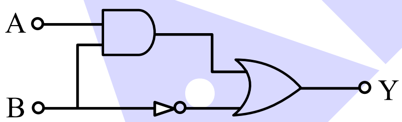
$$Q_1 - Q_2 = \frac{Q_2}{2} - Q_3$$

$$\Rightarrow \frac{2Q_1}{Q_2}+\frac{2Q_3}{Q_2} = 3$$

$$\Rightarrow \frac{2T_1}{T}+\frac{T_3}{T} = 3$$

$$\frac{2T_1}{3}+\frac{T_3}{3} = T$$

11. Find the truth table for the function Y of A and B represented in the following figure.



(1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

(3)

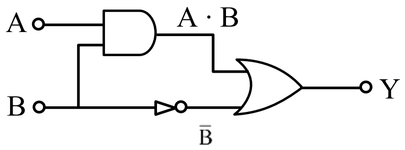
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Official Ans. by NTA (2)

Sol.



$$Y = A\cdot B+\overline{B}$$

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

12. Figure A and B shown two long straight wires of circular cross-section (a and b with a < b), carrying current I which is uniformly distributed across the cross-section. The magnitude of magnetic field B varies with radius r and can be represented as :

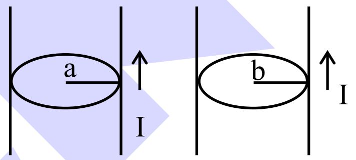
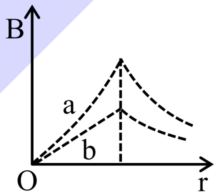
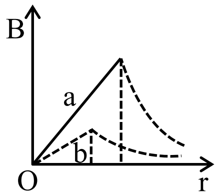


Fig. A Fig. B

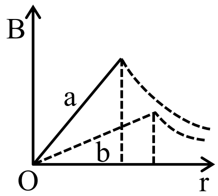
(1)



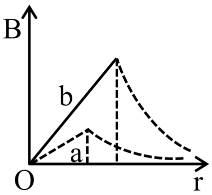
(2)



(3)

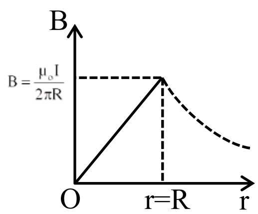


(4)



Official Ans. by NTA (3)

Sol. Graph for wire of radius R :



As b > a

$B_a > B_b$

$$B_a = \frac{\mu_0 i}{2\pi a}$$

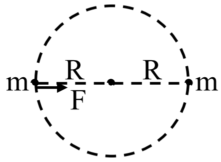
$$B_b = \frac{\mu_0 i}{2\pi b}$$

13. Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is :

(1) $\sqrt{\frac{G}{2R^3}}$ (2) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$ (3) $\frac{1}{2R}\sqrt{\frac{1}{G}}$ (4) $\sqrt{\frac{2G}{R^3}}$

Official Ans. by NTA (2)

Sol.



$$F = \frac{Gm^2}{(2R)^2} = mR\omega^2$$

$$\omega = \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

14. Consider the following statements :

A. Atoms of each element emit characteristics spectrum.

B. According to Bohr's Postulate, an electron in a hydrogen atom, revolves in a certain stationary orbit.

C. The density of nuclear matter depends on the size of the nucleus.

D. A free neutron is stable but a free proton decay is possible.

E. Radioactivity is an indication of the instability of nuclei.

Choose the correct answer from the options given below :

(1) A, B, C, D and E
(2) A, B and E only
(3) B and D only
(4) A, C and E only

Official Ans. by NTA (2)

Sol. (A) True, atom of each element emits characteristic spectrum.

(B) True, according to Bohr's postulates

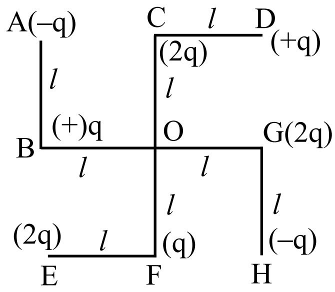
$mvr = \frac{nh}{2\pi}$ and hence electron resides into orbits of specific radius called stationary orbits.

(C) False, density of nucleus is constant

(D) False, A free neutron is unstable decays into proton and electron and antineutrino.

(E) True unstable nucleus show radioactivity.

15. What will be the magnitude of electric field at point O as shown in figure? Each side of the figure is l and perpendicular to each other?



(1) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$ (2) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l^2)}\left(2\sqrt{2}-1\right)$

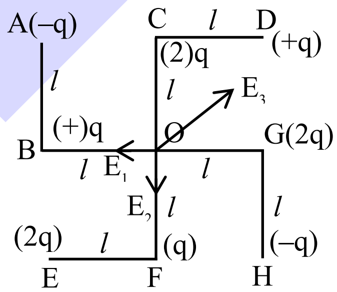
(3) $\frac{q}{4\pi\varepsilon_0(2l)^2}$ (4) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2l^2}\left(\sqrt{2}\right)$

Official Ans. by NTA (2)

Sol. $E_1 = \frac{kq}{\ell^2} = E_2$

$$E_3 = \frac{kq}{(\sqrt{2}\ell)^2} = \frac{kq}{2\ell^2}$$

$$E = \frac{\sqrt{2}kq}{\ell^2} - \frac{kq}{2\ell^2} = \frac{kq}{2\ell^2}\left(2\sqrt{2}-1\right)$$



16. A physical quantity 'y' is represented by the formula $y = m^2r^{-4}g^x l^{-\frac{3}{2}}$

If the percentage errors found in y, m, r, l and g are 18, 1, 0.5, 4 and p respectively, then find the value of x and p.

(1) 5 and ± 2 (2) 4 and ± 3

(3) $\frac{16}{3}$ and $\pm\frac{3}{2}$ (4) 8 and ± 2

Official Ans. by NTA (3)

Sol. $\frac{\Delta y}{y} = \frac{2\Delta m}{m} + \frac{4\Delta r}{r} + \frac{x\Delta g}{g} + \frac{3}{2}\frac{\Delta\ell}{\ell}$

$$18 = 2(1) + 4(0.5) + xp + \frac{3}{2}(4)$$

$8 = xp$

By checking from options.

$$x = \frac{16}{3},\ p = \pm\frac{3}{2}$$

17. An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by :

(1) $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (2) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$

(3) $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (4) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

Official Ans. by NTA (4)

Sol. P = const.

$$P = Fv = \frac{mv^2dv}{dx}$$

$$\int_0^x \frac{P}{m}dx = \int_0^v v^2 dv$$

$$\frac{Px}{m} = \frac{v^3}{3}$$

$$\left(\frac{3Px}{m}\right)^{1/3} = v = \frac{dx}{dt}$$

$$\left(\frac{3P}{m}\right)^{1/3} \int_0^t dt = \int_0^x x^{-1/3}dx$$

$$\Rightarrow x = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

18. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3 km. Find the mass of Mars.

$$\left\{\text{Given } \frac{4\pi^2}{G} = 6\times10^{11}\, N^{-1}\, m^{-2}\, kg^2\right\}$$

(1) 5.96×10^{19} kg (2) 3.25×10^{21} kg

(3) 7.02×10^{25} kg (4) 6.00×10^{23} kg

Official Ans. by NTA (4)

Sol. Option D is correct

$$T^2 = \frac{4\pi^2}{GM}\cdot r^3$$

$$M = \frac{4\pi^2}{G}\cdot\frac{r^3}{T^2}$$

by putting values

$M = 6 \times 10^{23}$

19. A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0\left[1 - \left(\frac{t-T}{T}\right)^2\right]$$

Where F_0 and T are constants. The force acts only for the time interval 2T. The velocity v of the particle after time 2T is :

(1) $2F_0T / M$

(2) $F_0T / 2M$

(3) $4F_0T / 3M$

(4) $F_0T / 3M$

Official Ans. by NTA (3)

Sol. t = 0, u = 0

$$a = \frac{F_o}{M} - \frac{F_o}{MT^2}(t-T)^2 = \frac{dv}{dt}$$

$$\int_0^v dv = \int_{t=0}^{2T}\left(\frac{F_o}{M} - \frac{F_o}{MT^2}(t-T)^2\right)dt$$

$$V = \left[\frac{F_o}{M}t\right]_o^{2T} - \frac{F_o}{MT^2}\left[\frac{t^3}{3} - t^2T + T^2t\right]_0^{2T}$$

$$V = \frac{4F_oT}{3M}$$

20. The resistance of a conductor at 15°C is 16 Ω and at 100°C is 20Ω. What will be the temperature coefficient of resistance of the conductor?

(1) $0.010°C^{-1}$

(2) $0.033°C^{-1}$

(3) $0.003°C^{-1}$

(4) $0.042°C^{-1}$

Official Ans. by NTA (3)

Sol. $16 = R_o\,[1 + \alpha\,(15 - T_o)\,]$

$20 = R_o\,[1 + \alpha\,(100 - T_o)\,]$

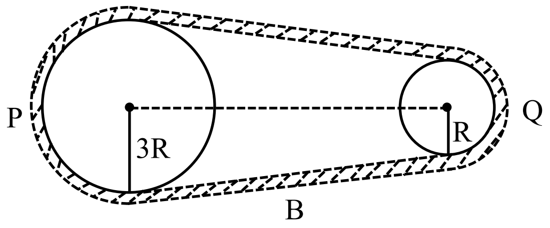
Assuming $T_o = 0°C$, as a general convention.

$$\Rightarrow \frac{16}{20} = \frac{1+\alpha\times15}{1+\alpha\times100}$$

$$\Rightarrow \alpha = 0.003\ °C^{-1}$$

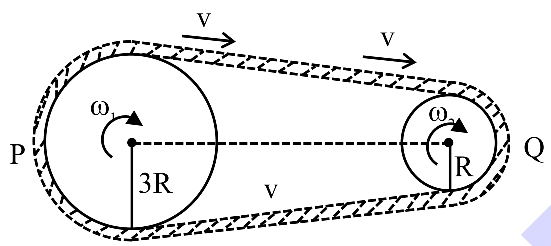
SECTION-B

1. In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_1}{I_2}\right)$ will be x : 1. The value of x will be _____.



Official Ans. by NTA (9)

Sol.



$$\frac{1}{2}I_1(\omega_1)^2 = \frac{1}{2}I_2(\omega_2)^2$$

$$I_1\left(\frac{v}{3R}\right)^2 = I_2\left(\frac{v}{R}\right)^2$$

$$\frac{I_1}{I_2} = \frac{9}{1}$$

2. The difference in the number of waves when yellow light propagates through air and vacuum columns of the same thickness is one. The thickness of the air column is ______ mm. [Refractive index of air = 1.0003, wavelength of yellow light in vacuum = 6000 Å]

Official Ans. by NTA (2)

Sol. Thickness t = nλ

So, n λ_{vac} = (n + 1) λ_{air}

$$n\,\lambda = (n+1)\,\frac{\lambda}{\mu_{air}}$$

$$n = \frac{1}{\mu_{air}-1} = \frac{10^4}{3}$$

$$t = n\lambda$$

$$= \frac{10^4}{3}\times 6000\text{Å}$$

$$= 2\text{ mm}$$

3. The maximum amplitude for an amplitude modulated wave is found to be 12V while the minimum amplitude is found to be 3V. The modulation index is 0.6x where x is ______.

Official Ans. by NTA (1)

Sol. $A_{max} = A_c + A_m = 12$

$A_{min} = A_c - A_m = 3$

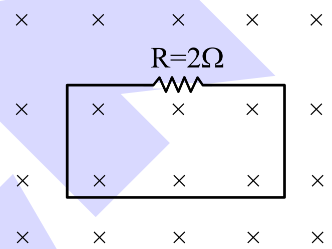
$$\Rightarrow A_c = \frac{15}{2} \;\&\; A_m = \frac{9}{2}$$

$$\text{modulation index} = \frac{A_m}{A_c} = \frac{9/2}{15/2} = 0.6$$

$\Rightarrow x = 1$

4. In the given figure the magnetic flux through the loop increases according to the relation $\phi_B(t) = 10t^2 + 20t$, where ϕ_B is in milliwebers and t is in seconds.

The magnitude of current through R = 2Ω resistor at t = 5 s is ______ mA.



Official Ans. by NTA (60)

Sol. $|\epsilon| = \frac{d\phi}{dt} = 20t + 20\text{ mV}$

$$|i| = \frac{|\epsilon|}{R} = 10t + 10\text{ mA}$$

at t = 5

|i| = 60 mA

5. A particle executes simple harmonic motion represented by displacement function as

$$x(t) = A\sin(\omega t + \phi)$$

If the position and velocity of the particle at t = 0 s are 2 cm and 2ω cm s^{-1} respectively, then its amplitude is $x\sqrt{2}$ cm where the value of x is ____.

Official Ans. by NTA (2)

Sol. x(t) = A sin (ωt + ϕ)

v(t) = Aω cos (ωt + ϕ)

2 = A sin ϕ ......(1)

2ω = Aω cos ϕ ......(2)

From (1) and (2)

tan ϕ = 1

ϕ = 45°

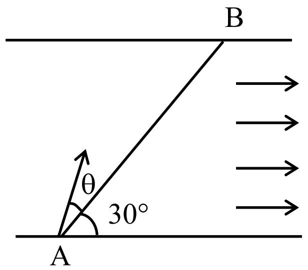
Putting value of ϕ in equation (1)

$$2 = A\left\{\frac{1}{\sqrt{2}}\right\}$$

$$A = 2\sqrt{2}$$

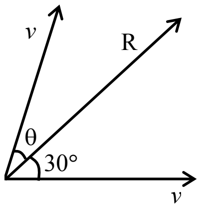
x = 2

6. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ____°, so that the swimmer reaches point B.



Official Ans. by NTA (30)

Sol.



Both velocity vectors are of same magnitude therefore resultant would pass exactly midway through them

$\theta = 30°$

7. For the circuit shown, the value of current at time t = 3.2 s will be ______ A.

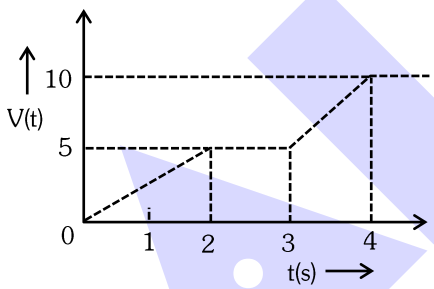


Figure 1

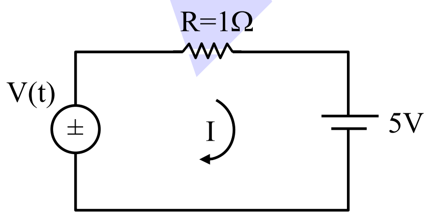
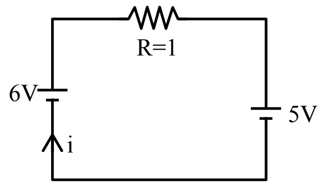


Figure-2

[Voltage distribution V(t) is shown by Fig. (1) and the circuit is shown in Fig. (2)]

Official Ans. by NTA (1)

Sol. From graph voltage at t = 3.2 sec is 6 volt.

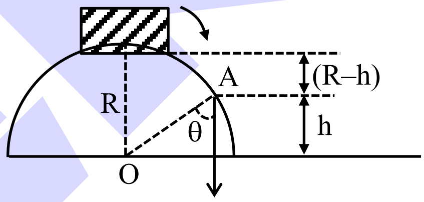


$$i = \frac{6-5}{1}$$

$$i = 1\ A$$

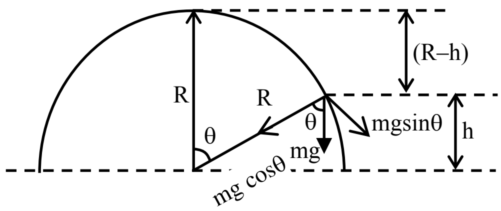
8. A small block slides down from the top of hemisphere of radius R = 3 m as shown in the figure. The height 'h' at which the block will lose contact with the surface of the sphere is ____m.

(Assume there is no friction between the block and the hemisphere)



Official Ans. by NTA (2)

Sol.



$$mg\cos\theta = \frac{mv^2}{R} \qquad ....(1)$$

$$\cos\theta = \frac{h}{R}$$

Energy conservation

$$mg\{R-h\} = \frac{1}{2}mv^2 \qquad ....(2)$$

$$\text{from (1) \& (2)} \Rightarrow mg\left\{\frac{h}{R}\right\} = \frac{2mg\{R-h\}}{R}$$

$$h = \frac{2R}{3} = 2m$$

9. The K_α X-ray of molybdenum has wavelength 0.071 nm. If the energy of a molybdenum atoms with a K electron knocked out is 27.5 keV, the energy of this atom when an L electron is knocked out will be ______ keV. (Round off to the nearest integer)

$[h = 4.14 \times 10^{-15}$ eVs, $c = 3 \times 10^{8}$ ms$^{-1}]$

Official Ans. by NTA (10)

Sol. $E_{k_\alpha} = E_k - E_L$

$$\frac{hc}{\lambda_{k_\alpha}} = E_k - E_L$$

$$E_L = E_k - \frac{hc}{\lambda_{k_\alpha}}$$

$$= 27.5\ \text{KeV} - \frac{12.42 \times 10^{-7}\ \text{eVm}}{0.071 \times 10^{-9}\ \text{m}}$$

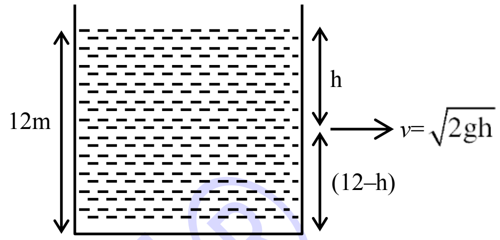
$E_L = (27.5 - 17.5)$ keV

$= 10$ keV

10. The water is filled upto height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth 'h' below the water level. The value of 'h' for which the emerging stream of water strikes the ground at the maximum range is ___ m.

Official Ans. by NTA (6)

Sol.



$$R = \sqrt{2gh} \times \sqrt{\frac{(12-h)\times 2}{g}}$$

$$\sqrt{4h(12-h)} = R$$

For maximum R

$$\frac{dR}{dh} = 0$$

$\Rightarrow h = 6$m

FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 27th July, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Which one of the following set of elements can be detected using sodium fusion extract ?

(1) Sulfur, Nitrogen, Phosphorous, Halogens

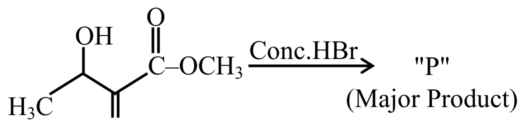
(2) Phosphorous, Oxygen, Nitrogen, Halogens

(3) Nitrogen, Phosphorous, Carbon, Sulfur

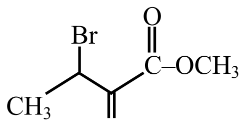
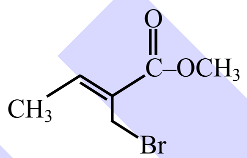
(4) Halogens, Nitrogen, Oxygen, Sulfur

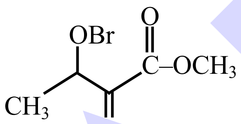
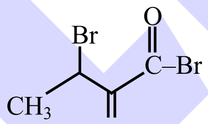
Official Ans. by NTA (1)

Sol. By sodium fusion extract we can detect sulphur, nitrogen,
Phosphorous and halogens, because they are converted in to their ionic form with sodium metal.

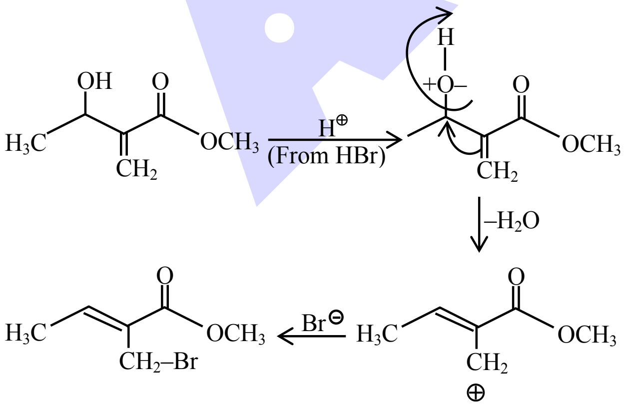
2. 

Consider the above reaction, the major product "P" formed is :-

(1)  (2) 

(3)  (4) 

Official Ans. by NTA (2)

Sol. 

3. The number of neutrons and electrons, respectively, present in the radioactive isotope of hydrogen is :-

(1) 1 and 1 (2) 3 and 1

(3) 2 and 1 (4) 2 and 2

Official Ans. by NTA (3)

Sol. Radioactive isotope of hydrogen is Tritium (^3_1T)

No. of neutrons (A–Z) = 3 – 1 = 2

No. of electrons = 1

4. Match List - I with List II :

List - I		List - II	
(a)	Li	(i)	photoelectric cell
(b)	Na	(ii)	absorbent of CO_2
(c)	K	(iii)	coolant in fast breeder nuclear reactor
(d)	Cs	(iv)	treatment of cancer
		(v)	bearings for motor engines

Choose the **correct** answer from the options given below :

(1) (a) - (v), (b) - (i), (c) - (ii), (d) - (iv)

(2) (a) - (v), (b) - (ii), (c) - (iv), (d) - (i)

(3) (a) - (iv), (b) - (iii), (c) - (i), (d) - (ii)

(4) (a) - (v), (b) - (iii), (c) - (ii), (d) - (i)

Official Ans. by NTA (4)

Sol. Li makes alloy with Lead to make white metal bearings for motor engines

Liquid Na metal is used as coolant in fast breeder nuclear reactor

K is a very absorbent of CO_2

Cs is used in making photoelectric cell

5. Given below are two statement : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : $SO_2(g)$ is adsorbed to a large extent than $H_2(g)$ on activated charcoal.

Reason R : $SO_2(g)$ has a higher critical temperature than $H_2(g)$.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both **A** and **R** are correct but **R** is not the correct explanation fo **A**

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(3) **A** is not correct but **R** is correct.

(4) **A** is correct but **R** is not correct.

Official Ans. by NTA (2)

Sol. Gases having higher critical temperature absorb to a greater extent.

6. The **CORRECT** order of first ionisation enthalpy is :

(1) Mg < S < Al < P (2) Mg < Al < S < P

(3) Al < Mg < S < P (4) Mg < Al < P < S

Official Ans. by NTA (3)

Sol. Mg Al P S $\rightarrow$ IE. order $\Rightarrow$ Al< Mg < S < P

$$\text{Valence } [N_e] : \overset{Mg}{3s^2} \quad \overset{A\ell}{3s^2 3p^1} \quad \overset{P}{3s^2 3p^3} \quad \overset{S}{3s^2 3p^4}$$

↑ Full Filled Stable (Mg) ↑ Half Filled Stable (P)

7. Given below are two statements :

Statement I : Hyperconjugation is a permanent effect.

Statement II : Hyperconjugation in ethyl cation $\left(CH_3 - \overset{+}{C}H_2\right)$ involves the overlapping of $C_{sp^2} - H_{1s}$ bond with empty 2p orbital of other carbon.

Choose the **correct** option :

(1) Both **statement I** and **statement II** are false

(2) **Statement I** is incorrect but **statement II** is true

(3) **Statement I** is correct but **statement II** is false

(4) Both **Statement I** and **statement II** are true.

Official Ans. by NTA (3)

Sol. **Statement I :** It is correct statement

Statement II : $CH_3 - \overset{\oplus}{C}H_2$ involve $C_{sp^3} - H_{1s}$ bond with empty 2p orbital hence given statement is false.

8. Given below are two **statements** :

Statement I : $[Mn(CN)_6]^{3-}$, $[Fe(CN)_6]^{3-}$ and $[Co(C_2O_4)_3]^{3-}$ are d^2sp^3 hybridised.

Statement II : $[MnCl_6]^{3-}$ and $[FeF_6]^{3-}$ are paramagnetic and have 4 and 5 unpaired electrons, respectively.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is correct but **statement II** is false

(2) Both **statement I** and **statement II** are false

(3) **Statement I** is incorrect but **statement II** is true

(4) Both **statement I** and **statement II** are are true

Official Ans. by NTA (4)

Sol. $\left[Mn(CN)_6\right]^{3-}$ $\left[Fe(CN)_6\right]^{3-}$ $\left[Co(C_2O_4)_3\right]^{3-}$

⇓

Mn^{3+} CN^- Fe^{3+}, CN^- Co3+, $C_2O_4^{2-}$

d^4 configuration, SFL d^5 configuration, SFL d^6 configuration, Chelating ligand

$\Rightarrow$ All will have larger splitting hence d^2sp^3 hybridisation

$[MnCl_6]^{3-}$ and $[FeF_6]^{3-}$

d^4 configuration, Cl^- WFL d^5 configuration, F^- WFL

↿ ―   ↿ ↿
↿ ↿ ↿   ↿ ↿ ↿
⇓   ⇓
4 unpaired electrons   5 unpaired electrons

9. To an aqueous solution containing ions such as Al^{3+}, Zn^{2+}, Ca^{2+}, Fe^{3+}, Ni^{2+}, Ba^{2+} and Cu^{2+} was added conc. HCl, followed by H_2S.

The total number of cations precipitated during this reaction is/are :

(1) 1 (2) 3 (3) 4 (4) 2

Official Ans. by NTA (1)

Sol. Al^{3+} and Fe^{3+} sulphides hydrolyse in water.

Ni^{2+} and Zn^{2+} require basic medium with H_2S to form ppt

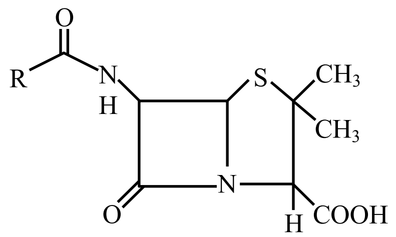
Ca^{2+} and Ba^{2+} sulphides are soluble

hence we will receive only CuS ppt.

10. Given below are two **statements** :

Statement I : Penicillin is a bacteriostatic type antibiotic.

Statement II : The general structure of Penicillin is:



Choose the correct option :

(1) Both **statement I** and **statement II** are false

(2) **Statement I** is incorrect but **statement II** is true

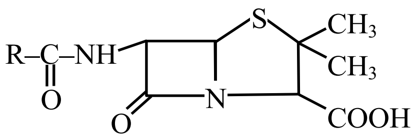
(3) Both **statement I** and **statement II** are true

(4) **Statement I** is correct but **statement II** is false

Official Ans. by NTA (2)

Sol. **Statement I** : Pencillin is bactericidal not bacteriostatic hence given statement is false.

Statement II : Structure of pencilline given is correct



11. Compound **A** gives D-Galactose and D-Glucose on hydrolysis. The compound **A** is :

(1) Amylose (2) Sucrose

(3) Maltose (4) Lactose

Official Ans. by NTA (4)

Sol. **Lactose :** It is a disaccharide of β–D–Galactose and β–D–Glucose with C_1 of galactose and C_4 of glucose link.

Lactose : β–D–Galactose + β–D–Glucose

12. $R-CN \xrightarrow[\text{(ii) } H_2O]{\text{(i) DIBAL–H}} R-Y$

Consider the above reaction and identify "Y"

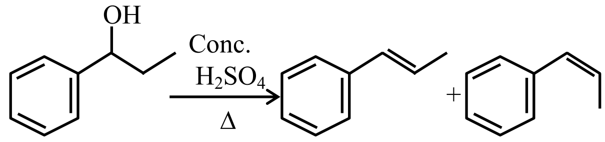
(1) $-CH_2NH_2$ (2) $-CONH_2$

(3) $-CHO$ (4) $-COOH$

Official Ans. by NTA (3)

Sol. $R-C\equiv N \xrightarrow[\text{(2) } H_2O]{\text{(1) DiBAL–H}} R-\underset{\underset{O}{\|}}{C}-H$

Here Y is $-\underset{\underset{O}{\|}}{C}-H$ Aldehyde

13. 

A B

consider the above reaction, and choose the correct statement :

(1) The reaction is not possible in acidic medium

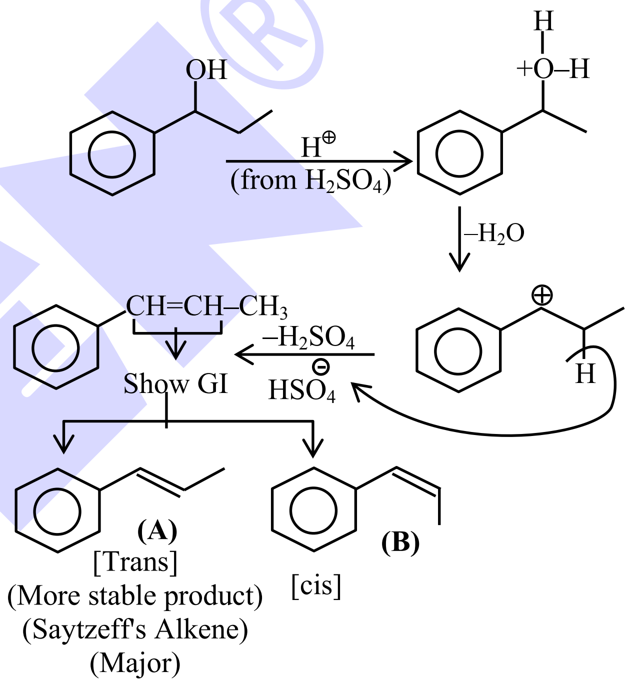
(2) Both compounds **A** and **B** are formed equally

(3) Compound **A** will be the major product

(4) Compound **B** will be the major product

Official Ans. by NTA (3)

Sol.



14. Match List - I with List - II :

List - I (compound)		List - II (effect/affected species)	
(a)	Carbon monoxide	(i)	Carcinogenic
(b)	Sulphur dioxide	(ii)	Metabolized by pyrus plants
(c)	Polychlorinated biphenyls	(iii)	Haemoglobin
(d)	Oxides of Nitrogen	(iv)	Stiffness of flower buds

Choose the **correct** answer from the options given below :

(1) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

(2) (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)

(3) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)

(4) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Official Ans. by NTA (1)

15. If the Thompson model of the atom was correct, then the result of Rutherford's gold foil experiment would have been :

(1) All of the α-particles pass through the gold foil without decrease in speed.

(2) α-Particles are deflected over a wide range of angles.

(3) All α-particles get bounced back by 180^o

(4) α-Particles pass through the gold foil deflected by small angles and with reduced speed.

Official Ans. by NTA (4)

Sol. As in Thomson model, protons are diffused (charge is not centred) α- particles deviate by small angles and due to repulsion from protons, their speed decreases.

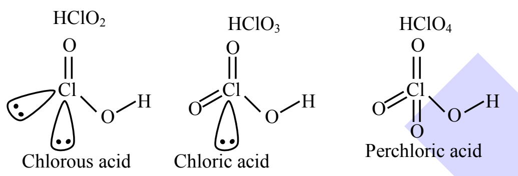
16. Number of Cl = O bonds in chlorous acid, chloric acid and perchloric acid respectively are :

(1) 3, 1 and 1 (2) 4, 1 and 0

(3) 1, 1 and 3 (4) 1, 2 and 3

Official Ans. by NTA (4)

Sol. Number of Cl = O bonds



17. Select the correct statements.

(A) Crystalline solids have long range order.

(B) Crystalline solids are isotropic.

(C) Amorphous solid are sometimes called pseudo solids.

(D) Amorphous solids soften over a range of temperatures.

(E) Amorphous solids have a definite heat of fusion.

Choose the most appropriate answer from the options given below.

(1) (A), (B), (E) only

(2) (B), (D) only

(3) (C), (D) only

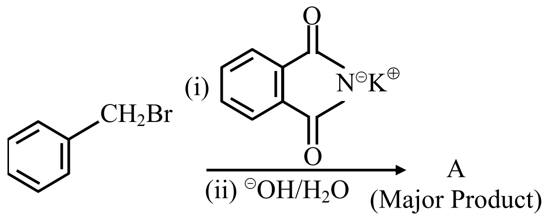
(4) (A), (C), (D) only

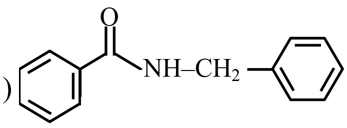
Official Ans. by NTA (4)

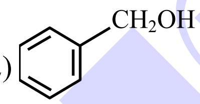
Sol. (A) Crystalline solids have definite arrangement of constituent particles and have long range order.

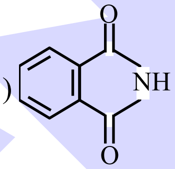
(C), (D) Different constituent particles of an amorphous solid have different bond strengths and soften over a range of temperatures.

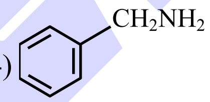
18. What is A in the following reaction ?



(1) 

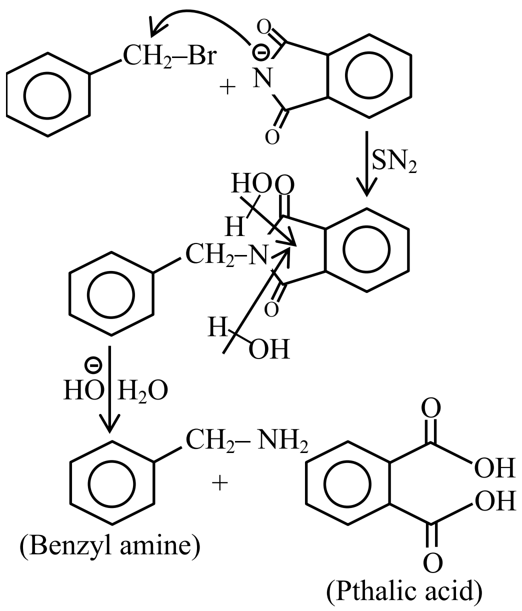
(2) 

(3) 

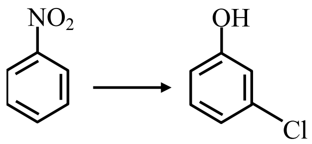
(4) 

Official Ans. by NTA (4)

Sol.



19. The correct sequence of correct reagents for the following transformation is :-



(1) (i) Fe, HCl (ii) Cl_2, HCl,
(iii) $NaNO_2$, HCl, 0°C (iv) H_2O/H^+

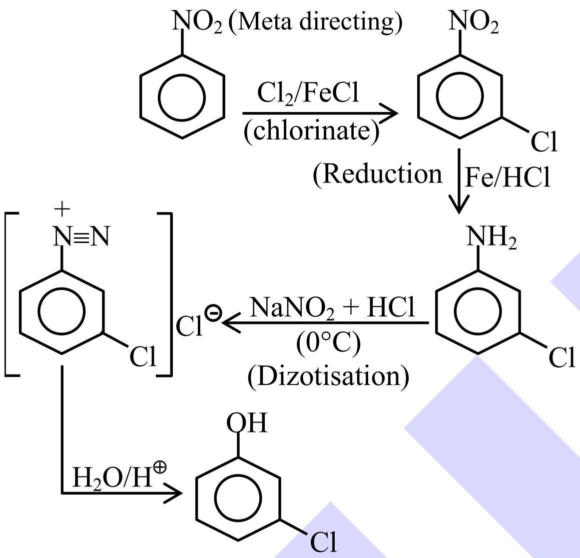
(2) (i) Fe, HCl (ii) $NaNO_2$, HCl, 0°C
(iii) H_2O/H^+ (iv) Cl_2, $FeCl_3$

(3) (i) Cl_2, $FeCl_3$ (ii) Fe, HCl
(iii) $NaNO_2$, HCl, 0°C (iv) H_2O/H^+

(4) (i) Cl_2, $FeCl_3$ (ii) $NaNO_2$, HCl, 0°C
(iii) Fe, HCl (iv) H_2O/H^+

Official Ans. by NTA (3)

Sol.



20. The addition of silica during the extraction of copper from its sulphide ore :-

(1) converts copper sulphide into copper silicate
(2) converts iron oxide into iron silicate
(3) reduces copper sulphide into metallic copper
(4) reduces the melting point of the reaction mixture

Official Ans. by NTA (2)

Sol. Silica is used to remove FeO impurity from the ore of copper

$$FeO + SiO_2 \rightarrow \underset{\text{(Slag)}}{\underset{\text{iron silicate}}{FeSiO_3}}$$

SECTION-B

1. The equilibrium constant for the reaction

$$A(s) \rightleftharpoons M(s) + \frac{1}{2}O_2(g)$$

is $K_p = 4$. At equilibrium, the partial pressure of O_2 is _____ atm. (Round off to the nearest integer)

Official Ans. by NTA (16)

Sol. $k_p = P_{O_2}^{1/2} = 4$

$\therefore\ P_{O_2} = 16$ bar $= 16$ atm

2. When 400 mL of 0.2M H_2SO_4 solution is mixed with 600 mL of 0.1 M NaOH solution, the increase in temperature of the final solution is _____ $\times 10^{-2}$ K. (Round off to the nearest integer).

[Use : $H^+(aq) + OH^-(aq) \rightarrow H_2O$:

$\Delta_\gamma H = -57.1$ kJ mol^{-1}]

Specific heat of $H_2O = 4.18$ J K^{-1} g^{-1}

density of $H_2O = 1.0$ g cm^{-3}

Assume no change in volume of solution on mixing.

Official Ans. by NTA (2)

ALLEN Ans. (82)

Sol. $n_{H^+} = \frac{400 \times 0.2}{1000} \times 2 = 0.16$

$n_{OH^-} = \frac{600 \times 0.1}{1000} = 0.06$ (L.R)

Now, heat liberated from reaction

= heat gained by solutions

or, $0.06 \times 57.1 \times 10^3$

$= (1000 \times 1.0) \times 4.18 \times \Delta T$

$\therefore\ \Delta T = 0.8196$ K

$= 81.96 \times 10^{-2}$ K $\approx 82 \times 10^{-2}$ K

3. $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$

The above reaction is carried out in a vessel starting with partial pressure $P_{SO_2} = 250\,\text{m bar}$, $P_{O_2} = 750\text{m}$ bar and $P_{SO_3} = 0$ bar. When the reaction is complete, the total pressure in the reaction vessel is _____ m bar. (Round off of the nearest integer).

Official Ans. by NTA (875)

Sol. $2SO_2\,(g) \quad + \quad O_2\,(g) \rightarrow \quad 2SO_3\,(g)$

	$2SO_2$	O_2	$2SO_3$
Initial	250 m bar (L. R.)	750 m bar	O
Final	–250 m bar	– 125 m bar	250 m bar
	0	625 m bar	250 m bar

$\therefore$ Final total pressure = 625 + 250 = 875 m bar

4. 10.0 mL of 0.05 M $KMnO_4$ solution was consumed in a titration with 10.0 mL of given oxalic acid dihydrate solution. The strength of given oxalic acid solution is .... $\times 10^{-2}$ g/L.

(Round off to the nearest integer)

Official Ans. by NTA (1575)

Sol. $n_{eq}\,KMnO_4 = n_{eq}\,H_2C_2O_4 . 2H_2O$

or , $\frac{10 \times 0.05}{1000} \times 5 = \frac{10 \times M}{1000} \times 2$

$\therefore$ Conc. of oxalic acid solution = 0.125 M

= 0.125 × 126 g /L = 15.75 g/L

= 1575 ×10^{-2} g/L

5. The total number of electrons in all bonding molecular orbitals of O_2^{2-} is ....

(Round off to the nearest integer)

Official Ans. by NTA (10)

Sol. M. O. Configuration of O_2^{2-} $((18\bar{e})$

$$\sigma 1s^2 \overset{*}{\sigma} 1s^2 \sigma 2s^2 \overset{*}{\sigma} 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2$$

$$\overset{*}{\pi} 2p_x^2 = \overset{*}{\pi} 2p_y^2$$

Total B.M.O electrons = 10

6. 3 moles of metal complex with formula $Co(en)_2Cl_3$ gives 3 moles of silver chloride on treatment with excess of silver nitrate. The secondary valency of Co in the complex is _____.

(Round off to the nearest integer)

Official Ans. by NTA (6)

Sol. $3\left[Co(en)_2 Cl_2\right]C\ell + \underset{(excess)}{AgNO_3} \rightarrow \underset{(white\ ppt.)}{3AgCl}$

Secondary valency of Co = 6

(C. N.)

7. In a solvent 50% of an acid HA dimerizes and the rest dissociates. The van't Hoff factor of the acid is _____ $\times 10^{-2}$.

(Round off to the nearest integer)

Official Ans. by NTA (125)

Sol. $2HA \rightleftharpoons H_2A_2 \quad HA \rightleftharpoons H^+ + A$

	2HA	H_2A_2	HA	H^+	A
Initial moles	$a \times \frac{50}{100}$	0	$a \times \frac{50}{100}$	0	0
Final moles	0	0.25 a	0	0.5a	0.5a

Now, $i = \frac{\text{final moles}}{\text{initial moles}} = \frac{0.25a + 0.5a + 0.5a}{0.5a + 0.5a}$

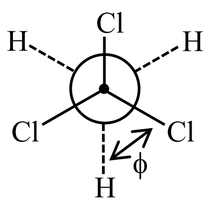
$= 1.25 = 125 \times 10^{-2}$

8. The dihedral angle in staggered form of Newman projection of 1, 1, 1-Trichloro ethane is .... degree. (Round off to the nearest integer)

(Round off to the nearest integer)

Official Ans. by NTA (60)

Sol. 1,1,1–Trichloro ethane [$CCl_3–CH_3$]



Dihedral angle(ϕ) = 60°

(Newmonns stqqared form)

9. For the first order reaction $A \rightarrow 2B$, 1 mole of reactant A gives 0.2 moles of B after 100 minutes. The half life of the reaction is ...... min. (Round off to the nearest integer).

[Use : $\ln 2 = 0.69$, $\ln 10 = 2.3$

Properties of logarithms : $\ln x^y = y \ln x$;

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y]$$

(Round off to the nearest integer)

Official Ans. by NTA (300)

Sol. $A \longrightarrow 2B$

	A	B
$t = 0$	1 mole	0
$t = 100$ min	$1 - x$	$2x$
	$= 0.9$mol	$= 0.2$mol

$$\text{Now, } t = \frac{t_{1/2}}{\ln 2} \times \frac{[A_0]}{[A_t]}$$

$$100 = \frac{t_{1/2}}{\ln 2} \times \ln \frac{1}{0.9} \Rightarrow t_{1/2} = 690 \text{ min.}$$

(taking $\ln 3 = 1.11$)

10. For the cell

$Cu(s) \mid Cu^{2+}(aq)\ (0.1M) \parallel Ag^{+}(aq)\ (0.01M) \mid Ag(s)$

the cell potential $E_1 = 0.3095$ V

For the cell

$Cu(s) \mid Cu^{2+}(aq)\ (0.01\ M) \parallel Ag^{+}(aq)\ (0.001\ M) \mid Ag(s)$

the cell potential = ______ $\times 10^{-2}$ V. (Round off the Nearest Integer).

[Use : $\frac{2.303\ RT}{F} = 0.059$]

Official Ans. by NTA (28)

Sol. Cell reaction is :

$Cu(s) + 2\ Ag^{+}(aq) \rightarrow Cu^{2+}(aq) + 2Ag(s)$

$$\text{Now, } E_{cell} = E^{o}_{Cell} - \frac{0.059}{2} \log \frac{[Cu^{2+}]}{[Ag^{+}]^2} \quad .... (1)$$

$$\therefore E_1 = 0.3095 = E^{o}_{Cell} - \frac{0.059}{2} . \log \frac{0.01}{(0.001)^2} \quad ... (2)$$

From (1) and (2) , $E_2 = 0.28$ V $= 28 \times 10^{-2}$ V

FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 27th July, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. The point P (a,b) undergoes the following three transformations successively :

(a) reflection about the line y = x.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to :

(1) 13 (2) 9 (3) 5 (4) 7

Official Ans. by NTA (2)

Sol. Image of A(a,b) along y = x is B(b,a). Translating it 2 units it becomes C(b + 2, a).

Now, applying rotation theorem

$$-\frac{1}{2}+\frac{7}{\sqrt{2}}i=((b+2)+ai)\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$\frac{-1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=\left(\frac{b+2}{\sqrt{2}}-\frac{a}{\sqrt{2}}\right)+i\left(\frac{b+2}{\sqrt{2}}+\frac{a}{\sqrt{2}}\right)$$

$\Rightarrow b - a + 2 = -1$ ....(i)

and $b + 2 + a = 7$ ....(ii)

$\Rightarrow a = 4; b = 1$

$\Rightarrow 2a + b = 9$

2. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is :

(1) 0 (2) –1 (3) 2 (4) 1

Official Ans. by NTA (4)

Sol. $^{10}C_8\left(25^{(x-1)}+7\right)\times\left(5^{(x-1)}+1\right)^{-1}=180$

$$\Rightarrow \frac{25^{x-1}+7}{5^{(x-1)}+1}=4$$

$$\Rightarrow \frac{t^2+7}{t+1}=4;$$

$\Rightarrow t = 1, 3 = 5^{x-1}$

$\Rightarrow x - 1 = 0$ (one of the possible value).

$\Rightarrow x = 1$

3. For real numbers α and β ≠ 0, if the point of intersection of the straight lines

$$\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3} \text{ and } \frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3},$$

lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to :

(1) 5 (2) 9 (3) 3 (4) 7

Official Ans. by NTA (4)

Sol. First line is $(\phi + \alpha, 2\phi + 1, 3\phi + 1)$

and second line is $(q\beta + 4, 3q + 6, 3q + 7)$.

For intersection $\phi + \alpha = q\beta + 4$ ...(i)

$2\phi + 1 = 3q + 6$ ...(i)

$3\phi + 1 = 3q + 7$ ...(iii)

for (ii) & (iii) $\phi = 1, q = -1$

So, from (i) $\alpha + \beta = 3$

Now, point of intersection is $(\alpha + 1,3,4)$

It lies on the plane.

Hence, $\alpha = 5$ & $\beta = -2$

4. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$f(x + y) + f(x - y) = 2 f(x) f(y)$, $f\left(\frac{1}{2}\right)=-1$. Then,

the value of $\sum_{k=1}^{20}\frac{1}{\sin(k)\sin(k+f(k))}$ is equal to :

(1) $\operatorname{cosec}^2(21)\cos(20)\cos(2)$

(2) $\sec^2(1)\sec(21)\cos(20)$

(3) $\operatorname{cosec}^2(1)\operatorname{cosec}(21)\sin(20)$

(4) $\sec^2(21)\sin(20)\sin(2)$

Official Ans. by NTA (3)

Sol. $f(x) = \cos \lambda x$

$\because f\left(\frac{1}{2}\right) = -1$

So, $-1 = \cos\frac{\lambda}{2}$

$\Rightarrow \lambda = 2\pi$

Thus $f(x) = \cos 2\pi x$

Now k is natural number

Thus $f(k) = 1$

$$\sum_{k=1}^{20}\frac{1}{\sin k \sin(k+1)} = \frac{1}{\sin 1}\sum_{k=1}^{20}\left[\frac{\sin((k+1)-k)}{\sin k\cdot\sin(k+1)}\right]$$

$$= \frac{1}{\sin 1}\sum_{k=1}^{20}(\cot k - \cot(k+1))$$

$$= \frac{\cot 1 - \cot 21}{\sin 1} = \operatorname{cosec}^2 1 \operatorname{cosec}(21)\cdot\sin 20$$

5. Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z-2| \le 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \ge 4\}$.

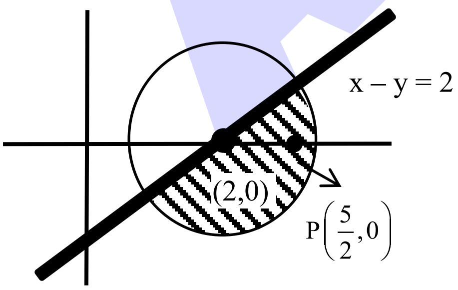
Then, the maximum value of $\left|z-\frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to :

(1) $\frac{3+2\sqrt{2}}{4}$ (2) $\frac{5+2\sqrt{2}}{2}$

(3) $\frac{3+2\sqrt{2}}{2}$ (4) $\frac{5+2\sqrt{2}}{4}$

Official Ans. by NTA (4)

Sol. $|t-2| \le 1$ Put $t = x + iy$



$(x-2)^2 + y^2 \le 1$

Also, $t(1+i) + \bar{t}(1-i) \ge 4$

Gives $x - y \ge 2$

Let point on circle be $A(2 + \cos\theta, \sin\theta)$

$\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$(AP)^2 = \left(2 + \cos\theta - \frac{5}{2}\right)^2 + \sin^2\theta$

$= \cos^2\theta - \cos\theta + \frac{1}{4} + \sin^2\theta$

$= \frac{5}{4} - \cos\theta$

For $(AP)^2$ maximum $\theta = -\frac{3\pi}{4}$

$(AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}+4}{4\sqrt{2}}$

6. A student appeared in an examination consisting of 8 true–false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is :

(1) 5 (2) 6 (3) 3 (4) 4

Official Ans. by NTA (1)

Sol. $P(E) < \frac{1}{2}$

$\Rightarrow \sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^{8-r}\left(\frac{1}{2}\right)^r < \frac{1}{2}$

$\Rightarrow \sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^8 < \frac{1}{2}$

$\Rightarrow {}^8C_n + {}^8C_{n+1} + ... + {}^8C_8 < 128$

$\Rightarrow 256 - \left({}^8C_0 + {}^8C_1 + ... + {}^8C_{n-1}\right) < 128$

$\Rightarrow {}^8C_0 + {}^8C_1 + ... + {}^8C_{n-1} > 128$

$\Rightarrow n - 1 \ge 4$

$\Rightarrow n \ge 5$

7. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to :

(1) 4 (2) 3 (3) 0 (4) 1

Official Ans. by NTA (3)

Sol. $x = \frac{1}{2}\left(\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right)$

and $2y = \tan\frac{\pi}{9} + \tan\frac{5\pi}{18}$

so, $x - 2y = \frac{1}{2}\left(\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right)$

$-\left(\tan\frac{\pi}{9} + \tan\frac{5\pi}{18}\right)$

$\Rightarrow |x - 2y| = \left|\frac{\cot\frac{\pi}{9} - \tan\frac{\pi}{9}}{2} - \tan\frac{5\pi}{18}\right|$

$= \left|\cot\frac{2\pi}{9} - \cot\frac{2\pi}{9}\right| = 0$

$\left(\text{as } \tan\frac{5\pi}{18} = \cot\frac{2\pi}{9}; \tan\frac{7\pi}{18} = \cot\frac{\pi}{9}\right)$

8. Let the mean and variance of the frequency distribution

x :	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f :	4	4	α	β

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be:

(1) 4 (2) 5

(3) $\frac{17}{3}$ (4) $\frac{16}{3}$

Official Ans. by NTA (3)

Sol. Given $32 + 8\alpha + 9\beta = (8 + \alpha + \beta) \times 6$

$\Rightarrow 2\alpha + 3\beta = 16$ ...(i)

Also, $4 \times 16 + 4 \times \alpha + 9\beta = (8 + \alpha + \beta) \times 6.8$

$\Rightarrow 640 + 40\alpha + 90\beta = 544 + 68\alpha + 68\beta$

$\Rightarrow 28\alpha - 22\beta = 96$

$\Rightarrow 14\alpha - 11\beta = 48$ ...(ii)

from (i) & (ii)

$\alpha = 5$ & $\beta = 2$

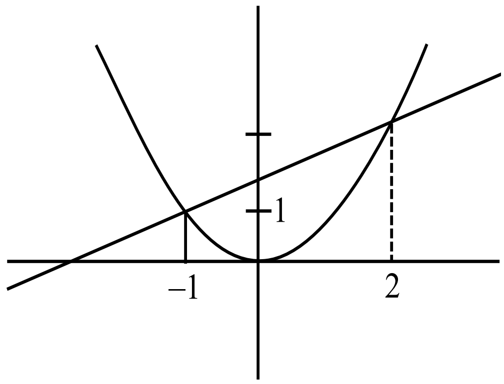
so, new mean $= \frac{32 + 35 + 18}{15} = \frac{85}{15} = \frac{17}{3}$

9. The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to :-

(1) $\frac{16}{3}$ (2) $\frac{2}{3}$ (3) $\frac{9}{2}$ (4) $\frac{4}{3}$

Official Ans. by NTA (3)

Sol.



$y - x = 2$, $x^2 = y$

Now, $x^2 = 2 + x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x + 1)(x - 2) = 0$

Area $= \int_{-1}^{2} (2 + x - x^2)$

$= \left|2x + \frac{x^2}{2} - \frac{x^3}{3}\right|_{-1}^{2}$

$= \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right)$

$= 6 - 3 + 2 - \frac{1}{2} = \frac{9}{2}$

10. Let $y = y(x)$ be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, $x > 2$. If $y(3) = 3$, then $y(4)$ is equal to :

(1) 4 (2) 12 (3) 8 (4) 16

Official Ans. by NTA (2)

Sol. $(x - x^3)dy = (y + yx^2 - 3x^4)dx$

$\Rightarrow xdy - ydx = (yx^2 - 3x^4)dx + x^3dy$

$\Rightarrow \frac{xdy - ydx}{x^2} = (ydx + xdy) - 3x^2dx$

$\Rightarrow d\left(\frac{y}{x}\right) = d(xy) - d(x^3)$

Integrate

$\Rightarrow \frac{y}{x} = xy - x^3 + c$

given $f(3) = 3$

$\Rightarrow \frac{3}{3} = 3 \times 3 - 3^3 + c$

$\Rightarrow c = 19$

$\therefore \frac{y}{x} = xy - x^3 + 19$

at $x = 4$, $\frac{y}{4} = 4y - 64 + 19$

$15y = 4 \times 45$

$\Rightarrow y = 12$

11. The value of $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to :

(1) 0 (2) 4 (3) – 4 (4) –1

Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$

$$= \lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$$

$$\left(\frac{\left(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}\right)}{\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}}\right)$$

$$\left(\frac{\left(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}\right)}{\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}}\right)$$

$$\left(\frac{\left(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}\right)}{\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{1-\sin x-(1+\sin x)}\right)$$

$$\left(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}\right)\left(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}\right)$$

$$\left(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}\right)$$

$$= \lim_{x\to 0}\frac{x}{(-2\sin x)}\left(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}\right)$$

$$\left(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}\right)\left(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}\right)$$

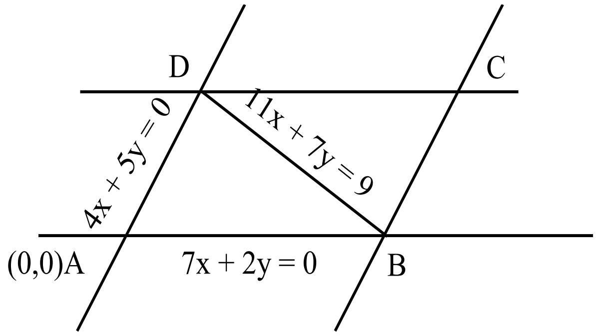
$$= \lim_{x\to 0}\left(-\frac{1}{2}\right)(2)\ (2)\ (2)\ \left\{\because \lim_{x\to 0}\frac{\sin x}{x}=1\right\} = -4$$

12. Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point :

(1) (1,2) (2) (2,2) (3) (2,1) (4) (1,3)

Official Ans. by NTA (2)

Sol. Both the lines pass through origin.



point D is equal of intersection of $4x + 5y = 0$ & $11x + 7y = 9$

So, coordinates of point $D = \left(\dfrac{5}{3}, -\dfrac{4}{3}\right)$

Also, point B is point of intersection of $7x + 2y = 0$ & $11x + 7y = 9$

So, coordinates of point $B = \left(-\dfrac{2}{3}, \dfrac{7}{3}\right)$

diagonals of parallelogram intersect at middle

let middle point of B,D

$$\Rightarrow \left(\frac{\frac{5}{3}-\frac{2}{3}}{2}, \frac{\frac{-4}{3}+\frac{7}{3}}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

equation of diagonal AC

$$\Rightarrow (y-0) = \frac{\frac{1}{\alpha}-0}{\frac{1}{\alpha}-0}(\pi - 0)$$

$y = x$

diagonal AC passes through (2, 2).

13. Let $\alpha = \max\limits_{x\in \mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$ and $\beta = \min\limits_{x\in \mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c - b$ is equal to :

(1) 42 (2) 47 (3) 43 (4) 50

Official Ans. by NTA (1)

Sol. $\alpha = \max\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$

$$= \max\{2^{6\sin 3x}\cdot 2^{8\cos 3x}\}$$

$$= \max\{2^{6\sin 3x + 8\cos 3x}\}$$

and $\beta = \min\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\} = \min\{2^{6\sin 3x+8\cos 3x}\}$

Now range of $6\sin 3x + 8\cos 3x$

$= \left[-\sqrt{6^2+8^2}, +\sqrt{6^2+8^2}\right] = [-10, 10]$

$\alpha = 2^{10}$ & $\beta = 2^{-10}$

So, $\alpha^{1/5} = 2^2 = 4$

$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$

quadratic $8x^2 + bx + c = 0$, $c - b =$

$8 \times [(\text{product of roots}] + (\text{sum of roots})$

$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4}\right] = 8 \times \left[\frac{21}{4}\right] = 42$

14. Let $f : [0, \infty) \to [0, 3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$$

Then which of the following is true ?

(1) f is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$

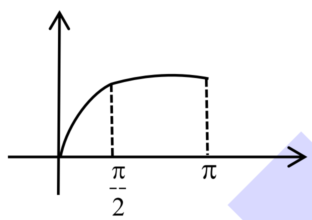
(2) f is differentiable everywhere in $(0, \infty)$

(3) f is not continuous exactly at two points in $(0, \infty)$

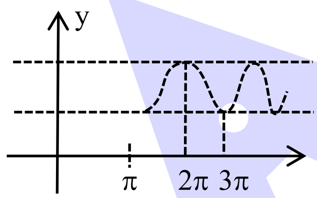
(4) f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$

Official Ans. by NTA (2)

Sol. Graph of $\max\{\sin t : 0 \le t \le x\}$ in $x \in [0, \pi]$

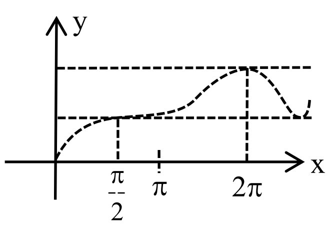


& graph of cos for $x \in [\pi, \infty)$



So graph of

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x, & 0 \le x \le \pi \\ 2 + \cos x & x > h \end{cases}$$



f(x) is differentiable everywhere in $(0, \infty)$

15. Let **N** be the set of natural numbers and a relation R on **N** be defined by

$R = \{(x,y) \in \mathbf{N} \times \mathbf{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$.

Then the relation R is :

(1) symmetric but neither reflexive nor transitive

(2) reflexive but neither symmetric nor transitive

(3) reflexive and symmetric, but not transitive

(4) an equivalence relation

Official Ans. by NTA (2)

Sol. $x^3 - 3x^2y - xy^2 + 3y^3 = 0$

$\Rightarrow x(x^2 - y^2) - 3y(x^2 - y^2) = 0$

$\Rightarrow (x - 3y)(x - y)(x + y) = 0$

Now, $x = y \ \forall (x,y) \in N \times N$ so reflexive

But not symmetric & transitive

See, (3,1) satisfies but (1,3) does not. Also (3,1) & (1,–1) satisfies but (3, –1) does not

16. Which of the following is the negation of the statement "for all $M > 0$, there exists $x \in S$ such that $x \ge M$" ?

(1) there exists $M > 0$, such that $x < M$ for all $x \in S$

(2) there exists $M > 0$, there exists $x \in S$ such that $x \ge M$

(3) there exists $M > 0$, there exists $x \in S$ such that $x < M$

(4) there exists $M > 0$, such that $x \ge M$ for all $x \in S$

Official Ans. by NTA (1)

Sol. P : for all $M > 0$, there exists $x \in S$ such that $x \ge M$.

$\sim P$: there exists $M > 0$, for all $x \in S$

Such that $x < m$

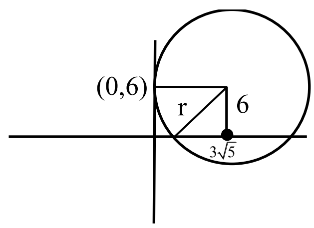
Negation of 'there exsits' is 'for all'.

17. Consider a circle C which touches the y-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to :

(1) $\sqrt{53}$ (2) 9 (3) 8 (4) $\sqrt{82}$

Official Ans. by NTA (2)

Sol.



$r = \sqrt{6^2 + (3\sqrt{5})^2}$

$= \sqrt{36 + 45} = 9$

18. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are $\sqrt{2}, 1$ and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta \left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to :

(1) $\sqrt{3}+1$ (2) 2

(3) 1 (4) $\frac{\sqrt{3}+1}{\sqrt{3}}$

Official Ans. by NTA (2)

Sol. $\vec{a} = (\vec{b}\cdot\vec{c})\vec{b} - (\vec{b}.\vec{b})\vec{c}$

$= 1.2\cos\theta\vec{b} - \vec{c}$

$\Rightarrow \vec{a} = 2\cos\theta\vec{b} - \vec{c}$

$|\vec{a}|^2 = (2\cos\theta)^2 + 2^2 - 2.2\cos\theta\vec{b}\cdot\vec{c}$

$\Rightarrow 2 = 4\cos^2\theta + 4 - 4\cos\theta \cdot 2\cos\theta$

$\Rightarrow -2 = -4\cos^2\theta$

$\Rightarrow \cos^2\theta = \frac{1}{2}$

$\Rightarrow \sec^2\theta = 2$

$\Rightarrow \tan^2\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

$1 + \tan\theta = 2.$

19. Let A and B be two 3×3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix A^3+B^3 is equal to :

(1) 2 (2) 4 (3) 1 (4) 0

Official Ans. by NTA (4)

Sol. $C = A^2 - B^2; |C| \neq 0$

$A^5 = B^5$ and $A^3B^2 = A^2B^2$

Now, $A^5 - A^3B^2 = B^5 - A^2B^3$

$\Rightarrow A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$

$\Rightarrow (A^3 + B^3)(A^2 - B^2) = 0$

Post multiplying inverse of $A^2 - B^2$:

$A^3 + B^3 = 0$

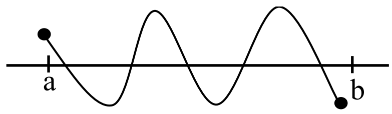
20. Let $f : (a,b) \to \mathbf{R}$ be twice differentiable function such that $f(x) = \int_a^x g(t)\,dt$ for a differentiable function g(x). If $f(x) = 0$ has exactly five distinct roots in (a, b), then $g(x)g'(x) = 0$ has at least :

(1) twelve roots in (a, b) (2) five roots in (a, b)

(3) seven roots in (a, b) (4) three roots in (a, b)

Official Ans. by NTA (3)

Sol.



$f(x) = \int_a^x g(t)dt$

$f(x) \to 5$

$f'(x) \to 4$

$g(x) \to 4$

$g'(x) \to 3$

SECTION-B

1. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a}\cdot\vec{b} = -1$ and $\vec{b}\cdot\vec{c} = 10$, then $(\vec{a}\times\vec{b})\cdot\vec{c}$ is equal to____ .

Official Ans. by NTA (9)

Sol. $\vec{a} = (1, -\alpha, \beta)$

$\vec{b} = (3, \beta, -\alpha)$

$\vec{c} = (-\alpha, -2, 1);\ \alpha, \beta \in I$

$\vec{a}.\vec{b} = -1 \Rightarrow 3 - \alpha\beta - \alpha\beta = -1$

$\Rightarrow \alpha\beta = 2$

1	2
2	1
−1	−2
−2	−1

$\vec{b}\cdot\vec{c} = 10$

$\Rightarrow -3\alpha - 2\beta - \alpha = 10$

$\Rightarrow 2\alpha + \beta + 5 = 0$

$\therefore \alpha = -2; \beta = -1$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$$

$= 1(-1 + 4) - 2(3 - 4) - 1(-6 + 2)$

$= 3 + 2 + 4 = 9$

2. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points. Q(3, –4, –5) and R(2, –3, 1) and the plane 2x + y + z = 7, is equal to______.

Official Ans. by NTA (7)

Sol. $\overrightarrow{QR} :- \frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+5}{-6} = r$

$\Rightarrow$ (x,y,z) $\equiv$ (r + 3, –r – 4, –6r – 5)

Now, satisfying it in the given plane.

We get r = –2.

so, required point of intersection is T(1,–2,7).

Hence, PT = 7.

3. If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to______.

Official Ans. by NTA (1)

Sol. Re (z) = $\frac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$

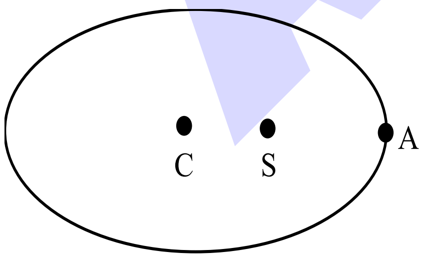
$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $\sin^2 3\theta + \cos^2\theta = 1$.

4. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, – 4), one focus at (4, – 4) and one vertex at (5, – 4). If mx – y = 4, m > 0 is a tangent to the ellipse E, then the value of $5m^2$ is equal to______.

Official Ans. by NTA (3)

Sol. Given C(3,–4), S(4,–4)



and A(5,–4)

Hence, a = 2 & ae = 1

$\Rightarrow e = \frac{1}{2}$

$\Rightarrow b^2 = 3.$

So, $E : \frac{(x-3)^2}{4} + \frac{(y+4)^2}{3} = 1$

Intersecting with given tangent.

$$\frac{x^2-6x+9}{4} + \frac{m^2x^2}{3} = 1$$

Now, D = 0 (as it is tangent)

So, $5m^2 = 3$.

5. If $\int_0^{\pi}(\sin^3 x)e^{-\sin^2 x}\,dx = \alpha - \frac{\beta}{e}\int_0^1 \sqrt{t}\,e^t dt$, then α + β is equal to _____ .

Official Ans. by NTA (5)

Sol. $I = 2\int_0^{\pi/2} \sin^3 x\, e^{-\sin^2 x} dx$

$$= 2\int_0^{\pi/2} \sin x\, e^{-\sin^2 x} dx + \int_0^{\pi/2} \underset{\mathrm{I}}{\cos x}\, \underbrace{e^{-\sin^2 x}(-\sin 2x)}_{\mathrm{II}}\, dx$$

$$= 2\int_0^{\pi/2} \sin x e^{-\sin^2 x} dx + \left[\cos x\, e^{-\sin^2 x}\right]_0^{\pi/2}$$

$$+ \int_0^{\pi/2} \sin x\, e^{-\sin^2 x}\, dx$$

$$= 3\int_0^{\pi/2} \sin x\, e^{-\sin^2 x} dx - 1$$

$$= \frac{3}{2}\int_{-1}^{0} \frac{e^{\alpha}d\alpha}{\sqrt{1+\alpha}} - 1 \quad (\text{Put } -\sin^2 x = t)$$

$$= \frac{3}{2e}\int_0^1 \frac{e^x}{\sqrt{x}} dx - 1 \quad (\text{put } 1 + \alpha = x)$$

$$= \frac{3}{2e}\int_0^1 \underset{\mathrm{II}}{e^x}\, \underset{\mathrm{II}}{\frac{1}{\sqrt{x}}} dx - 1$$

$$= 2 - \frac{3}{e}\int_0^1 e^x\sqrt{x}\, dx$$

Hence, $\alpha + \beta = \boxed{5}$

6. The number of real roots of the equation

$e^{4x} - e^{3x} - 4e^{2x} - e^{x} + 1 = 0$ is equal to____ .

Official Ans. by NTA (2)

Sol. $t^4 - t^3 - 4t^2 - t + 1 = 0,\ e^x = t > 0$

$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$

$\Rightarrow\ \alpha^2 - \alpha - 6 = 0,\ \alpha = t + \frac{1}{t} \geq 2$

$\Rightarrow \alpha = 3, -2$ (reject)

$\Rightarrow t + \frac{1}{t} = 3$

$\Rightarrow$ The number of real roots = 2

7. Let y= y(x) be the solution of the differential equation $dy = e^{\alpha x + y}\,dx$; $\alpha \in \mathbf{N}$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to____.

Official Ans. by NTA (2)

Sol. $\int e^{-y}dy = \int e^{\alpha x}\,dx$

$\Rightarrow e^{-y} = \frac{e^{\alpha x}}{\alpha} + c$ ...(i)

Put $(x,y) = (\ell n2, \ell n2)$

$\frac{-1}{2} = \frac{2^{\alpha}}{\alpha} + C$ ...(ii)

Put $(x,y) \equiv (0, -\ell n2)$ in (i)

$-2 = \frac{1}{\alpha} + C$ ...(iii)

(ii) – (iii)

$\frac{2^{\alpha} - 1}{\alpha} = \frac{3}{2}$

$\Rightarrow\ \alpha = 2$ (as $\alpha \in \mathbb{N}$)

8. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10}.(11)^{11}.(13)^{13}$ is equal to_____ .

Official Ans. by NTA (924)

Sol. $N = 2^{10} \times 5^{10} \times 11^{11} \times 13^{13}$

Now, power of 2 must be zero,

power of 5 can be anything,

power of 13 can be anything.

But, power of 11 should be even.

So, required number of divisors is

$1 \times 11 \times 14 \times 6 = 924$

9. Let $A = \{n \in \mathbf{N} \mid n^2 \leq n + 10{,}000\}$, $B = \{3k + 1 \mid k \in \mathbf{N}\}$ and $C = \{2k \mid k \in \mathbf{N}\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to_______ .

Official Ans. by NTA (832)

Sol. $B - C \equiv \{7, 13, 19, ... 97, ...\}$

Now, $n^2 - n \leq 100 \times 100$

$\Rightarrow n(n-1) \leq 100 \times 100$

$\Rightarrow A = \{1, 2, ..., 100\}$.

So, $A \cap (B - C) = \{7, 13, 19, ..., 97\}$

Hence, sum $= \frac{16}{2}(7 + 97) = 832$

10. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + ... + A^{20}$, then the sum of all the elements of the matrix M is equal to______.

Official Ans. by NTA (2020)

Sol. $A^n = \begin{bmatrix} 1 & n & \frac{n^2 + n}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$

So, required sum

$= 20 \times 3 + 2 \times \left(\frac{20 \times 21}{2}\right) + \sum_{r=1}^{20}\left(\frac{r^2 + r}{2}\right)$

$= 60 + 420 + 105 + 35 \times 41 = 2020$