

FINAL JEE-MAIN EXAMINATION – JULY, 2021

(Held On Sunday 25th July, 2021)

TIME : 2 : 00 PM to 5 : 00 PM

PHYSICS

SECTION-A

1. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is : (When v stands for velocity)

- (1) $2mv^3$ (2) $2mnv^3$
(3) $2nv^3$ (4) $2n^2v^3$

Official Ans. by NTA (1)

Sol. $t = mx^2 + nx$
 $\frac{1}{v} = \frac{dt}{dx} = 2mx + n$

$$v = \frac{1}{2mx + n}$$

$$\frac{dv}{dt} = -\frac{2m}{(2mx + n)^2} \left(\frac{dx}{dt} \right)$$

$$a = -(2m)v^3$$

2. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.

- (1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Official Ans. by NTA (2)

Sol. $K = \frac{1}{2} m \omega^2 (A^2 - x^2)$

$$= \frac{1}{2} m \omega^2 \left(A^2 - \frac{A^2}{4} \right)$$

$$= \frac{1}{2} m \omega^2 \left(\frac{3A^2}{4} \right)$$

$$K = \frac{3}{4} \left(\frac{1}{2} m \omega^2 A^2 \right)$$

3. A force $\vec{F} = (40\hat{i} + 10\hat{j})\text{N}$ acts on a body of mass 5 kg. If the body starts from rest, its position vector \vec{r} at time $t = 10$ s, will be :

- (1) $(100\hat{i} + 400\hat{j})\text{m}$ (2) $(100\hat{i} + 100\hat{j})\text{m}$
(3) $(400\hat{i} + 100\hat{j})\text{m}$ (4) $(400\hat{i} + 400\hat{j})\text{m}$

Official Ans. by NTA (3)

TEST PAPER WITH SOLUTION

Sol. $\frac{d\vec{v}}{dt} = \vec{a} = \frac{\vec{F}}{m} = (8\hat{i} + 2\hat{j})\text{m/s}^2$

$$\frac{d\vec{r}}{dt} = \vec{v} = (8t\hat{i} + 2t\hat{j})\text{m/s}$$

$$\vec{r} = (8\hat{i} + 2\hat{j}) \frac{t^2}{2} \text{m}$$

At $t = 10$ sec

$$\vec{r} = [(8\hat{i} + 2\hat{j})50]\text{m}$$

$$\Rightarrow \vec{r} = (400\hat{i} + 100\hat{j})\text{m}$$

4. A prism of refractive index μ and angle of prism A is placed in the position of minimum angle of deviation. If minimum angle of deviation is also A , then in terms of refractive index

(1) $2\cos^{-1}\left(\frac{\mu}{2}\right)$ (2) $\sin^{-1}\left(\frac{\mu}{2}\right)$

(3) $\sin^{-1}\left(\sqrt{\frac{\mu-1}{2}}\right)$ (4) $\cos^{-1}\left(\frac{\mu}{2}\right)$

Official Ans. by NTA (1)

Sol. $\mu = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\mu = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\mu = \frac{\sin A}{\sin \frac{A}{2}} = 2 \cos \frac{A}{2}$$

$$A = 2 \cos^{-1}\left(\frac{\mu}{2}\right)$$

5. A heat engine has an efficiency of $\frac{1}{6}$. When the temperature of sink is reduced by 62°C , its efficiency get doubled. The temperature of the source is :

- (1) 124°C (2) 37°C
(3) 62°C (4) 99°C

Official Ans. by NTA (4)

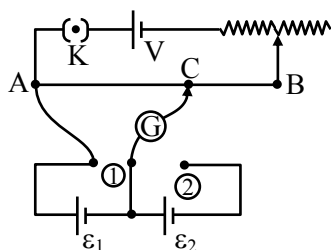
Sol. $\eta = 1 - \frac{T_L}{T_H} \dots (i)$

$$2\eta = 1 - \frac{(T_L - 62)}{T_H} = 1 - \frac{T_L}{T_H} + \frac{62}{T_H}$$

$$\Rightarrow \eta = \frac{62}{T_H} \Rightarrow \frac{1}{6} = \frac{62}{T_H} \Rightarrow T_H = 6 \times 62 = 372\text{K}$$

$$\text{In } ^\circ\text{C} \Rightarrow 372 - 273 = 99^\circ\text{C}$$

6. In the given potentiometer circuit arrangement, the balancing length AC is measured to be 250 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ratio of the emf of two cells, $\frac{\epsilon_1}{\epsilon_2}$ is :



- (1) $\frac{5}{3}$ (2) $\frac{8}{5}$ (3) $\frac{4}{3}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

Sol. $E_1 = k\ell_1 \dots (i)$

$$E_1 + E_2 = k\ell_2 \dots (ii)$$

$$\frac{E_1}{E_1 + E_2} = \frac{\ell_1}{\ell_2} = \frac{250}{400} = \frac{5}{8}$$

$$8E_1 = 5E_1 + 5E_2$$

$$3E_1 = 5E_2$$

$$\frac{E_1}{E_2} = \frac{5}{3}$$

7. Two ions having same mass have charges in the ratio 1 : 2. They are projected normally in a uniform magnetic field with their speeds in the ratio 2 : 3. The ratio of the radii of their circular trajectories is :
- (1) 1 : 4 (2) 4 : 3 (3) 3 : 1 (4) 2 : 3

Official Ans. by NTA (2)

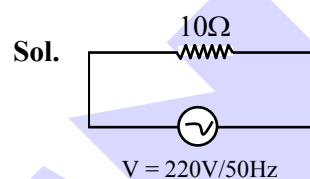
Sol. $R = \frac{mv}{qB} \Rightarrow \frac{R_1}{R_2} = \frac{\frac{mv_1}{q_1B}}{\frac{mv_2}{q_2B}} = \frac{v_1}{q_1} \times \frac{q_2}{v_2} = \frac{q_2}{q_1} \times \frac{v_1}{v_2}$

$$= \frac{2}{1} \times \left(\frac{2}{3}\right) = \frac{4}{3}$$

8. A 10Ω resistance is connected across 220V – 50Hz AC supply. The time taken by the current to change from its maximum value to the rms value is:

- (1) 2.5 ms (2) 1.5 ms
(3) 3.0 ms (4) 4.5 ms

Official Ans. by NTA (1)



$$\Rightarrow i = i_0 \sin \omega t$$

$$\text{When } i = i_0$$

$$i_0 = i_0 \sin \omega t_1 \Rightarrow \omega t_1 = \frac{\pi}{2} \dots (i)$$

$$\text{When } i = \frac{i_0}{\sqrt{2}}$$

$$\frac{i_0}{\sqrt{2}} = i_0 \sin \omega t_2 \Rightarrow \omega t_2 = \frac{\pi}{4} \dots (ii)$$

Time taken by current from maximum value to rms value

$$\Rightarrow (t_1 - t_2) = \frac{\pi}{2\omega} - \frac{\pi}{4\omega} = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi f}$$

$$= \frac{1}{8 \times 50}$$

$$= \frac{1}{400} \text{ sec}$$

$$= 2.5 \text{ ms}$$

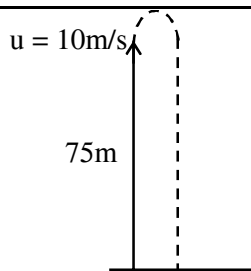
9. A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around :

(takes the value of g as 10 m/s^2)

- (1) 300 m (2) 200 m
(3) 125 m (4) 250 m

Official Ans. by NTA (3)

Sol.



Object is projected as shown so as per motion under gravity

$$S = ut + \frac{1}{2}at^2$$

$$-75 = +10t + \frac{1}{2}(-10)t^2 \Rightarrow t = 5 \text{ sec}$$

Object takes $t = 5$ s to fall on ground

Height of balloon from ground

$$H = 75 + ut$$

$$= 75 + 10 \times 5 = 125 \text{ m}$$

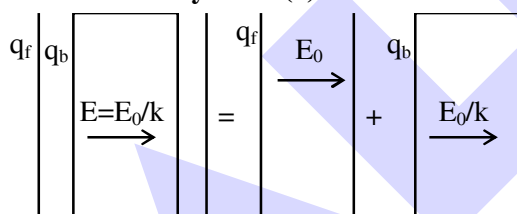
10. If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant k placed between the capacitor plates, then bound charge q_b can be expressed as :

$$(1) q_b = q_f \left(1 - \frac{1}{\sqrt{k}}\right) \quad (2) q_b = q_f \left(1 - \frac{1}{k}\right)$$

$$(3) q_b = q_f \left(1 + \frac{1}{\sqrt{k}}\right) \quad (4) q_b = q_f \left(1 + \frac{1}{k}\right)$$

Official Ans. by NTA (2)

Sol.



When a dielectric is inserted in a capacitor

Due to free charge $\vec{E} = \vec{E}_0$ only

$$\text{After dielectric } E' = \frac{E_0}{k}$$

$$q_B = q_f \left(1 - \frac{1}{k}\right)$$

11. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W , the weight of the same object on that planet will be :

$$(1) 2W \quad (2) W \quad (3) 2^{\frac{1}{3}}W \quad (4) \sqrt{2}W$$

Official Ans. by NTA (3)

Sol. Density is same

$$M = \frac{4}{3}\pi R^3 \rho, \quad 2m = \frac{4}{3}\pi R'^3 \rho$$

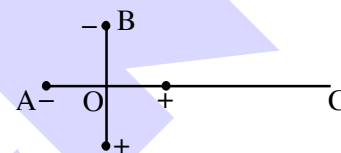
$$R' = 2^{1/3}R$$

$$\omega = \frac{GMm}{R^2}$$

$$\omega_2 = \frac{G2Mm}{R'^2}$$

$$\omega_2 = 2^{1/3}\omega$$

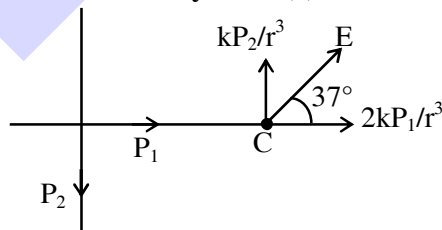
12. Two ideal electric dipoles A and B, having their dipole moment p_1 and p_2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of 37° with the axis. The ratio of the dipole moment of A and B, $\frac{p_1}{p_2}$ is : (take $\sin 37^\circ = \frac{3}{5}$)



$$(1) \frac{3}{8} \quad (2) \frac{3}{2} \quad (3) \frac{2}{3} \quad (4) \frac{4}{3}$$

Official Ans. by NTA (3)

Sol.



$$\tan 37^\circ = \frac{3}{4} = \frac{\frac{kP_2}{r^3}}{\frac{2kP_1}{r^3}} = \frac{P_2}{2P_1} = \frac{3}{4}$$

$$\frac{P_2}{P_1} = \frac{3}{2}$$

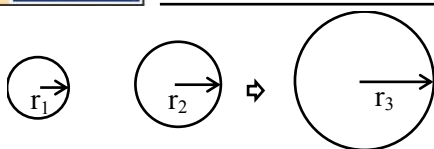
$$\frac{P_1}{P_2} = \frac{2}{3}$$

13. Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions. The resulting bubble has a radius equal to :

$$(1) \frac{r_1 r_2}{r_1 + r_2} \quad (2) \sqrt{r_1 r_2} \\ (3) \sqrt{r_1^2 + r_2^2} \quad (4) \frac{r_1 + r_2}{2}$$

Official Ans. by NTA (3)

Sol.



no. of moles is conserved

$$n_1 + n_2 = n_3$$

$$P_1 V_1 + P_2 V_2 = P_3 V_3$$

$$\frac{4S}{r_1} \left(\frac{4}{3} \pi r_1^3 \right) + \frac{4S}{r_2} \left(\frac{4}{3} \pi r_2^3 \right) = \frac{4S}{r_3} \left(\frac{4}{3} \pi r_3^3 \right)$$

$$r_1^2 + r_2^2 = r_3^2$$

$$r_3 = \sqrt{r_1^2 + r_2^2}$$

14. The force is given in terms of time t and displacement x by the equation

$$F = A \cos Bx + C \sin Dt$$

The dimensional formula of $\frac{AD}{B}$ is :

- (1) $[M^0 L T^{-1}]$ (2) $[M L^2 T^{-3}]$
(3) $[M^1 L^1 T^{-2}]$ (4) $[M^2 L^2 T^{-3}]$

Official Ans. by NTA (2)

Sol. $[A] = [MLT^{-2}]$

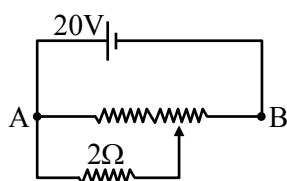
$$[B] = [L^{-1}]$$

$$[D] = [T^{-1}]$$

$$\left[\frac{AD}{B} \right] = \frac{[MLT^{-2}][T^{-1}]}{[L^{-1}]}$$

$$\left[\frac{AD}{B} \right] = [ML^2 T^{-3}]$$

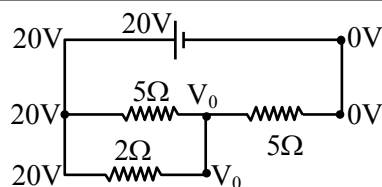
15. The given potentiometer has its wire of resistance 10Ω . When the sliding contact is in the middle of the potentiometer wire, the potential drop across 2Ω resistor is :



- (1) 10 V (2) 5 V (3) $\frac{40}{9}$ V (4) $\frac{40}{11}$ V

Official Ans. by NTA (3)

Sol.



$$\frac{20 - V_0}{5} + \frac{0 - V_0}{5} + \frac{20 - V_0}{2} = 0$$

$$4 + 10 = \frac{2V_0}{5} + \frac{V_0}{2}$$

$$14 = \frac{4V_0 + 5V_0}{10}$$

$$V_0 = \frac{140}{9} \text{ Volt}$$

Potential difference across 2Ω resistor is $20 - V_0$

$$\text{That is } \left(20 - \frac{140}{9} \right) \text{ Volt}$$

$$\text{Hence answer is } \left(\frac{40}{9} \right) \text{ Volt}$$

16. An electron moving with speed v and a photon moving with speed c , have same D-Broglie wavelength. The ratio of kinetic energy of electron to that of photon is :

- (1) $\frac{3c}{v}$ (2) $\frac{v}{3c}$ (3) $\frac{v}{2c}$ (4) $\frac{2c}{v}$

Official Ans. by NTA (3)

Sol. $\lambda_e = \lambda_{ph}$

$$\frac{h}{p_e} = \frac{h}{p_{ph}}$$

$$\sqrt{2mk_e} = \frac{E_{ph}}{c}$$

$$2mk_e = \frac{(E_{ph})^2}{c^2}$$

$$\frac{k_e}{E_{ph}} = \frac{E_{ph}}{c^2} \left(\frac{1}{2m} \right)$$

$$= \frac{p_{ph}}{c} \left(\frac{1}{2m} \right)$$

$$= \frac{p_e}{c} \left(\frac{1}{2m} \right)$$

$$= \frac{mv}{c} \frac{1}{2m}$$

$$= \frac{v}{2c}$$

17. The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1s and 2s is :

- (1) $3\alpha + 7\beta$ (2) $\frac{3}{2}\alpha + \frac{7}{3}\beta$
(3) $\frac{\alpha}{2} + \frac{\beta}{3}$ (4) $\frac{3}{2}\alpha + \frac{7}{2}\beta$

Official Ans. by NTA (2)

Sol. $V = \alpha t + \beta t^2$

$$\frac{ds}{dt} = \alpha t + \beta t^2$$

$$\int_{s_1}^{s_2} ds = \int_1^2 (\alpha t + \beta t^2) dt$$

$$S_2 - S_1 = \left[\frac{\alpha t^2}{2} + \frac{\beta t^3}{3} \right]_1^2$$

As particle is not changing direction

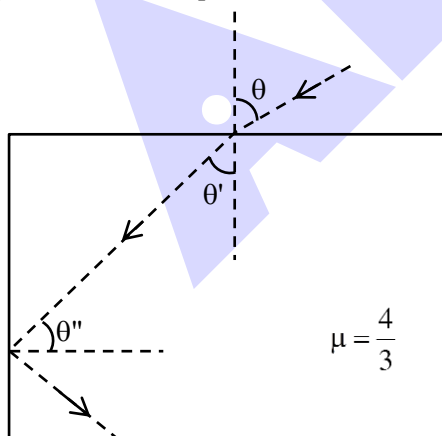
So distance = displacement.

$$\text{Distance} = \left[\frac{\alpha [4-1]}{2} + \frac{\beta [8-1]}{3} \right]$$

$$= \frac{3\alpha}{2} + \frac{7\beta}{3}$$

18. A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure.

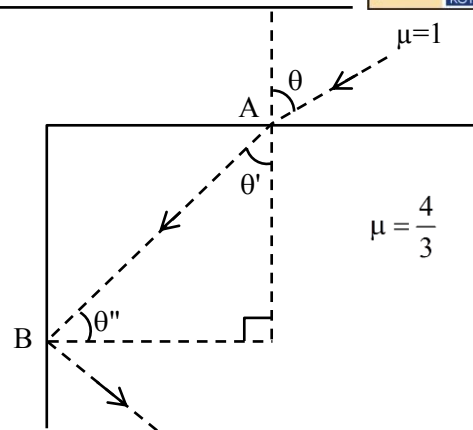
The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle θ should be equal to :



- (1) $\sin^{-1} \frac{\sqrt{7}}{3}$ (2) $\sin^{-1} \frac{\sqrt{5}}{4}$
(3) $\sin^{-1} \frac{\sqrt{7}}{4}$ (4) $\sin^{-1} \frac{\sqrt{5}}{3}$

Official Ans. by NTA (1)

Sol.



At maximum angle θ ray at point B goes in grazing emergence, at all less values of θ , TIR occurs.

At point B

$$\frac{4}{3} \times \sin \theta'' = 1 \times \sin 90^\circ$$

$$\theta'' = \sin^{-1} \left(\frac{3}{4} \right)$$

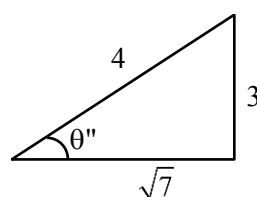
$$\theta' = \left(\frac{\pi}{2} - \theta'' \right)$$

At point A

$$1 \times \sin \theta = \frac{4}{3} \times \sin \theta'$$

$$\sin \theta = \frac{4}{3} \times \sin \left(\frac{\pi}{2} - \theta'' \right)$$

$$\sin \theta = \frac{4}{3} \cos \left[\cos^{-1} \frac{\sqrt{7}}{4} \right]$$



$$\sin \theta = \frac{4}{3} \times \frac{\sqrt{7}}{4}$$

$$\theta = \sin^{-1} \left(\frac{\sqrt{7}}{3} \right)$$

19. When radiation of wavelength λ is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8 V. If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6 V. The threshold wavelength of the metal is :

- (1) 2λ (2) 4λ (3) 8λ (4) 6λ

Official Ans. by NTA (2)

Sol. $V_s = h\nu - \phi$

$$4.8 = \frac{hc}{\lambda} - \phi \quad \dots (i)$$

$$1.6 = \frac{hc}{2\lambda} - \phi \quad \dots (ii)$$

Using above equation (i) – (ii)

$$3.2 = \frac{hc}{\lambda} - \frac{hc}{2\lambda}$$

$$3.2 = \frac{hc}{2\lambda} \quad \dots (iii)$$

$$\left[\lambda = \frac{hc}{6.4} \right]$$

Put in equation (ii)

$$\phi = 1.6$$

$$\frac{hc}{\lambda_{th}} = 1.6$$

$$\lambda_{th} = \frac{hc}{1.6}$$

$$= \left(\frac{hc}{6.4} \right) \times 4 = 4\lambda$$

- 20.** Two vectors \vec{X} and \vec{Y} have equal magnitude. The magnitude of $(\vec{X} - \vec{Y})$ is n times the magnitude of $(\vec{X} + \vec{Y})$. The angle between \vec{X} and \vec{Y} is :

$$(1) \cos^{-1} \left(\frac{-n^2 - 1}{n^2 - 1} \right) \quad (2) \cos^{-1} \left(\frac{n^2 - 1}{-n^2 - 1} \right)$$

$$(3) \cos^{-1} \left(\frac{n^2 + 1}{-n^2 - 1} \right) \quad (4) \cos^{-1} \left(\frac{n^2 + 1}{n^2 - 1} \right)$$

Official Ans. by NTA (2)

Sol. Given $X = Y$

$$\sqrt{X^2 + Y^2 - 2 \times Y \cos \theta}$$

$$= n \sqrt{X^2 + Y^2 + 2 \times Y \cos \theta}$$

Square both sides

$$2X^2(1 - \cos \theta) = n^2 \cdot 2X^2(1 + \cos \theta)$$

$$1 - \cos \theta = n^2 + n^2 \cos \theta$$

$$\cos \theta = \frac{1 - n^2}{1 + n^2}$$

$$\theta = \cos^{-1} \left[\frac{n^2 - 1}{-n^2 - 1} \right]$$

SECTION-B

- 1.** A system consists of two types of gas molecules A and B having same number density $2 \times 10^{25} / \text{m}^3$. The diameter of A and B are 10 \AA and 5 \AA respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collision is $\underline{\hspace{2cm}} \times 10^{-2}$

Official Ans. by NTA (25)

Sol. \therefore mean free path

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{d_2^2 n_2}{d_1^2 n_1}$$

$$= \left(\frac{5}{10} \right)^2 = 0.25 = 25 \times 10^{-2}$$

- 2.** A light beam of wavelength 500 nm is incident on a metal having work function of 1.25 eV , placed in a magnetic field of intensity B . The electrons emitted perpendicular to the magnetic field B , with maximum kinetic energy are bent into circular arc of radius 30 cm . The value of B is $\underline{\hspace{2cm}} \times 10^{-7} \text{ T}$.

Given $hc = 20 \times 10^{-26} \text{ J-m}$, mass of electron $= 9 \times 10^{-31} \text{ kg}$

Official Ans. by NTA (125)

Sol. By photoelectric equation

$$\frac{hc}{\lambda} - \phi = k_{\max}$$

$$k_{\max} = \frac{1240}{500} - 1.25 \approx 1.25$$

$$r = \frac{\sqrt{2mk}}{eB}$$

$$B = \frac{\sqrt{2mk}}{er}$$

$$= 125 \times 10^{-7} \text{ T}$$

- 3.** A message signal of frequency 20 kHz and peak voltage of 20 volt is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volt . The modulation index will be :

Official Ans. by NTA (1)

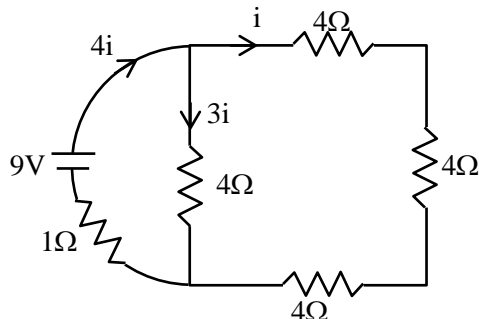
Sol. Modulation index

$$\mu = \frac{A_m}{A_c} = \frac{20}{20} = 1$$

4. A $16\ \Omega$ wire is bent to form a square loop. A 9 V supply having internal resistance of $1\ \Omega$ is connected across one of its sides. The potential drop across the diagonals of the square loop is $\times 10^{-1}\text{ V}$

Official Ans. by NTA (45)

Sol. here assume current as



By KVL in outer loop

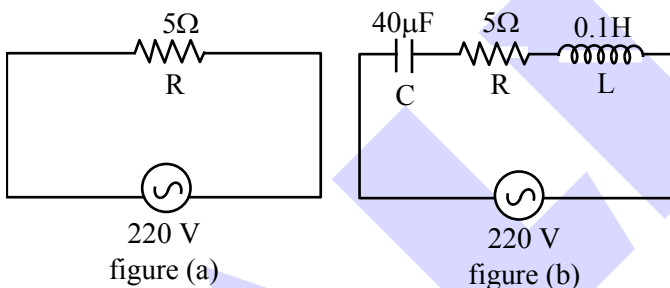
$$9 - 12i - 4i = 0$$

$$16i = 9$$

$$8i = \frac{9}{2} = 4.5$$

$$= 45 \times 10^{-1}$$

5. Two circuits are shown in the figure (a) & (b). At a frequency of _____ rad/s the average power dissipated in one cycle will be same in both the circuits.



Official Ans. by NTA (500)

Sol. For figure (a)

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R}$$

$$\frac{V_{\text{rms}}^2}{Z^2} \times R = \frac{V_{\text{rms}}^2}{R} \times 1$$

$$R^2 = Z^2$$

$$25 = (\sqrt{(x_c - x_L)^2 + 5^2})^2$$

$$25 = (x_c - x_L)^2 + 25$$

$$x_c = x_L \Rightarrow \frac{1}{\omega C} = \omega L$$

$$\omega^2 = \frac{1}{LC} = \frac{10^6}{0.1 \times 40}$$

$$\omega = 500$$

6. From the given data, the amount of energy required to break the nucleus of aluminium $^{27}_{13}\text{Al}$ is $\times 10^{-3}\text{ J}$.

Mass of neutron = 1.00866 u

Mass of proton = 1.00726 u

Mass of Aluminium nucleus = 27.18846 u

(Assume 1 u corresponds to $x\text{ J}$ of energy)

(Round off to the nearest integer)

Official Ans. by NTA (27)

Sol. $\Delta m = (Zm_p + (A - Z)m_n) - M_{\text{Al}}$
 $= (13 \times 1.00726 + 14 \times 1.00866) - 27.18846$
 $= 27.21562 - 27.18846$
 $= 0.02716\text{ u}$
 $E = 27.16 \times 10^{-3}\text{ J}$

7. A force of $F = (5y + 20)\hat{j}\text{ N}$ acts on a particle. The workdone by this force when the particle is moved from $y = 0\text{ m}$ to $y = 10\text{ m}$ is _____ J.

Official Ans. by NTA (450)

Sol. $F = (5y + 20)\hat{j}$
 $W = \int F dy = \int_0^{10} (5y + 20) dy$
 $= \left(\frac{5y^2}{2} + 20y \right)_0^{10}$
 $= \frac{5}{2} \times 100 + 20 \times 10$
 $= 250 + 200 = 450\text{ J}$

8. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm , about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is $\pi \times 10^{-1}\text{ Nm}$.

Official Ans. by NTA (4)

Sol. $\tau = \frac{\Delta L}{\Delta t} = \frac{I(\omega_f - \omega_i)}{\Delta t}$
 $\tau = \frac{\frac{mR^2}{2} \times [0 - \omega]}{\Delta t}$
 $= \frac{10 \times (20 \times 10^{-2})^2}{2} \times \frac{600 \times \pi}{30 \times 10}$
 $= 0.4\pi = 4\pi \times 10^{-2}$

9. In a semiconductor, the number density of intrinsic charge carriers at 27°C is $1.5 \times 10^{16} / \text{m}^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22} / \text{m}^3$. The electron density in the doped semiconductor is $\times 10^9 / \text{m}^3$.

Official Ans. by NTA (5)

Sol. $n_e n_h = n_i^2$

$$n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}}$$

$$= \frac{1.5 \times 1.5 \times 10^{32}}{4.5 \times 10^{22}}$$

$$5 \times 10^9 / \text{m}^3$$

10. The nuclear activity of a radioactive element becomes $\left(\frac{1}{8}\right)^{\text{th}}$ of its initial value in 30 years. The half-life of radioactive element is _____ years.

Official Ans. by NTA (10)

Sol. $A = A_0 e^{-\lambda t}$

$$\frac{A_0}{8} = A_0 e^{-\lambda t} \Rightarrow \lambda t = \ln 8$$

$$\lambda t = 3 \ln 2$$

$$\frac{\ln 2}{\lambda} = \frac{t}{3} = \frac{30}{3} = 10 \text{ years}$$

FINAL JEE-MAIN EXAMINATION – JULY, 2021

(Held On Sunday 25th July, 2021)

TIME : 3 : 00 PM to 6 : 00 PM

CHEMISTRY

SECTION-A

1. In the following the correct bond order sequence is:

- (1) $O_2^{2-} > O_2^+ > O_2^- > O_2$ (2) $O_2^+ > O_2^- > O_2^{2-} > O_2$
(3) $O_2^+ > O_2 > O_2^- > O_2^{2-}$ (4) $O_2 > O_2^- > O_2^{2-} > O_2^+$

Official Ans. by NTA (3)

Sol. O_2 (16 electrons)

$$\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \sigma_{2p_z}^2$$

$$\pi_{2p_x}^2 = \pi_{2p_y}^2, \pi_{2p_x}^{*1} = \pi_{2p_y}^{*1}, \sigma_{2p_z}^*$$

Bond order of $O_2 \Rightarrow 2$

Bond order of $O_2^- \Rightarrow 1.5$

Bond order of $O_2^{2-} \Rightarrow 1$

Bond order of $O_2^+ \Rightarrow 2.5$

2. A biodegradable polyamide can be made from:

- (1) Glycine and isoprene
(2) Hexamethylene diamine and adipic acid
(3) Glycine and aminocaproic acid
(4) Styrene and caproic acid

Official Ans. by NTA (3)

Sol. A biodegradable polyamide nylon-2-Nylon-6 is made from glycine and amino caproic acid

3. Match List I with List II :

	List-I Elements		List-II Properties
(a)	Li	(i)	Poor water solubility of Γ salt
(b)	Na	(ii)	Most abundant element in cell fluid
(c)	K	(iii)	Bicarbonate salt used in fire extinguisher
(d)	Cs	(iv)	Carbonate salt decomposes easily on heating

Choose the correct answer from the options given below :

- (1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)
(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
(4) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

Official Ans. by NTA (1)

TEST PAPER WITH SOLUTION

Sol. (a) CsI salt is poor water soluble due to its low hydration energy

(b) $NaHCO_3$ is used in fire extinguisher

(c) K is most abundant element in cell fluid

(d) Li_2CO_3 decomposes easily due to high covalent character caused by small size Li^+ cation.

4. Which one of the following metal complexes is most stable?

(1) $[Co(en)(NH_3)_4]Cl_2$

(2) $[Co(en)_3]Cl_2$

(3) $[Co(en)_2(NH_3)_2]Cl_2$

(4) $[Co(NH_3)_6]Cl_2$

Official Ans. by NTA (2)

Sol. Complex $[Co(en)_3]Cl_2$ is most stable complex among the given complex compounds because more number of chelate rings are present in this complex as compare to others.

(1) $[Co(en)(NH_3)_4]Cl_2$ 1 chelate ring

(2) $[Co(en)_3]Cl_2$ 3 chelate ring

(3) $[Co(en)_2(NH_3)_2]Cl_2$ 2 chelate ring

(4) $[Co(NH_3)_6]Cl_2$ 0 chelate ring

5. Match List I with List II : (Both having metallurgical terms)

	List-I		List-II
(a)	Concentration of Ag ore	(i)	Reverberatory furnace
(b)	Blast furnace	(ii)	Pig iron
(c)	Blister copper	(iii)	Leaching with dilute NaCN solution
(d)	Froth floatation method	(iv)	Sulfide ores

Choose the correct answer from the options given below :

(1) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)

(4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Official Ans. by NTA (1)

- Sol.** (a) Concentration of Ag is performed by leaching with dilute NaCN solution
 (b) Pig iron is formed in blast furnace
 (c) Blister Cu is produced in Bessemer converter
 (d) Froth floatation method is used for sulphide ores.

Note : During extraction of Cu reverberatory furnace is involved.

6. The ionic radii of F^- and O^{2-} respectively are 1.33 Å and 1.4 Å, while the covalent radius of N is 0.74 Å.

The correct statement for the ionic radius of N^{3-} from the following is :

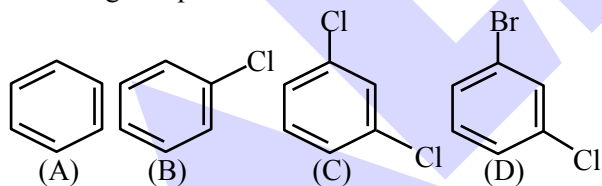
- (1) It is smaller than F^- and N
- (2) It is bigger than O^{2-} and F^-
- (3) It is bigger than F^- and N, but smaller than of O^{2-}
- (4) It is smaller than O^{2-} and F^- , but bigger than of N

Official Ans. by NTA (2)

- Sol.** F^- , O^{2-} and N^{3-} all are isoelectronic species in which N^{3-} have least number of protons due to which it's size increases as least nuclear attraction is experienced by the outer shell electrons.

Size order $N^{3-} > O^{2-} > F^-$

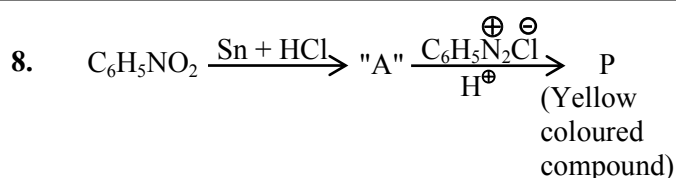
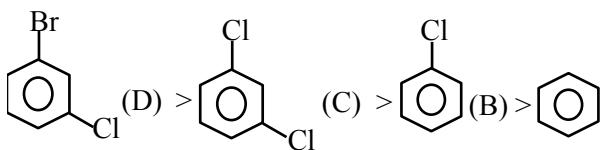
7. The correct decreasing order of densities of the following compounds is :



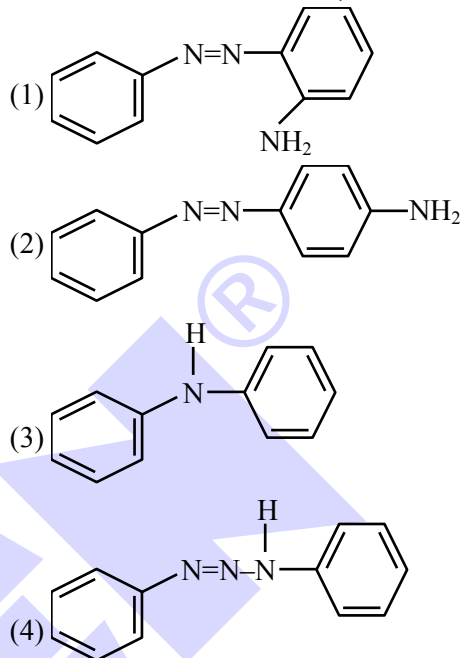
- (1) (D) > (C) > (B) > (A)
- (2) (C) > (D) > (A) > (B)
- (3) (C) > (B) > (A) > (D)
- (4) (A) > (B) > (C) > (D)

Official Ans. by NTA (1)

7. The density order

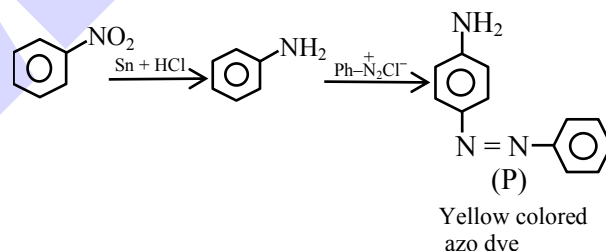


Consider the above reaction, the Product "P" is :



Official Ans. by NTA (2)

Sol.

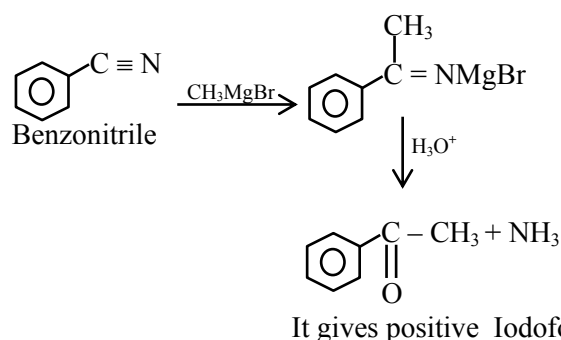


9. A reaction of benzonitrile with one equivalent CH_3MgBr followed by hydrolysis produces a yellow liquid "P". The compound "P" will give positive_____.

- (1) Iodoform test
- (2) Schiff's test
- (3) Ninhydrin's test
- (4) Tollen's test

Official Ans. by NTA (1)

Sol.



10. The spin only magnetic moments (in BM) for free Ti^{3+} , V^{2+} and Sc^{3+} ions respectively are (At.No. Sc : 21, Ti : 22, V : 23)

(1) 3.87, 1.73, 0 (2) 1.73, 3.87, 0
(3) 1.73, 0, 3.87 (4) 0, 3.87, 1.73

Official Ans. by NTA (2)

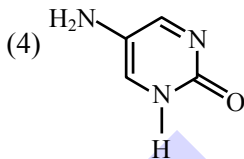
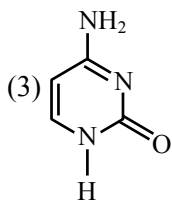
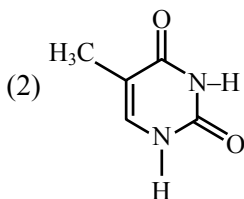
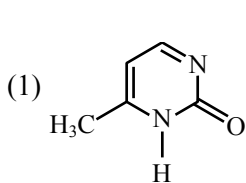
Sol. $\mu = \sqrt{n(n+2)} \text{ BM}$

$Ti^{3+} = [Ar]3d^1$ $n = 1$ $\mu = 1.73 \text{ BM}$

$V^{2+} = [Ar]3d^3$ $n = 3$ $\mu = 3.87 \text{ BM}$

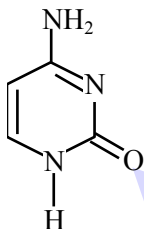
$Sc^{3+} = [Ar]3d^0 4s^0$ $n = 0$ $\mu = 0$

11. Which one of the following is correct structure for cytosine ?



Official Ans. by NTA (3)

Sol. The correct structure of cytosine

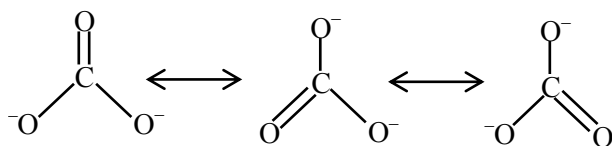


12. Identify the species having one π -bond and maximum number of canonical forms from the following :

(1) SO_3 (2) O_2 (3) SO_2 (4) CO_3^{2-}

Official Ans. by NTA (4)

Sol. Among SO_3 , O_2 , SO_2 and CO_3^{2-} , only O_2 and CO_3^{2-} has only one π -bond

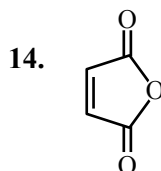


13. Which one of the following metals forms interstitial hydride easily ?

(1) Cr (2) Fe (3) Mn (4) Co

Official Ans. by NTA (1)

Sol. Elements of group 7,8,9 do not form hydrides thus Cr will only form hydride among the given elements (Fe, Mn, Co)



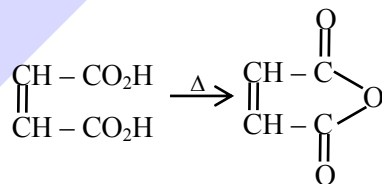
Maleic anhydride

Maleic anhydride can be prepared by :

- (1) Heating trans-but-2-enedioic acid
(2) Heating cis-but-2-enedioic acid
(3) Treating cis-but-2-enedioic acid with alcohol and acid
(4) Treating trans-but-2-enedioic acid with alcohol and acid

Official Ans. by NTA (2)

Sol. Cis but 2-enoic acid



Maleic anhydride

15. Given below are two statements :

Statement I : Chlorofluoro carbons breakdown by radiation in the visible energy region and release chlorine gas in the atmosphere which then reacts with stratospheric ozone.

Statement II : Atmospheric ozone reacts with nitric oxide to give nitrogen and oxygen gases, which add to the atmosphere.

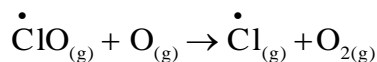
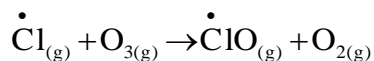
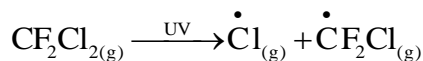
For the above statements choose the correct answer from the options given below :

- (1) **Statement I** is incorrect but **statement II** is true
(2) Both **statement I** and **II** are false
(3) **Statement I** is correct but **statement II** is false
(4) Both **statement I** and **II** are correct

Official Ans. by NTA (2)

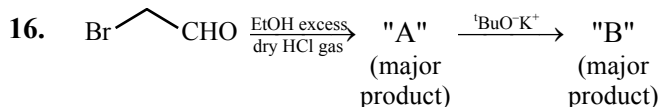
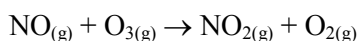
Sol. Statement (1)

CFCs are broken down by powerful UV radiation and releases chlorine free radical which reacts with ozone and start chain reaction.



Statement (2)

Atmosphere ozone reacts with nitric oxide to produce nitrogen dioxide and oxygen.



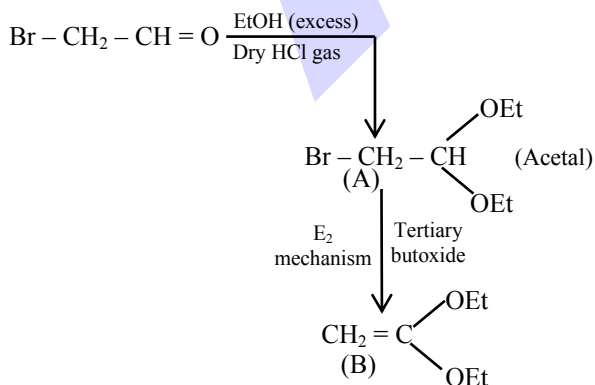
[where Et \Rightarrow $-\text{C}_2\text{H}_5$ $t\text{Bu} \Rightarrow (\text{CH}_3)_3\text{C}-$]

Consider the above reaction sequence, Product "A" and Product "B" formed respectively are :

- (1) $\text{Br}-\text{CH}_2-\text{CH}(\text{OEt})_2$, $\text{H}_2\text{C}=\text{CH}(\text{OEt})_2$
- (2) $\text{EtO}-\text{CH}_2-\text{CHO}$, $\text{EtO}-\text{CH}_2-\text{CH}(\text{OH})-\text{O}^t\text{Bu}$
- (3) $\text{EtO}-\text{CH}_2-\text{CH}(\text{OEt})_2$, $\text{H}_2\text{C}=\text{CH}(\text{OEt})_2$
- (4) $\text{Br}-\text{CH}_2-\text{CH}(\text{OEt})_2$, $t\text{BuO}-\text{CH}_2-\text{CH}(\text{OEt})_2$

Official Ans. by NTA (1)

Sol.



17. Match List I with List II :

List-I	List-II
Example of colloids	Classification

- | | |
|------------------|------------------------------------|
| (a) Cheese | (i) dispersion of liquid in liquid |
| (b) Pumice stone | (ii) dispersion of liquid in gas |
| (c) Hair cream | (iii) dispersion of gas in solid |
| (d) Cloud | (iv) dispersion of liquid in solid |
- Choose the most appropriate answer from the options given below

- (1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
 (2) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)
 (3) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
 (4) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Official Ans. by NTA (4)

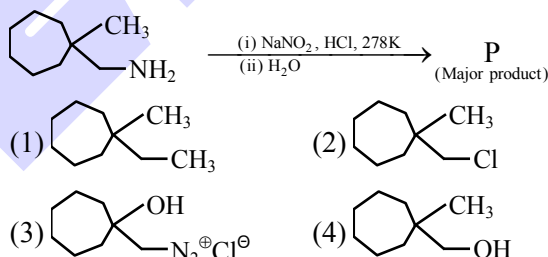
17. Cheese \rightarrow liquid in solid

Pumice stone \rightarrow gas in solid

Hair cream \rightarrow liquid in liquid

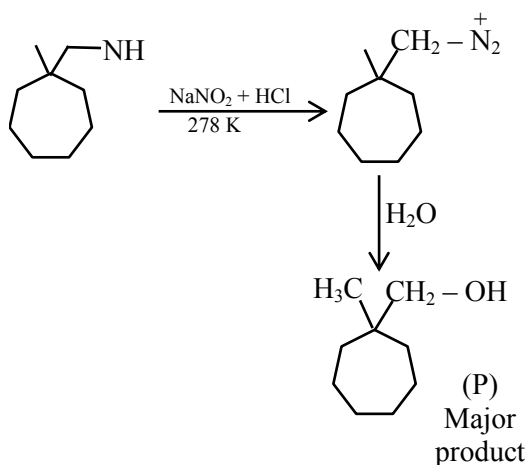
Cloud \rightarrow liquid in gas

18. What is the major product "P" of the following reaction ?



Official Ans. by NTA (4)

Sol.



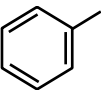


19. Identify the process in which change in the oxidation state is five :

- (1) $\text{Cr}_2\text{O}_7^{2-} \rightarrow 2\text{Cr}^{3+}$ (2) $\text{MnO}_4^- \rightarrow \text{Mn}^{2+}$
(3) $\text{CrO}_4^{2-} \rightarrow \text{Cr}^{3+}$ (4) $\text{C}_2\text{O}_4^{2-} \rightarrow 2\text{CO}_2$


Official Ans. by NTA (2)

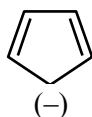
Sol. $\text{MnO}_4^- + 5\text{e}^- \rightarrow \text{Mn}^{+2}$

20. Which among the following is the strongest acid ?

- (1) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3$ (2) 
(3)  (4) 

Official Ans. by NTA (4)

Sol.  ; because its conjugate base is aromatic
Strongest acid



SECTION-B

1. A system does 200 J of work and at the same time absorbs 150 J of heat. The magnitude of the change in internal energy is _____ J. (Nearest integer)

Official Ans. by NTA (50)

1. $w = -200 \text{ J}$, $q = +150$: $\Delta U = q + w$

$$\Delta U = 150 - 200 = -50 \text{ J} : \text{magnitude} = 50 \text{ J} = |\Delta U|$$

2. An accelerated electron has a speed of $5 \times 10^6 \text{ ms}^{-1}$ with an uncertainty of 0.02%. The uncertainty in finding its location while in motion is $x \times 10^{-9} \text{ m}$. The value of x is _____. (Nearest integer)

[Use mass of electron = $9.1 \times 10^{-31} \text{ kg}$,

$$h = 6.63 \times 10^{-34} \text{ Js}, \pi = 3.14]$$

Official Ans. by NTA (58)

$$2. \Delta v = \frac{0.02}{100} \times 5 \times 10^6 = 10^3 \text{ m/s}$$

$$\Delta x \cdot \Delta v = \frac{h}{4\pi m}$$

$$x \times 10^{-9} \times 10^3 = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$x \times 10^{-9} \times 10^3 = 0.058 \times 10^{-3}$$

$$x = \frac{0.058 \times 10^{-6}}{10^{-9}} = 58$$

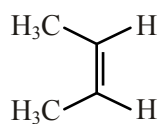
3. Number of electrons present in 4f orbital of Ho^{3+} ion is _____. (Given Atomic No. of Ho = 67)

Official Ans. by NTA (10)

3. $\text{Ho} = [\text{Xe}]4f^{11}6s^2$

$$\text{Ho}^{3+} = [\text{Xe}] 4f^{10}$$

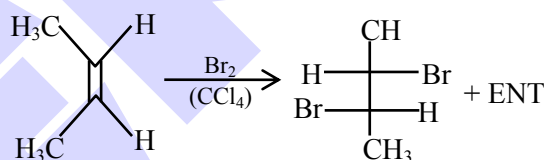
so number of e^- present in 4f is 10.

4.  + $\text{Br}_2 \xrightarrow{\text{CCl}_4}$ Product "P"

Consider the above chemical reaction. The total number of stereoisomers possible for Product 'P' is _____.

Official Ans. by NTA (2)

Sol.



The total number of products possible = 2

5. For a chemical reaction $A \rightarrow B$, it was found that concentration of B is increased by 0.2 mol L^{-1} in 30 min. The average rate of the reaction is _____ $\times 10^{-1} \text{ mol L}^{-1} \text{ h}^{-1}$. (in nearest integer)

Official Ans. by NTA (4)

$A \longrightarrow B$
5. $t=0$ 0
 $t=30 \text{ min}$ 0.2M

$$\text{Av. rate of reaction} = -\frac{\Delta[A]}{\Delta t} = \frac{\Delta[B]}{\Delta t} = \frac{(0.2-0)}{\frac{1}{2}}$$

$$= 0.4 = 4 \times 10^{-1} \text{ mol/L} \times \text{hr}$$

6. The number of significant figures in 0.00340 is _____.

Official Ans. by NTA (3)

6. Number of significant figures = 3
7. Assuming that $\text{Ba}(\text{OH})_2$ is completely ionised in aqueous solution under the given conditions the concentration of H_3O^+ ions in 0.005 M aqueous solution of $\text{Ba}(\text{OH})_2$ at 298 K is _____ $\times 10^{-12} \text{ mol L}^{-1}$. (Nearest integer)

Official Ans. by NTA (1)

7. $\text{Ba}(\text{OH})_2 \rightarrow \text{Ba}^{+2} + 2\text{OH}^-$
- ↓
- $2 \times 0.005 = 0.01 = 10^{-2}$
- At 298 K : in aq. solution $[\text{H}_3\text{O}^+][\text{OH}^-] = 10^{-14}$
- $[\text{H}_3\text{O}^+] = \frac{10^{-14}}{10^{-2}} = 10^{-12}$

8. 0.8 g of an organic compound was analysed by Kjeldahl's method for the estimation of nitrogen. If the percentage of nitrogen in the compound was found to be 42%, then _____ mL of 1 M H_2SO_4 would have been neutralized by the ammonia evolved during the analysis.

Official Ans. by NTA (12)

8. Organic compound : 0.8 gm
- wt. of N = $\left(\frac{42}{100} \times 0.8 \right) \text{ gm}$
- mole of N = $\frac{42 \times 0.8}{100 \times 14} = \frac{2.4}{100} \text{ mol}$
- moles of $\text{NH}_3 = \frac{2.4}{100}$
- $2\text{NH}_3 + \text{H}_2\text{SO}_4 \longrightarrow (\text{NH}_4)_2\text{SO}_4$
- ↓
- $\frac{2.4}{100} \text{ mole} \quad \frac{1.2}{100} \text{ mole}$
- $\frac{1.2}{100} = 1 \times V(\ell)$
- $\Rightarrow V_{\text{H}_2\text{SO}_4} = \frac{1.2}{100} \ell = 12 \text{ ml}$

9. When 3.00 g of a substance 'X' is dissolved in 100 g of CCl_4 , it raises the boiling point by 0.60 K. The molar mass of the substance 'X' is _____ g mol^{-1} . (Nearest integer).

[Given K_b for CCl_4 is $5.0 \text{ K kg mol}^{-1}$]

Official Ans. by NTA (250)

9. $\Delta T_b = K_b \times \text{molality}$
- $0.60 = 5 \times \left(\frac{3/M}{100/100} \right)$

$M = 250$

10. An LPG cylinder contains gas at a pressure of 300 kPa at 27°C . The cylinder can withstand the pressure of $1.2 \times 10^6 \text{ Pa}$. The room in which the cylinder is kept catches fire. The minimum temperature at which the bursting of cylinder will take place is _____ $^\circ\text{C}$. (Nearest integer)

Official Ans. by NTA (927)

10. $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{300 \times 10^3}{300} = \frac{1.2 \times 10^6}{T_2}$
- $\Rightarrow T_2 = 1200 \text{ K}$
- $T_2 = 927^\circ\text{C}$

FINAL JEE-MAIN EXAMINATION – JULY, 2021

(Held On Sunday 25th July, 2021)

TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The sum of all those terms which are rational numbers in the expansion of $(2^{1/3} + 3^{1/4})^{12}$ is:
(1) 89 (2) 27 (3) 35 (4) 43

Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^{12}C_r (2^{1/3})^r (3^{1/4})^{12-r}$

T_{r+1} will be rational number

when $r = 0, 3, 6, 9, 12$

& $r = 0, 4, 8, 12$

$\Rightarrow r = 0, 12$

$$T_1 + T_{13} = 1 \times 3^3 + 1 \times 2^4 \times 1$$

$$= 24 + 16 = 43$$

2. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is :

- (1) 8 (2) 6 (3) 4 (4) 5

Official Ans. by NTA (3)

Sol. $n_1 = 100$

$m = 250$

$\bar{X}_1 = 15$

$\bar{X} = 15.6$

$V_1(x) = 9$

$\text{Var}(x) = 13.44$

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1n_2}{(n_1 + n_2)^2}(\bar{X}_1 - \bar{X}_2)^2$$

$n_2 = 150, \bar{X}_2 = 16, V_2(x) = \sigma_2^2$

$$13.44 = \frac{100 \times 9 + 150 \times \sigma_2^2}{250} + \frac{100 \times 150}{(250)^2} \times 1$$

$\Rightarrow \sigma_2^2 = 16 \Rightarrow \sigma_2 = 4$

3. If $f(x) = \begin{cases} \int_0^x (5 + |1-t|) dt, & x > 2 \\ 5x + 1, & x \leq 2 \end{cases}$, then

(1) $f(x)$ is not continuous at $x = 2$

(2) $f(x)$ is everywhere differentiable

(3) $f(x)$ is continuous but not differentiable at $x = 2$

(4) $f(x)$ is not differentiable at $x = 1$

Official Ans. by NTA (3)

Sol. $f(x) = \int_0^1 (5 + (1-t)) dt + \int_1^x (5 + (t-1)) dt$

$$= 6 - \frac{1}{2} + \left(4t + \frac{t^2}{2} \right) \Big|_1^x$$

$$= \frac{11}{2} + 4x + \frac{x^2}{2} - 4 - \frac{1}{2}$$

$$= \frac{x^2}{2} + 4x + 1$$

$f(2^+) = 2 + 8 + 1 = 11$

$f(2) = f(2^-) = 5 \times 2 + 1 = 11$

\Rightarrow continuous at $x = 2$

Clearly differentiable at $x = 1$

$Lf'(2) = 5$

$Rf'(2) = 6$

\Rightarrow not differentiable at $x = 2$

4. If the greatest value of the term independent of 'x' in the expansion of $\left(x \sin \alpha + a \frac{\cos \alpha}{x} \right)^{10}$ is $\frac{10!}{(5!)^2}$,

then the value of 'a' is equal to:

- (1) -1 (2) 1 (3) -2 (4) 2

Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^{10}C_r (x \sin \alpha)^{10-r} \left(\frac{a \cos \alpha}{x} \right)^r$

$r = 0, 1, 2, \dots, 10$

T_{r+1} will be independent of x

when $10 - 2r = 0 \Rightarrow r = 5$

$$T_6 = {}^{10}C_5 (x \sin \alpha)^5 \times \left(\frac{a \cos \alpha}{x} \right)^5$$

$$= {}^{10}C_5 \times a^5 \times \frac{1}{2^5} (\sin 2\alpha)^5$$

will be greatest when $\sin 2\alpha = 1$

$\Rightarrow {}^{10}C_5 \frac{a^5}{2^5} = {}^{10}C_5 \Rightarrow a = 2$

5. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following:

- (1) The match will not be played and weather is not good and ground is wet.
- (2) If the match will not be played, then either weather is not good or ground is wet.
- (3) The match will be played and weather is not good or ground is wet.
- (4) The match will not be played or weather is good and ground is not wet.

Official Ans. by NTA (3)

Sol. p : weather is food

q : ground is not wet

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

\equiv weather is not good or ground is wet

6. The value of $\cot \frac{\pi}{24}$ is:

- (1) $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$
- (2) $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$
- (3) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$
- (4) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

Official Ans. by NTA (2)

Sol. $\cot \theta = \frac{1 + \cos 2\theta}{\sin 2\theta} = \frac{1 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$\theta = \frac{\pi}{24}$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{(2\sqrt{2} + \sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$$

$$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2$$

7. The lowest integer which is greater than

$\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$ is _____.

- (1) 3
- (2) 4
- (3) 2
- (4) 1

Official Ans. by NTA (1)

Sol. Let $P = \left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$,

Let $x = 10^{100}$

$\Rightarrow P = \left(1 + \frac{1}{x}\right)^x$

$\Rightarrow P = 1 + (x) \left(\frac{1}{x}\right) + \frac{(x)(x-1)}{2!} \cdot \frac{1}{x^2}$
 $+ \frac{(x)(x-1)(x-2)}{3!} \cdot \frac{1}{x^3} + \dots$

(upto $10^{100} + 1$ terms)

$\Rightarrow P = 1 + 1 + \left(\frac{1}{2} - \frac{1}{2x^2}\right) + \left(\frac{1}{3} - \dots\right) + \dots$ so on

$\Rightarrow P = 2 + \left(\text{Positive value less than } \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots\right)$

Also $e = 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$

$\Rightarrow \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots = e - 2$

$\Rightarrow P = 2 + (\text{positive value less than } e - 2)$

$\Rightarrow P \in (2, 3)$

\Rightarrow least integer value of P is 3

8. The value of the integral $\int_{-1}^1 \log(x + \sqrt{x^2 + 1}) dx$ is:

- (1) 2
- (2) 0
- (3) -1
- (4) 1

Official Ans. by NTA (2)

Sol. Let $I = \int_{-1}^1 \log(x + \sqrt{x^2 + 1}) dx$

$\because \log(x + \sqrt{x^2 + 1})$ is an odd function

$\therefore I = 0$

9. Let a , b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to:

- (1) $\frac{2}{\frac{1}{a} + \frac{1}{b}}$
- (2) $\frac{a+b}{2}$
- (3) $\frac{1}{a} + \frac{1}{b}$
- (4) \sqrt{ab}

Official Ans. by NTA (4)

Sol. Because vectors are coplanar

$$\text{Hence } \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$\Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

10. If $[x]$ be the greatest integer less than or equal to x ,

then $\sum_{n=8}^{100} \left[\frac{(-1)^n n}{2} \right]$ is equal to:

- (1) 0 (2) 4 (3) -2 (4) 2

Official Ans. by NTA (2)

Sol. $\sum_{n=8}^{100} \left[\frac{(-1)^n n}{2} \right]$

$$= 4 - 5 + 5 - 6 + 6 + \dots - 50 + 50 = 4$$

11. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in the interval}$$

$$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4} \text{ is:}$$

- (1) 4 (2) 1 (3) 2 (4) 3

Sol. $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

Apply : $R_1 \rightarrow R_1 - R_2$ & $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$$

$$\therefore x = \frac{\pi}{4}$$

Official Ans. by NTA (2)

12. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to:

- (1) 6 (2) 4 (3) 3 (4) 5

Official Ans. by NTA (1)

Sol. $|\vec{a}| = 2, |\vec{b}| = 5$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta = \pm 8$$

$$\sin \theta = \pm \frac{4}{5}$$

$$\therefore \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$= 10 \cdot \left(\pm \frac{3}{5} \right) = \pm 6$$

$$|\vec{a} \cdot \vec{b}| = 6$$

13. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is:

- (1) 2 (2) 3 (3) 1 (4) 4

Official Ans. by NTA (1)

Sol. $|x|^2 - |x| - 12 = 0$

$$(|x| + 3)(|x| - 4) = 0$$

$$|x| = 4 \Rightarrow x = \pm 2$$

14. Consider function $f: A \rightarrow B$ and

$g: B \rightarrow C$ ($A, B, C \subseteq \mathbf{R}$) such that $(g \circ f)^{-1}$ exists, then:

- (1) f and g both are one-one
(2) f and g both are onto
(3) f is one-one and g is onto
(4) f is onto and g is one-one

Official Ans. by NTA (3)

Sol. $\therefore (g \circ f)^{-1}$ exist $\Rightarrow g \circ f$ is bijective

$\Rightarrow 'f'$ must be one-one and ' g ' must be ONTO

15. If $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then P^{50} is:

- (1) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$
(3) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

Official Ans. by NTA (1)

Sol. $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

∴

$$\therefore P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

16. Let x be a random variable such that the probability function of a distribution is given by

$$P(X=0) = \frac{1}{2}, P(X=j) = \frac{1}{3^j} \quad (j = 1, 2, 3, \dots, \infty).$$

Then the mean of the distribution and $P(X \text{ is positive and even})$ respectively are:

- (1) $\frac{3}{8}$ and $\frac{1}{8}$ (2) $\frac{3}{4}$ and $\frac{1}{8}$
(3) $\frac{3}{4}$ and $\frac{1}{9}$ (4) $\frac{3}{4}$ and $\frac{1}{16}$

Official Ans. by NTA (2)

Sol. mean = $\sum x_i p_i = \sum_{r=0}^{\infty} r \cdot \frac{1}{3^r} = \frac{3}{4}$

$$p(x \text{ is even}) = \frac{1}{3^2} + \frac{1}{3^4} + \dots \infty$$

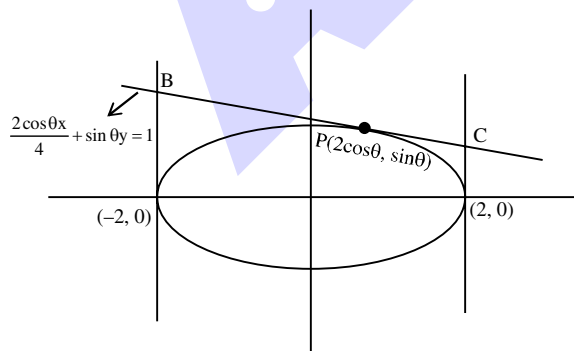
$$= \frac{\frac{1}{9}}{1 - \frac{1}{9}} = \frac{1/9}{8/9} = \frac{1}{8}$$

17. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point :

- (1) $(\sqrt{3}, 0)$ (2) $(\sqrt{2}, 0)$ (3) $(1, 1)$ (4) $(-1, 1)$

Official Ans. by NTA (1)

Sol.



$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Equation of tangent is $(\cos\theta)x + 2\sin\theta y = 2$

$$B\left(-2, \frac{1+\cos\theta}{\sin\theta}\right), \quad C\left(2, \frac{1-\cos\theta}{\sin\theta}\right)$$

$$B\left(-2, \cot\frac{\theta}{2}\right) \quad C\left(2, \tan\frac{\theta}{2}\right)$$

Equation of circle is

$$(x+2)(x-2) + \left(y - \cot\frac{\theta}{2}\right)\left(y - \tan\frac{\theta}{2}\right) = 0$$

$$x^2 - 4 + y^2 - \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)y + 1 = 0$$

so, $(\sqrt{3}, 0)$ satisfying option (1)

18. Let the equation of the pair of lines, $y = px$ and $y = qx$, can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

- (1) $x^2 - 3xy + y^2 = 0$ (2) $x^2 + 4xy - y^2 = 0$
(3) $x^2 + 3xy - y^2 = 0$ (4) $x^2 - 3xy - y^2 = 0$

Official Ans. by NTA (3)

Sol. $\frac{x^2 - y^2}{1 - (-5)} = \frac{xy}{-2}$

$$\frac{x^2 - y^2}{6} = \frac{xy}{-2}$$

$$\Rightarrow x^2 - y^2 = -3xy$$

$$\Rightarrow x^2 + 3xy - y^2 = 0$$

19. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to:

- (1) 1 (2) 4 (3) 2 (4) 3

Official Ans. by NTA (3)

Sol. ${}^nP_r = {}^nP_{r+1} \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$$\Rightarrow (n-r) = 1 \quad \dots(1)$$

$${}^nC_r = {}^nC_{r-1}$$

$$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\Rightarrow \frac{1}{r(n-r)!} = \frac{1}{(n-r+1)(n-r)!}$$

$$\Rightarrow n-r+1 = r$$

$$\Rightarrow n+1 = 2r \quad \dots(2)$$

$$(1) \Rightarrow 2r - 1 - r = 1 \Rightarrow r = 2$$

20. Let $y = y(x)$ be the solution of the differential equation $xdy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then

$y\left(\frac{\pi}{2}\right)$ is equal to:

- (1) $\frac{\pi^2}{4} + \frac{\pi}{2}$ (2) $\frac{\pi^2}{2} + \frac{\pi}{4}$
(3) $\frac{\pi^2}{2} - \frac{\pi}{4}$ (4) $\frac{\pi^2}{4} - \frac{\pi}{2}$

Official Ans. by NTA (1)

Sol. $xdy = (y + x^3 \cos x)dx$
 $xdy = ydx + x^3 \cos x dx$
 $\frac{xdy - ydx}{x^2} = \frac{x^3 \cos x dx}{x^2}$
 $\frac{d}{dx}\left(\frac{y}{x}\right) = \int x \cos x dx$
 $\Rightarrow \frac{y}{x} = x \sin x - \int 1 \cdot \sin x dx$
 $\frac{y}{x} = x \sin x + \cos x + C$
 $\Rightarrow 0 = -1 + C \Rightarrow C = 1, x = \pi, y = 0$
 so $\frac{y}{x} = x \sin x + \cos x + 1$
 $y = x^2 \sin x + x \cos x + x \quad x = \frac{\pi}{2}$
 $y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

SECTION-B

1. Let $n \in \mathbb{N}$ and $[x]$ denote the greatest integer less than or equal to x . If the sum of $(n+1)$ terms ${}^nC_0, 3 \cdot {}^nC_1, 5 \cdot {}^nC_2, 7 \cdot {}^nC_3, \dots$ is equal to $2^{100} \cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to _____.

Official Ans. by NTA (98)

Sol. $1 \cdot {}^nC_0 + 3 \cdot {}^nC_1 + 5 \cdot {}^nC_2 + \dots + (2n+1) \cdot {}^nC_n$
 $T_r = (2r+1) \cdot {}^nC_r$
 $S = \sum T_r$
 $S = \sum (2r+1) \cdot {}^nC_r = \sum 2r \cdot {}^nC_r + \sum {}^nC_r$
 $S = 2(n \cdot 2^{n-1}) + 2^n = 2^n(n+1)$
 $2^n(n+1) = 2^{100} \cdot 101 \Rightarrow n = 100$

$$2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 98$$

2. Consider the function $f(x) = \frac{P(x)}{\sin(x-2)}$, $x \neq 2$
 $= 7$, $x = 2$

Where $P(x)$ is a polynomial such that $P''(x)$ is always a constant and $P(3) = 9$. If $f(x)$ is continuous at $x = 2$, then $P(5)$ is equal to _____.

Official Ans. by NTA (39)

Sol. $f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)}, & x \neq 2 \\ 7, & x = 2 \end{cases}$

$P''(x) = \text{const.} \Rightarrow P(x)$ is a 2 degree polynomial

$f(x)$ is cont. at $x = 2$

$$f(2^+) = f(2^-)$$

$$\lim_{x \rightarrow 2^+} \frac{P(x)}{\sin(x-2)} = 7$$

$$\lim_{x \rightarrow 2^+} \frac{(x-2)(ax+b)}{\sin(x-2)} = 7 \Rightarrow \boxed{2a+b=7}$$

$$P(x) = (x-2)(ax+b)$$

$$P(3) = (3-2)(3a+b) = 9 \Rightarrow \boxed{3a+b=9}$$

$$\boxed{a=2, b=3}$$

$$P(5) = (5-2)(2.5+3) = 3.13 = 39$$

3. The equation of a circle is

$$\text{Re}(z^2) + 2(\text{Im}(z))^2 + 2\text{Re}(z) = 0, \text{ where } z = x + iy.$$

A line which passes through the center of the given circle and the vertex of the parabola,

$$x^2 - 6x - y + 13 = 0, \text{ has y-intercept equal to } \underline{\hspace{2cm}}.$$

Official Ans. by NTA (1)

Sol. Equation of circle is $(x^2 - y^2) + 2y^2 + 2x = 0$

$$x^2 + y^2 + 2x = 0$$

$$\text{Centre} : (-1, 0)$$

$$\text{Parabola} : x^2 - 6x - y + 13 = 0$$

$$(x-3)^2 = y - 4$$

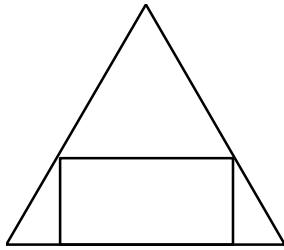
$$\text{Vertex} : (3, 4)$$

$$\text{Equation of line} \equiv y - 0 = \frac{4-0}{3+1}(x+1)$$

$$y = x + 1$$

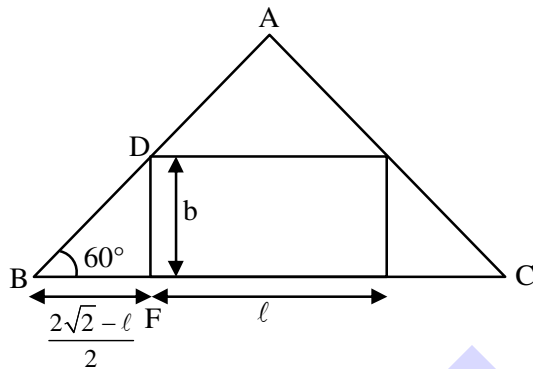
$$\text{y-intercept} = 1$$

4. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is _____.



Official Ans. by NTA (3)

Sol.



In $\triangle DBF$

$$\tan 60^\circ = \frac{2b}{2\sqrt{2}-\ell} \Rightarrow b = \frac{\sqrt{3}(2\sqrt{2}-\ell)}{2}$$

$A = \text{Area of rectangle} = \ell \times b$

$$A = \ell \times \frac{\sqrt{3}}{2}(2\sqrt{2}-\ell)$$

$$\frac{dA}{d\ell} = \frac{\sqrt{3}}{2}(2\sqrt{2}-\ell) - \frac{\ell \cdot \sqrt{3}}{2} = 0$$

$$\boxed{\ell = \sqrt{2}}$$

$$A = \ell \times b = \sqrt{2} \times \frac{\sqrt{3}}{2}(\sqrt{2}) = \sqrt{3}$$

$$\Rightarrow \boxed{A^2 = 3}$$

5. If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between \vec{a} and \vec{b} (in degrees) is _____.

Official Ans. by NTA (60)

Sol. $(\vec{a} + 3\vec{b}) \perp (7\vec{a} - 5\vec{b})$

$$(\vec{a} + 3\vec{b}) \cdot (7\vec{a} - 5\vec{b}) = 0$$

$$7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a} \cdot \vec{b} = 0 \quad \dots(1)$$

$$(\vec{a} - 4\vec{b}) \cdot (7\vec{a} - 2\vec{b}) = 0$$

$$7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a} \cdot \vec{b} = 0 \quad \dots(2)$$

from (1) & (2)

$$|\vec{a}| = |\vec{b}|$$

$$\cos \theta = \frac{|\vec{b}|}{2|\vec{a}|} \therefore \theta = 60^\circ$$

6. Let a curve $y = f(x)$ pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x . Then the value of $f(e)$ is equal to _____.

Official Ans. by NTA (1)

Sol. $y' = \frac{2y}{x \ln x}$

$$\Rightarrow \frac{dy}{y} = \frac{2dx}{x \ln x}$$

$$\Rightarrow \ln|y| = 2\ln|\ln x| + C$$

$$\text{put } x = 2, y = (\ln 2)^2$$

$$\Rightarrow C = 0$$

$$\Rightarrow y = (\ln x)^2$$

$$\Rightarrow f(e) = 1$$

7. If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$, then the value of $a^4 + b^4 + c^4$ is equal to _____.

Official Ans. by NTA (13)

Sol. $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\sum ab = -3$

$$(ab + bc + ca)^2 = \sum (ab)^2 + 2abc\sum a$$

$$\Rightarrow \sum (ab)^2 = -2$$

$$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - 2\sum (ab)^2$$

$$= 9 - 2(-2) = 13$$

8. A fair coin is tossed n -times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is _____.

Official Ans. by NTA (4)

Sol. $P(\text{Head}) = \frac{1}{2}$

$$1 - P(\text{All tail}) \geq 0.9$$

$$1 - \left(\frac{1}{2}\right)^n \geq 0.9$$

$$\Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{10}$$

$$\Rightarrow n_{\min} = 4$$

9. If the co-efficient of x^7 and x^8 in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal to _____.

Official Ans. by NTA (55)

Sol. ${}^nC_7 2^{n-7} \frac{1}{3^7} = {}^nC_8 2^{n-8} \frac{1}{3^8}$

$$\Rightarrow n - 7 = 48 \Rightarrow n = 55$$

10. If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and

$\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is _____.

Official Ans. by NTA (1)

Sol. $\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$

$$(k+1)[2-6] - 4[1-9] + 6[2-6] = 0$$

$$k = 1$$