FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 20th July, 2021) **TIME : 2 : 00 PM to 5 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be :

(1) 100% (2) 200%

(3) 300% (4) 400%

Official Ans. by NTA (1)

Sol. $K_2 = 4K_1$

$$\frac{1}{2}mv_2^2 = 4\frac{1}{2}mv_1^2$$

$$v_2 = 2v_1$$

$$P = mv$$

$$P_2 = mv_2 = 2mv_1$$

$$P_1 = mv_1$$

$$\% \text{ change } = \frac{\Delta P}{P_1}\times 100 = \frac{2mv_1 - mv_1}{mv_1}\times 100 = 100\%$$

2. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be :

(1) $\frac{t_1t_2}{t_2-t_1}$ (2) $\frac{t_1+t_2}{2}$ (3) $\frac{t_1t_2}{t_2+t_1}$ (4) $t_2 - t_1$

Official Ans. by NTA (3)

Sol. L = Length of escalator

$$V_{b/esc} = \frac{L}{t_1}$$

When only escalator is moving.

$$V_{esc} = \frac{L}{t_2}$$

when both are moving

$$V_{b/g} = V_{b/esc} + V_{esc}$$

$$V_{b/g} = \frac{L}{t_1} + \frac{L}{t_2} \Rightarrow \left[t = \frac{L}{V_{b/g}} = \frac{t_1t_2}{t_1+t_2}\right]$$

3. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is :

(1) 1.5 (2) 2.0

(3) 0.7 (4) 3.0

Official Ans. by NTA (4)

Sol. $T^2 \propto R^3$

$$T = kR^{3/2}$$

$$\frac{dT}{T} = \frac{3}{2}\frac{dR}{R}$$

$$= \frac{3}{2}\times 0.02 = 0.03$$

% Change = 3%

4. With what speed should a galaxy move outward with respect to earth so that the sodium-D line at wavelength 5890 Å is observed at 5896 Å ?

(1) 306 km/sec (2) 322 km/sec

(3) 296 km/sec (4) 336 km/sec

Official Ans. by NTA (1)

Sol. $f = f_0\sqrt{\frac{1+\beta}{1-\beta}}$ $\beta = \frac{v}{c}$

$$\frac{f}{f_0}\sqrt{\frac{1+\beta}{1-\beta}}$$

$$\left(1+\frac{\Delta f}{f_0}\right)^2 = (1+\beta)(1-\beta)^{-1}$$

β is small compared to 1

$$\left(1+\frac{2\Delta f}{f_0}\right) = (1+2\beta)$$

$$\beta = \frac{\Delta f}{f_0} = \frac{v}{c}$$

$$v = 6\times\frac{c}{5890} = 305.6 \text{ km/s}$$

5. The length of a metal wire is ℓ_1, when the tension in it is T_1 and is ℓ_2 when the tension is T_2. The natural length of the wire is :

(1) $\sqrt{\ell_1\ell_2}$ (2) $\dfrac{\ell_1T_2-\ell_2T_1}{T_2-T_1}$

(3) $\dfrac{\ell_1T_2+\ell_2T_1}{T_2+T_1}$ (4) $\dfrac{\ell_1+\ell_2}{2}$

Official Ans. by NTA (2)

Sol. $T_1=k(\ell_1-\ell_0)$

$T_2=k(\ell_2-\ell_0)$

$$\frac{T_1}{T_2}=\frac{\ell_1-\ell_0}{\ell_2-\ell_0}$$

$$\frac{T_1\ell_2-T_2\ell_1}{T_1-T_2}=\ell_0$$

6. In an electromagnetic wave the electric field vector and magnetic field vector are given as $\vec{E}=E_0\,\hat{i}$ and $\vec{B}=B_0\hat{k}$ respectively. The direction of propagation of electromagnetic wave is along :

(1) $(\hat{k})$

(2) $\hat{j}$

(3) $(-\hat{k})$

(4) $(-\hat{j})$

Official Ans. by NTA (4)

Sol. Direction of propagation $=\vec{E}\times\vec{B}=\hat{i}\times\hat{k}=-\hat{j}$

7. For a series LCR circuit with R = 100 Ω, L = 0.5 mH and C = 0.1 pF connected across 220 V–50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is :

(1) 0°, resistive circuit

(2) ≈ 90°, predominantly inductive circuit

(3) 0°, resonance circuit

(4) ≈ 90°, predominantly capacitive circuit

Official Ans. by NTA (4)

Sol. $R=100\Omega$

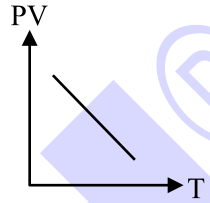
$X_L=\omega L=50\pi\times10^{-3}$

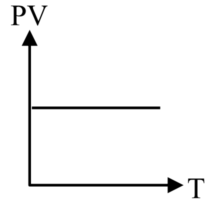
$$X_C=\frac{1}{\omega C}=\frac{10^{11}}{100\pi}$$

$X_C>>X_L$

$\&\ |X_C-X_L|>>R$

8. Which of the following graphs represent the behavior of an ideal gas ? Symbols have their usual meaning.

(1)


(2)


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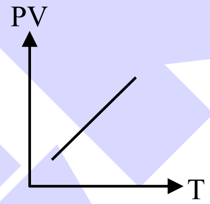

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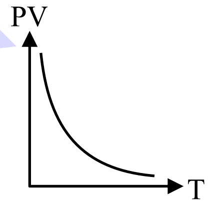


Official Ans. by NTA (3)

Sol. $PV=nRT$

$PV\propto T$

Straight line with positive slope (nR)

9. A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as :

(1) $T=2\pi\sqrt{\dfrac{x_2^2+x_1^2}{v_1^2-v_2^2}}$ (2) $T=2\pi\sqrt{\dfrac{x_2^2+x_1^2}{v_1^2+v_2^2}}$

(3) $T=2\pi\sqrt{\dfrac{x_2^2-x_1^2}{v_1^2+v_2^2}}$ (4) $T=2\pi\sqrt{\dfrac{x_2^2-x_1^2}{v_1^2-v_2^2}}$

Official Ans. by NTA (4)

Sol. $v^2=\omega^2(A^2-x^2)$

$$A^2=x_1^2+\frac{v_1^2}{\omega^2}=x_2^2+\frac{v_2^2}{\omega^2}$$

$$\omega^2=\frac{v_2^2-v_1^2}{x_1^2-x_2^2}$$

$$T=2\pi\sqrt{\frac{x_1^2-x_2^2}{v_2^2-v_1^2}}$$

10. An electron having de-Broglie wavelength λ is incident on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is :

(1) 0
(2) $\frac{2m^2c^2\lambda^2}{h^2}$
(3) $\frac{2mc\lambda^2}{h}$
(4) $\frac{hc}{mc}$

Official Ans. by NTA (3)

Sol. $\lambda = \frac{h}{mv}$

kinetic energy, $\frac{P^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{hc}{\lambda_c}$

$\lambda_C = \frac{2m\lambda^2 c}{h}$

11. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

(1) Solid sphere
(2) Solid cylinder
(3) Hollow cylinder
(4) Ring

Official Ans. by NTA (2)

Sol. $\frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{1}{2}mv^2$

$I = \frac{1}{2}mR^2$

Body is solid cylinder

12. If time (t), velocity (v), and angular momentum (l) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v and l is :

(1) $[t^{-1} v^1 l^{-2}]$
(2) $[t^1 v^2 l^{-1}]$
(3) $[t^{-2} v^{-1} l^1]$
(4) $[t^{-1} v^{-2} l^1]$

Official Ans. by NTA (4)

Sol. $m \propto t^a v^b \ell^c$

$m \propto [T]^a [LT^{-1}]^b [ML^2T^{-1}]^c$

$M^1L^0T^0 = M^cL^{b+2c}T^{a-b-c}$

comparing powers

c = 1, b = –2, a = –1

$m \propto t^{-1}v^{-2}\ell^1$

13. The correct relation between the degrees of freedom f and the ratio of specific heat γ is :

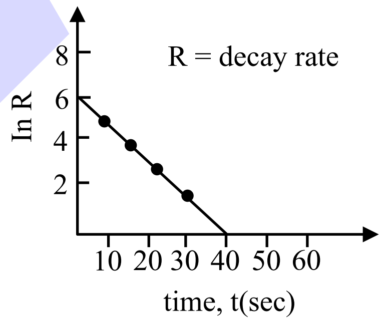
(1) $f = \frac{2}{\gamma - 1}$
(2) $f = \frac{2}{\gamma + 1}$
(3) $f = \frac{\gamma + 1}{2}$
(4) $f = \frac{1}{\gamma + 1}$

Official Ans. by NTA (1)

Sol. $\gamma = 1 + \frac{2}{f}$

$f = \frac{2}{\gamma - 1}$

14. For a certain radioactive process the graph between In R and t(sec) is obtained as shown in the figure. Then the value of half life for the unknown radioactive material is approximately :



(1) 9.15 sec
(2) 6.93 sec
(3) 2.62 sec
(4) 4.62 sec

Official Ans. by NTA (4)

Sol. $R = R_0 e^{-\lambda t}$

$\ell nR = \ell nR_0 - \lambda t$

$-\lambda$ is slope of straight line

$\lambda = \frac{3}{20}$

$t_{1/2} = \frac{\ell n2}{\lambda} = 4.62$

15. Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then :

(1) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$

(2) $T_A = T_B$

(3) $T_A > T_B$ (if $m_A > m_B$)

(4) $T_A > T_B$ (if $r_A > r_B$)

Official Ans. by NTA (2)

Sol. $T_A = T_B$ (since $\omega_A = \omega_B$)

16. At an angle of 30° to the magnetic meridian, the apparent dip is 45°. Find the true dip :

(1) $\tan^{-1}\sqrt{3}$ (2) $\tan^{-1}\frac{1}{\sqrt{3}}$

(3) $\tan^{-1}\frac{2}{\sqrt{3}}$ (4) $\tan^{-1}\frac{\sqrt{3}}{2}$

Official Ans. by NTA (4)

Sol. $A\tan\delta = \tan\delta'\cos\theta$

$= \tan 45°\cos 30°$

$\tan\delta = 1 \times \frac{\sqrt{3}}{2}$

$\delta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

17. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time *'t'* is proportional to :

(1) $t^{\frac{3}{2}}$ (2) $t^{\frac{1}{2}}$ (3) $t^{\frac{1}{4}}$ (4) $t^{\frac{3}{4}}$

Official Ans. by NTA (1)

Sol. P = constant

$\frac{1}{2}mv^2 = Pt$

$\Rightarrow v \propto \sqrt{t}$

$\frac{dx}{dt} = C\sqrt{t}$ C = constant

by integration.

$x = C\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$x \propto t^{3/2}$

18. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P} + \vec{Q}$ is *n* times the magnitude of $\vec{P} - \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is :

(1) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$ (2) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

(3) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (4) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

Official Ans. by NTA (4)

Sol. $|\vec{P}| = |\vec{Q}| = x$ ...(i)

$|\vec{P} + \vec{Q}| = n|\vec{P} - \vec{Q}|$

$\mathbf{P^2 + Q^2 + 2PQ\cos\theta = n^2(P^2 + Q^2 - 2PQ\cos\theta)}$

Using (i) in above equation

$\cos\theta = \frac{n^2-1}{1+n^2}$

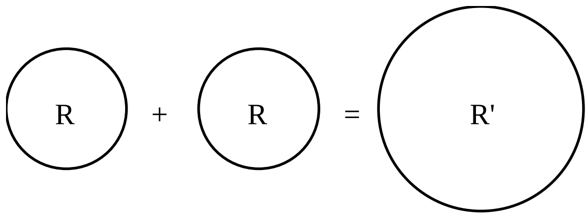
$\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

19. Two small drops of mercury each of radius R coalesce to form a single large drop. The ratio of total surface energy before and after the change is :

(1) $2^{\frac{1}{3}}:1$ (2) $1:2^{\frac{1}{3}}$ (3) 2 : 1 (4) 1 : 2

Official Ans. by NTA (1)

Sol.



$\frac{4}{3}\pi R^3 + \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3$

$R' = 2^{\frac{1}{3}}R$ ....(i)

$A_i = 2[4\pi R^2]$

$A_f = 4\pi R'^2$

$\frac{U_i}{U_f} = \frac{A_i}{A_f} = \frac{2R^2}{2^{2/3}R^2} = 2^{1/3}$

20. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi \times 10^{-7}$ H/m. Absolute permeability of the material of the rod is :

(1) $4\pi \times 10^{-4}$ H/m

(2) $2\pi \times 10^{-4}$ H/m

(3) $3\pi \times 10^{-4}$ H/m

(4) $\pi \times 10^{-4}$ H/m

Official Ans. by NTA (2)

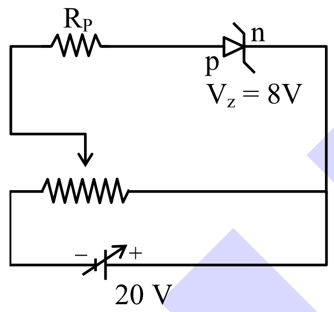
Sol. $\mu = \mu_0 (1 + x_m)$

$= 4\pi \times 10^{-7} \times 500$

$= 2\pi \times 10^{-4}$ H/m

SECTION-B

1. A zener diode having zener voltage 8 V and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance R_p is ....Ω.



Official Ans. by NTA (192)

Sol. $P = Vi$

$0.5 = 8i$

$i = \frac{1}{16}A$

$E = 20 = 8 + i R_P$

$R_P = 12 \times 16 = 192\Omega$

2. A body of mass '*m*' is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is _____.

Official Ans. by NTA (3)

Sol. $t_a = \frac{1}{2}t_d$

$$\sqrt{\frac{2s}{a_a}} = \frac{1}{2}\sqrt{\frac{2s}{a_d}} \qquad \ldots(i)$$

$a_a = g\sin\theta + \mu g\cos\theta$

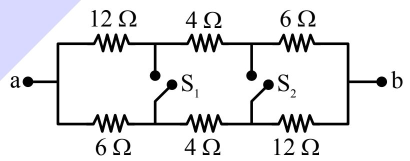
$= \frac{g}{2} + \frac{\sqrt{3}}{2}\mu g$

$a_d = g\sin\theta - \mu g\cos\theta$

$= \frac{g}{2} - \frac{\sqrt{3}}{2}\mu g$

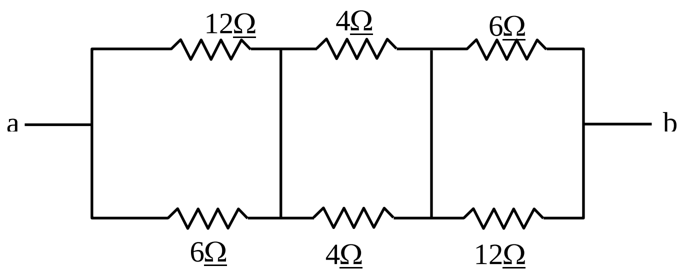
using the above values of a_a and a_d and putting in eqution (i) we will gate $\mu = \frac{\sqrt{3}}{5}$

3. In the given figure switches S_1 and S_2 are in open condition. The resistance across ab when the switches S_1 and S_2 are closed is __________Ω.



Official Ans. by NTA (10)

Sol. when switch S_1 and S_2 are closed



$$\frac{12\times 6}{12+6} + 2 + \frac{6\times 12}{6+12}$$

$$\frac{72}{18} + 2 + \frac{72}{18} = 4 + 2 + 4 = 10\Omega$$

4. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is _____.

Official Ans. by NTA (3)

Sol. I in both cases is about point of contact

Ring

$$mgh = \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}(2mR^2)\frac{v_R^2}{R^2}$$

$$v_R = \sqrt{gh}$$

Solid cylinder

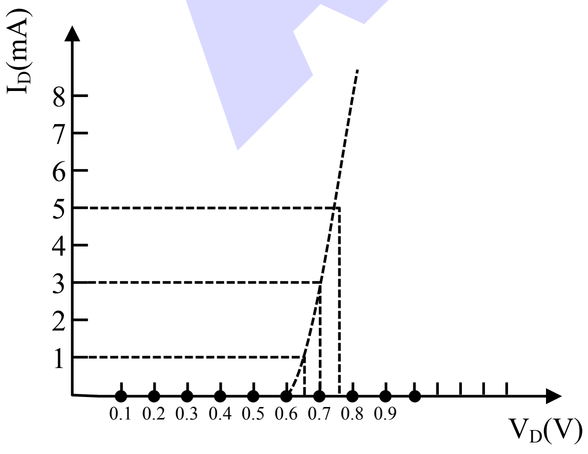
$$mgh = \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}\left(\frac{3}{2}mR^2\right)\frac{v_C^2}{R^2}$$

$$v_C = \sqrt{\frac{4gh}{3}}$$

$$\frac{v_R}{v_C} = \frac{\sqrt{3}}{2}$$

5. For the forward biased diode characteristics shown in the figure, the dynamic resistance at $I_D = 3$ mA will be_______Ω.



Official Ans. by NTA (25)

Sol. $$R_d = \frac{dV}{di} = \frac{1}{\frac{di}{dv}} = \frac{1}{\frac{5-1\times10^{-3}}{0.75-0.65}}$$

$$\frac{100}{4} = 25\Omega$$

6. A series LCR circuit of R = 5Ω, L = 20 mH and C = 0.5 μF is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is_____× 10^2 W.

Official Ans. by NTA (125)

Sol. $X_L = X_C$ (due to resonance)

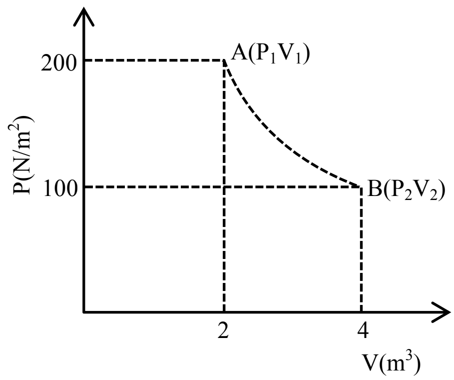
$$Z = R \text{ so } i_{rms} = \frac{V}{Z} = \frac{V}{R}$$

$$\frac{V^2}{R} = \frac{250\times250}{5} = 125\times10^2\text{ W}$$

7. One mole of an ideal gas at 27ºC is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be____ × 10^{-1} J.

[Given : R = 8.3 J / mole K, ln2 = 0.6931]

(Round off to the nearest integer)



Official Ans. by NTA (17258)

Sol. Process of isothermal

$$W = nRT\ell n\left(\frac{V_2}{V_1}\right)$$

$$= 1 \times 8.3 \times 300 \times \ln 2$$

$$= 17258 \times 10^{-1}\text{ J}$$

8. A certain metallic surface is illuminated by monochromatic radiation of wavelength λ. The stopping potential for photoelectric current for this radiation is $3V_0$. If the same surface is illuminated with a radiation of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is_____λ.

Official Ans. by NTA (4)

Sol. $KE = \frac{hc}{\lambda} - \phi\, hc$

$$e(3V_0) = \frac{hc}{\lambda_0} - \phi \qquad \ldots(i)$$

$$eV_0 = \frac{hc}{2\lambda_0} - \phi \qquad \ldots(ii)$$

Using (i) & (ii)

$$\phi = \frac{hc}{4\lambda_0} = \frac{hc}{\lambda_t}$$

$\lambda_t = 4\lambda_0$

9. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is___.

Official Ans. by NTA (200)

Sol. $\omega_f = \omega_0 + \alpha t$

$\alpha = 1200 \times 6$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 600 \times \frac{10}{60} + \frac{1}{2} \times 1200 \times 6 \times \frac{1}{36}$$

$\theta = 200$

10. A radioactive substance decays to $\left(\frac{1}{16}\right)^{th}$ of its initial activity in 80 days. The half life of the radioactive substance expressed in days is____.

Official Ans. by NTA (20)

Sol. $N_0 \xrightarrow{t_{\frac{1}{2}}} \frac{N_0}{2} \xrightarrow{t_{\frac{1}{2}}} \frac{N_0}{4} \xrightarrow{t_{\frac{1}{2}}} \frac{N_0}{8} \xrightarrow{t_{\frac{1}{2}}} \frac{N_0}{16}$

$4 \times t_{1/2} = 80$

$t_{1/2} = 20$ days

FINAL JEE–MAIN EXAMINATION – JULY, 2021

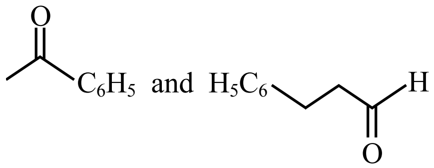
(Held On Tuesday 20th July, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

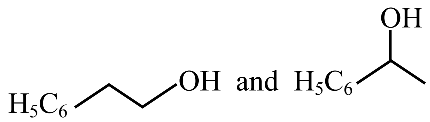
CHEMISTRY — TEST PAPER WITH SOLUTION

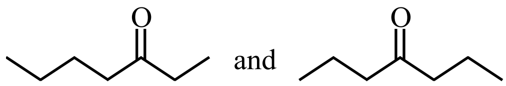
SECTION-A

1. Which one of the following pairs of isomers is an example of metamerism ?

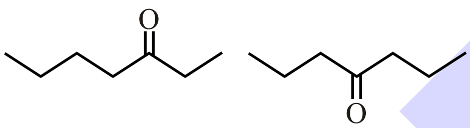
(1) $CH_3CH_2CH_2CH_2CH_3$ and $H_3C\text{–}\underset{CH_3}{\overset{CH}{C}}\text{–}CH_3$

(2) 

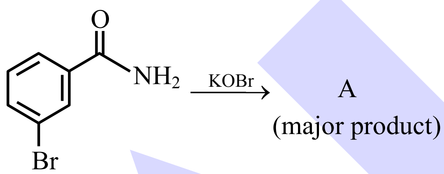
(3) 

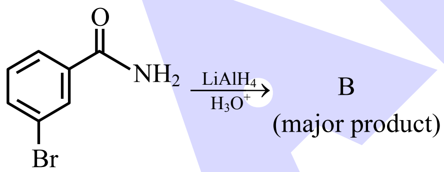
(4) 

Official Ans. by NTA (4)

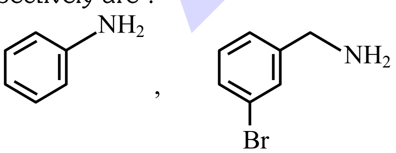
Sol. 

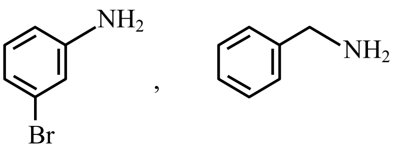
are metamers.

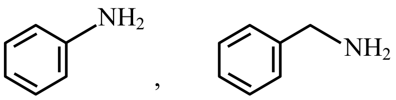
2. 

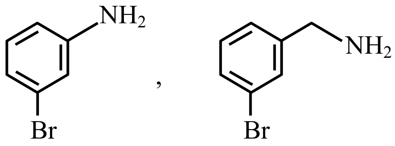


In the above reactions, product A and product B respectively are :

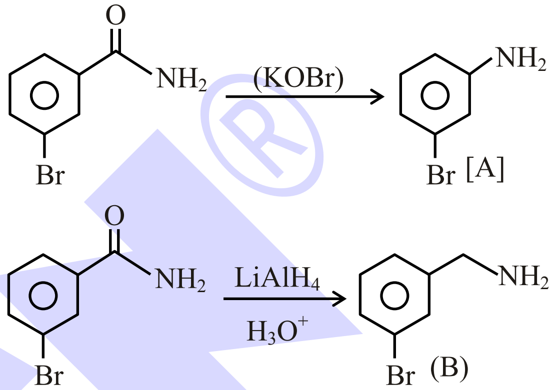
(1) 

(2) 

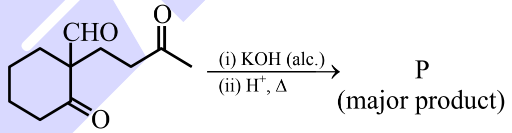
(3) 

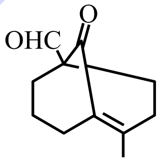
(4) 

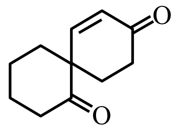
Official Ans. by NTA (4)

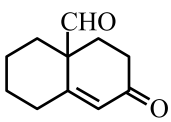
Sol. 

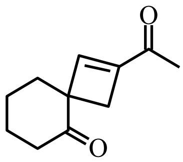
3. The major product (P) in the following reaction is :



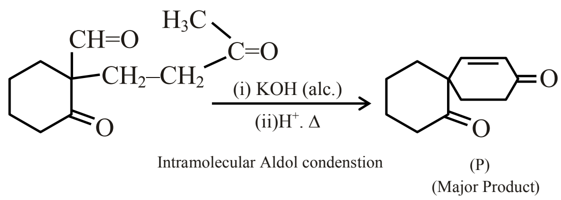
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. 

4. The single largest industrial application of dihydrogen is :

(1) Manufacture of metal hydrides

(2) Rocket fuel in space research

(3) In the synthesis of ammonia

(4) In the synthesis of nitric acid

Official Ans. by NTA (3)

Sol. Informative, according to ncert uses of di hydrogen.

In fact NH_3 largest production in used to manufacture nitrogenous fertilisers.

5. Consider two chemical reactions (A) and (B) that take place during metallurgical process :

(A) $ZnCO_{3(s)} \xrightarrow{\Delta} ZnO_{(s)} + CO_{2(g)}$

(B) $2ZnS_{(s)} + 3O_{2(g)} \xrightarrow{\Delta} 2ZnO_{(s)} + 2SO_{2(g)}$

The **correct** option of names given to them respectively is :

(1) (A) is calcination and (B) is roasting

(2) Both (A) and (B) are producing same product so both are roasting

(3) Both (A) and (B) are producing same product so both are calcination

(4) (A) is roasting and (B) is calcination

Official Ans. by NTA (1)

Sol. (A) $ZnCO_3$ (S) $\xrightarrow{\Delta}$ ZnO(s) + CO_2(g)

Heating in absence of oxygen in calcination.

(B) 2Zns(s) + $3O_2$(g) $\rightarrow$ 2ZnO(g) + $2SO_2$(g)

heating in presence of oxygen in roasting

Hence (A) is calcination while (B) in roasting.

6. A solution is 0.1 M in Cl^- and 0.001 M in CrO_4^{2-}. Solid $AgNO_3$ is gradually added to it

Assuming that the addition does not change in volume and $K_{sp}(AgCl) = 1.7 \times 10^{-10}\ M^2$ and $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12}\ M^3$.

Select **correct** statement from the following :

(1) AgCl precipitates first because its K_{sp} is high.

(2) Ag_2CrO_4 precipitates first as its K_{sp} is low.

(3) Ag_2CrO_4 precipitates first because the amount of Ag^+ needed is low.

(4) AgCl will precipitate first as the amount of Ag^+ needed to precipitate is low.

Official Ans. by NTA (4)

Sol. **(i)** $[Ag^+]$ required to ppt AgCl(s)

$Ksp = IP = [Ag^+][Cl^-] = 1.7 \times 10^{-10}$

$[Ag^+] = 1.7 \times 10^{-9}$

(ii) $[Ag^+]$ required to ppt Ag_2CrO_4(s)

$Ksp = IP = [Ag+]^2 [CrO_4^{-2}] = 1.9 \times 10^{-12}$

$[Ag^+] = 4.3 \times 10^{-5}$

$[Ag^+]$ required to ppt AgCl is low so AgCl will ppt 1st.

7. Outermost electronic configuration of a group 13 element, E, is $4s^2, 4p^1$. The electronic configuration of an element of p-block period-five placed diagonally to element, E is :

(1) $[Kr]\ 3d^{10}\ 4s^2\ 4p^2$ (2) $[Ar]\ 3d^{10}\ 4s^2\ 4p^2$

(3) $[Xe]\ 5d^{10}\ 6s^2\ 6p^2$ (4) $[Kr]\ 4d^{10}\ 5s^2\ 5p^2$

Official Ans. by NTA (4)

Sol. The element E is Ga and the diagonal element of 5^{th} period is $_{50}Sn$ having outer electronic configuration will be $[Kr]\ 5s^2\ 4d^{10}\ 5\ p^2$.

8. Metallic sodium does not react normally with :

(1) gaseous ammonia (2) But-2-yne

(3) Ethyne (4) tert-butyl alcohol

Official Ans. by NTA (2)

Sol. Metallic sodium does not react with 2-butyne because 2-butyne does not have acidic hydrogen.

9. Spin only magnetic moment of an octahedral complex of Fe^{2+} in the presence of a strong field ligand in BM is :

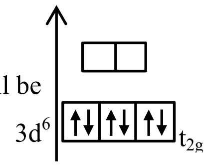
(1) 4.89 (2) 2.82 (3) 0 (4) 3.46

Official Ans. by NTA (3)

Sol. In presence of SFL $\Delta_0 > P$ means pairing occurs therefore

For $Fe^{+2} \rightarrow 3d^6$

Splitting will be



$\therefore$ No of unpaired e^- (s) = 0

$\therefore\ \mu = \sqrt{n(n+2)}\ BM = 0$

[n = No of unpaired e^-(s)]

In $NiCl_2$ Ni^{+2} is having configuration $3d^8$

$\therefore$ Number of unpaired electron = 2

After formation of oxidised product

$[Ni(CN)_6]^{-2}$ Ni^{+4} is obtained

$Ni^{+4} \Rightarrow 3d^6$ and CN^- is strong field ligand

$\therefore$ number of unpaired electrons = 0

$\therefore$ The charge is 2 – 0 = 2

10. Which one of the following species **doesn't** have a magnetic moment of 1.73 BM, (spin only value) ?

(1) O_2^+ (2) CuI

(3) $[Cu(NH_3)_4]Cl_2$ (4) O_2^-

Official Ans. by NTA (2)

Sol. Species must not contain single unpaired

(1)$O_2^+ \rightarrow \sigma_{1s}^2 < \sigma_{1s}^{*2} < \sigma_{2s}^2 < \sigma_{2s}^{*2} < \sigma_{2pz}^2 < \pi_{2px}^2 = \pi_{2py}^2 < \pi_{2px}^{*1} = \pi_{2py}^*$

unpaired $e^- = 1 \therefore \mu = 1.73$ BM

(1) Cu^+I^- $Cu^+ \rightarrow [Ar]3d^{10} \therefore$ unpaired $e^- = 0$

$I^- \rightarrow [Xe] \therefore$ unpaired $e^- = 0$

therefore $\mu = 0$

3. $[Cu(NH_3)_4]Cl_2$

$Cu \rightarrow [A]\ 3d^3 \therefore$ unpaired $= 1 \therefore \mu = 1.73$ BM

4. $O_2^- \rightarrow d$

$\sigma_{1s}^2 < \sigma_{1s}^{*2} < \sigma_{2s}^2 < \sigma_{2s}^{*2} < \sigma_{2px}^2 < \pi_{2px}^2 = \pi_{2py}^2 < \pi_{2px}^{*1} = \pi_{2py}^*$

$(11e^-)$

$\therefore$ unpaired $\therefore \mu = 1.73$ BM

11. Which one of the following statements is not true about enzymes ?

(1) Enzymes are non-specific for a reaction and substrate.

(2) Almost all enzymes are proteins.

(3) Enzymes work as catalysts by lowering the activation energy of a biochemical reaction.

(4) The action of enzymes is temperature and pH specific

Official Ans. by NTA (1)

Sol. Fact

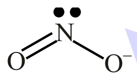
12. The hybridisations of the atomic orbitals of nitrogen in NO_2^-, NO_2^+ and NH_4^+ respectively are.

(1) sp^3, sp^2 and sp (2) sp, sp^2 and sp^3

(3) sp^3, sp and sp^2 (4) sp^2, sp and sp^3

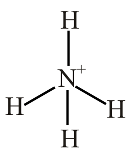
Official Ans. by NTA (4)

Sol.



$2\sigma + 1$ lp $\therefore sp^2$

$O = \overset{\oplus}{N} = O$ $2\sigma + 0$ lp $\therefore$ sp



$4\sigma + 0$ $\therefore sp^3$

13. Bakelite is a cross-linked polymer of formaldehyde and :

(1) PHBV (2) Buna-S (3) Novolac (4) Dacron

Official Ans. by NTA (3)

Sol. Novolac (phenol formaldehyde Resin) $\rightarrow$ Bakelite

14. Benzene on nitration gives nitrobenzene in presence of HNO_3 and H_2SO_4 mixture, where :

(1) both H_2SO_4 and HNO_3 act as a bases

(2) HNO_3 acts as an acid and H_2SO_4 acts as a base

(3) both H_2SO_4 and HNO_3 act as an acids

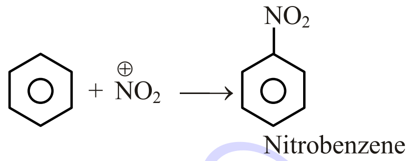
(4) HNO_3 acts as a base and H_2SO_4 acts as an acid

Official Ans. by NTA (4)

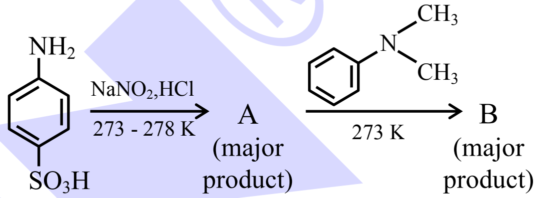
Sol. Reagent for nitration of Benzene

$$\underset{(Acid)}{H_2SO_4} + \underset{(Base)}{HNO_3} \rightleftarrows HSO_4^{\ominus} + H_2\overset{\oplus}{N}O_3$$

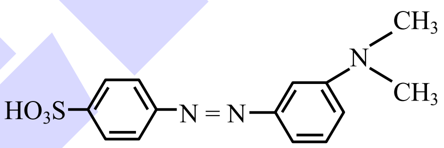
$$H_2\overset{\oplus}{N}O_3 \rightleftarrows H_2O + \overset{\oplus}{N}O_2$$

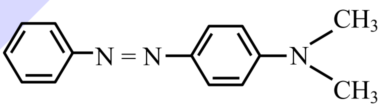


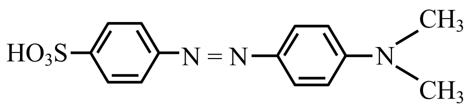
15.

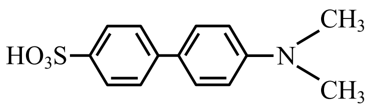


Consider the above reaction, compound B is :

(1) 

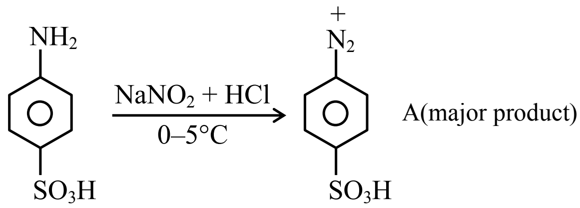
(2) 

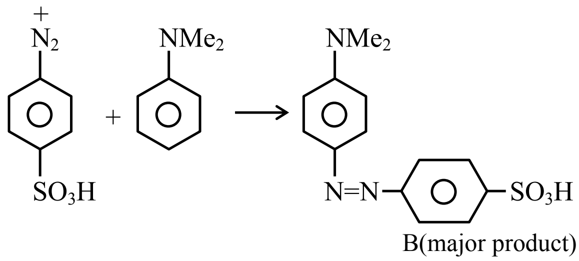
(3) 

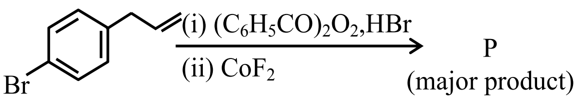
(4) 

Official Ans. by NTA (3)

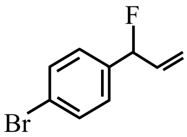
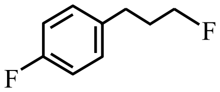
Sol.

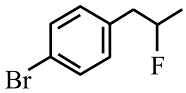
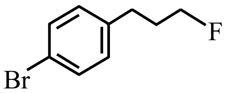




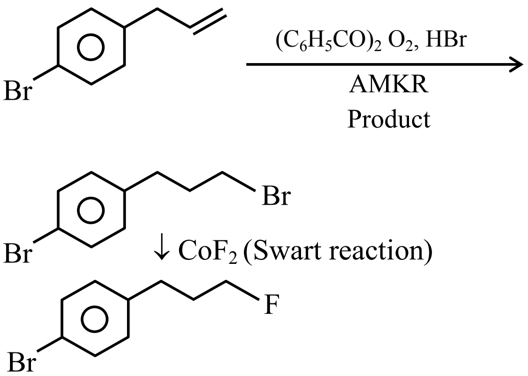
16. 

Major product P of above reaction, is :

(1)  (2) 

(3)  (4) 

Official Ans. by NTA (4)

Sol. 

17. Cu^{2+} salt reacts with potassium iodide to give

(1) Cu_2I_2 (2) Cu_2I_3 (3) CuI (4) $Cu(I_3)_2$

Official Ans. by NTA (1)

Sol. $2Cu^{+2} + 4I^- \longrightarrow Cu_2I_2(s) + I_2$

$2Cu^{+2} + 3I^- \longrightarrow 2CuI + I_2$

18. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of :

(1) HNO_3 (2) $AgNO_3$ (3) $CuSO_4$ (4) $BaSO_4$

Official Ans. by NTA (2)

Sol. Organic compound is heated with fuming nitric acid in the presence of silver nitrate in carius method.

Lunar caustic ($AgNO_3$) is used as reagent hare to distinguish Cl^-, Br and I^- respectively as follows.

$Cl^-(aq) \xrightarrow{AgNO_3} AgCl\downarrow_{ppt}$ white

$Br^-(aq) \xrightarrow{AgNO_3} AgBr\downarrow_{ppt}$ pale yellow

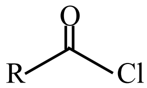
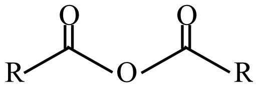
$I^-(aq) \xrightarrow{AgNO_3} AgI\downarrow_{ppt}$ Dark yellow

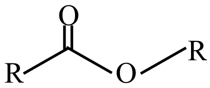
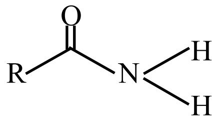
19. Which one of the following gases is reported to retard photosynthesis ?

(1) CO (2) CFCs (3) CO_2 (4) NO_2

Official Ans. by NTA (4)

Sol. According to NCERT only NO_2 from the given options can retard the photosynthesis process in plants.

20. (A)  (B) 

(C)  (D) 

The **correct** order of their reactivity towards hydrolysis at room temperature is :

(1) (A) > (B) > (C) > (D)

(2) (D) > (A) > (B) > (C)

(3) (D) > (B) > (A) > (C)

(4) (A) > (C) > (B) > (D)

Official Ans. by NTA (1)

Sol. $R{-}\underset{\underset{O}{\|}}{C}{-}Cl > R{-}\underset{\underset{O}{\|}}{C}{-}O{-}\underset{\underset{O}{\|}}{C}{-}R > R{-}\underset{\underset{O}{\|}}{C}{-}OR > R{-}\underset{\underset{O}{\|}}{C}{-}NH_2$

Reactivity towards Hydrolysis A > B > C > D

SECTION-B

1. For a given chemical reaction A → B at 300 K the free energy change is –49.4 kJ mol^{-1} and the enthalpy of reaction is 51.4 kJ mol^{-1}. The entropy change of the reaction is ______ J K^{-1} mol^{-1}.

Official Ans. by NTA (360)

Sol. Given chemical reaction:

$A \xrightarrow[T\ 300K]{} B \quad [\Delta G]_{P,T} = -49.4$ kJ/mol

$\Delta H_{rxn} = 51.4$ kJ/mol

$\Delta S_{rxn} = ?$

$\Rightarrow$ From the relation $[\Delta G]_{P,T} = \Delta H - T\Delta S$

$$\Rightarrow \Delta S_{rxn} = \frac{\Delta H_{rxn} - [\Delta G]_{P,T}}{T}$$

$$= \frac{[51.4 - (-49.4)] \times 1000}{300} \frac{J}{mol K}$$

$$\Rightarrow \Delta S_{rxn} = 336 \frac{J}{mol K}$$

2. The wavelength of electrons accelerated from rest through a potential difference of 40 kV is x × 10^{-12} m. The value of x is ______. (Nearest integer)

Given : Mass of electron = 9.1×10^{-31} kg

Charge on an electron = 1.6×10^{-19} C

Planck's constant = 6.63×10^{-34} Js

Official Ans. by NTA (6)

Sol. De-broglie-wave length of electron:

$$\lambda_e = \frac{h}{\sqrt{2m(KE)}} \left\{ \begin{array}{l} \because e^- \text{ is accelerated} \\ \text{from rest} \\ \Rightarrow KE = q \times V \end{array} \right.$$

$$\lambda = \frac{h}{\sqrt{2mqv}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-31} \times 40 \times 10^3}}$$

$= 0.614 \times 10^{-11}$ m

$= 6.14 \times 10^{-12}$ m

Nearest integer = 6

OR

$$\lambda = \frac{12.3}{\sqrt{V}} \text{Å}$$

$$= \frac{12.3}{200} = 6.15 \times 10^{-12}\text{ m}$$

Ans. is 6

3. The vapour pressures of A and B at 25°C are 90 mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is $x \times 10^{-1}$. The value of x is ______. (Nearest integer)

Official Ans. by NTA (1)

Sol. Given $P_A° = 90$ mm Hg, at 25°C

$P_B° = 15$ mm Hg

and $\left.\begin{array}{l} X_A = 0.6 \\ X_B = 0.4 \end{array}\right\} P_T = X_A P_A^o + X_B P_B^o$

$= (0.6 \times 90) + (0.4 \times 15)$

$= 54 + 6 = 60$ mm

Now mol fraction of B in the vapour phase

i.e. $Y_B = \frac{P_B}{P_T} = \frac{X_B P_B^o}{60} = 0.1 = 1 \times 10^{-1}$

therefore: $x = 1$

4. 4g equimolar mixture of NaOH and Na_2CO_3 contains x g of NaOH and y g of Na_2CO_3. The value of x is _______ g. (Nearest integer)

Official Ans. by NTA (1)

Sol. Total mass = 4g

NaOH : a mol

Na_2CO_3 : 'a' mol

Now

$W_{NaOH} + W_{Na_2CO_3} = 4$

$\Rightarrow 40a + 106\,a = 4$

$\Rightarrow a = \frac{4}{146}$ mol

$\Rightarrow$ therefore mass of NaOH is : $\frac{4}{146} \times 40$g

$= 1.095 \approx 1$

5. When 0.15 g of an organic compound was analyzed using Carius method for estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is _______. (Nearest integer)

[Atomic mass : Silver = 108, Bromine = 80]

Official Ans. by NTA (68)

Sol. Moles of Br = Moles of AgBr obtained

$\Rightarrow$ Mass of Br = $\frac{0.2397}{188} \times 80$g

therefore % Br in the organic compound

$$= \frac{W_{Br}}{W_T} \times 100$$

$$= \frac{0.2397 \times 80}{188 \times 0.15} \times 100 = 0.85 \times 80$$

$= 68$

$\Rightarrow$ Nearest integer is '68'

6. 100 ml of 0.0018% (w/v) solution of Cl^- ion was the minimum concentration of Cl^- required to precipitate a negative sol in one h. The coagulating value of Cl^- ion is _______ (Nearest integer)

Official Ans. by NTA (1)

7. $PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$

In the above first order reaction the concentration of PCl_5 reduces from initial concentration 50 mol L^{-1} to 10 mol L^{-1} in 120 minutes at 300 K. The rate constant for the reaction at 300 K is x × 10^{-2} min^{-1}. The value of x is _______.
[Given log5 = 0.6989]

Official Ans. by NTA (1)

Sol. $PCl_{5(g)} \xrightarrow[300K]{\text{I order}} PCl_{3(g)} + Cl_{2(g)}$

t = 0 50 M

t = 120 min 10 M

$$\Rightarrow K = \frac{2.303}{t}\log\frac{[A_0]}{[A_t]}$$

$$\Rightarrow K = \frac{2.303}{120}\log\frac{50}{10}$$

$$\Rightarrow K = \frac{2.303}{120}\times 0.6989 = 0.013413\ min^{-1}$$

$$= 1.3413 \times 10^{-2}\ min^{-1}$$

1.34 ⇒ Nearest integer = 1

8. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is _______.

Official Ans. by NTA (8)

Sol. Carbon atoms occupy FCC lattice points as well as half of the tetrahedral voids

therefore number of carbon atoms atoms per unit cell = 8

9. An aqueous solution of $NiCl_2$ was heated with excess sodium cyanide in presence of strong oxidizing agent to form $[Ni(CN)_6]^{2-}$. The total change in number of unpaired electrons on metal centre is _______.

Official Ans. by NTA (2)

Sol. $[Ni(CN)_6]^{2-}$

$Ni^{+4} \rightarrow d^6$ strong field ligand

↑↓	↑↓	↑↓		

Pairing will be there zero unpaired electron

$NiCl_2 \rightarrow Ni^{2+} \rightarrow d^8$

↑↓	↑↓	↑↓	↑	↑

→ two unpaired e^-

Change = 2

10. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.

$6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

A current of xA has to be passed for 10h to produce 10.0g of potassium chlorate. the value of x is _______. (Nearest integer)

(Molar mass of $KClO_3$ = 122.6 g mol^{-1},
F = 96500 C)

Official Ans. by NTA (1)

Sol. Given balanced equation is

$$6\overset{\ominus}{O}H + \overset{\ominus}{Cl} \longrightarrow ClO_3^- + 3H_2O + 6e^-$$

$\rightarrow$ 10g $KClO_3 \Rightarrow \frac{10}{122.6}$ mol KCO_3 in obtained

$\rightarrow$ from the above reaction, it is concluded that by 6F charge 1 mol $KClO_3$ is obtained.

$\rightarrow$ By the passage of 6F charge = 1 mol $KClO_3$

$\therefore$ By the passage of $\frac{x\times 10\times 60\times 60}{96500}$ F charge

$$= \frac{1}{6}\times\frac{x\times 10\times 60\times 60}{96500}$$

Now $\frac{x\times 10\times 60\times 60}{6\times 96500} = \frac{10}{122.6}$

$$\Rightarrow x = \frac{10\times 965}{60\times 122.6} = \frac{965}{735.6} = 1.311 \simeq 1$$

OR

$$W = \frac{E}{F}\times I\times t$$

$$10 = \frac{122.6}{96500\times 6}\times x\times 10\times 3600$$

X = 1.311

Ans.(1)

FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 20th July, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. For the natural numbers m, n, if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots + a_{m+n}y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m + n)$ is equal to :

(1) 88 (2) 64

(3) 100 (4) 80

Official Ans. by NTA (4)

Sol. $(1-y)^m(1+y)^n$

Coefficient of y $(a_1) = 1.{}^nC_1 + {}^mC_1(-1)$

$= n - m = 10$ .....(1)

Coefficient of y^2 (a_2)

$= 1.\,{}^nC_2 - {}^mC_1.\,{}^nC_1. + 1.{}^mC_2 = 10$

$$= \frac{n(n-1)}{2} - m.n + \frac{m(m-1)}{2} = 10$$

$m^2 + n^2 - 2mn - (n + m) = 20$

$(n - m)^2 - (n + m) = 20$

$n + m = 80$ ..... (2)

By equation (1) & (2)

$m = 35, n = 45$

2. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to :

(1) $\frac{-181}{69}$ (2) $\frac{220}{21}$

(3) $\frac{-291}{76}$ (4) $\frac{151}{63}$

Official Ans. by NTA (2)

Sol. $\underbrace{\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{5}}_{x>0,y>0,xy<1} + \tan^{-1}\frac{5}{12}$

$$\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}} = \underbrace{\tan^{-1}\frac{15}{8} + \tan^{-1}\frac{5}{12}}_{x>0,y>0,xy<1}$$

$$\tan^{-1}\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\cdot\frac{5}{12}} = \tan^{-1}\frac{220}{21}$$

$$\tan\left(\tan^{-1}\frac{220}{21}\right) = \frac{220}{21}$$

3. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4,1)$ and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$.

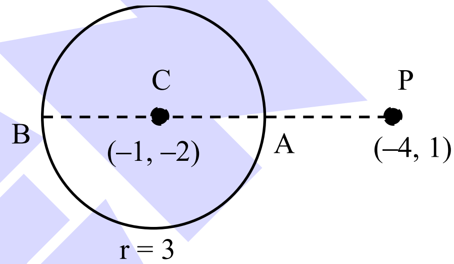
If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a + b$ is equal to :

(1) 3 (2) 11

(3) 5 (4) 7

Official Ans. by NTA (3)

Sol.



Centre of smallest circle is A

Centre of largest circle is B

$r_2 = |CP - CA| = 3\sqrt{2} - 3$

$r_1 = CP + CB = 3\sqrt{2} + 3$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = \frac{(3\sqrt{2}+3)^2}{9} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$$

$a = 3, b = 2$

4. Consider the following three statements :

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 = 17.

Then, which of the following statements is correct ?

(1) (A) is false, but (B) and (C) are true

(2) (A) and (C) are true while (B) is false

(3) (A) is true while (B) and (C) are false

(4) (A) and (B) are false while (C) is true

Official Ans. by NTA (2)

Sol. Truth Table

P	q	P→q
T	T	T
T	F	F
F	T	T
F	F	T

5. The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, $(ab \neq 0)$ are coplanar, if :

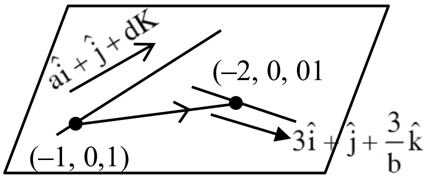
(1) $b = 1,\ a \in R - \{0\}$ (2) $a = 1,\ b \in R - \{0\}$

(3) $a = 2, b = 2$ (4) $a = 2, b = 3$

Official Ans. by NTA (1)

Sol. $\dfrac{x+1}{a} = y = \dfrac{z-1}{a}$

$\dfrac{x+2}{3} = y = \dfrac{z}{3/b}$



lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b} - a\right) - 1(a-3) = 0$$

$$a - \frac{3}{b} - a + 3 = 0$$

$$b = 1, a \in R - \{0\}$$

6. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2} [[x] - \sin x]dx$ is equal to :

(1) $-\pi$ (2) π (3) 0 (4) 1

Official Ans. by NTA (1)

Sol. $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} ([x] + [-\sin x])dx$ .....(i)

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} ([-x] + [\sin x])dx$ .....(2)

(King property)

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\underbrace{[x]+[-x]}_{-1}\right) + \left(\underbrace{[\sin x]+[-\sin x]}_{-1}\right)dx$$

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (-2)dx = -2(\pi)$$

$$I = -\pi$$

7. If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^{\theta} \sin x\, dx$ is equal to :

(1) 1 (2) 2 (3) –1 (4) 0

Official Ans. by NTA (1)

Sol. $z = \dfrac{1}{1 - \cos\theta + 2i\sin\theta}$

$$= \frac{2\sin^2\frac{\theta}{2} - 2i\sin\theta}{(1-\cos\theta)^2 + 4\sin^2\theta}$$

$$= \frac{\sin\frac{\theta}{2} - 2i\cos\frac{\theta}{2}}{4\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)}$$

$$\text{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$$

$$\sin\frac{2\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$$

$$1 - \cos^2\frac{\theta}{2} + 4\cos\frac{\theta}{2} = \frac{5}{2}$$

$$3\cos^2\frac{\theta}{2} = \frac{3}{2}$$

$$\cos^2\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

$$\theta \in (0, \pi)$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta - [-\cos\theta]_0^{\frac{\pi}{2}}$$

$= -(0 - 1)$

$= 1$

8. Let $f : \mathbf{R} - \left\{\frac{\alpha}{6}\right\} \to \mathbf{R}$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$.

Then the value of α for which (fof)(x) = x, for all $x \in \mathbf{R} - \left\{\frac{\alpha}{6}\right\}$, is :

(1) No such α exists (2) 5

(3) 8 (4) 6

Official Ans. by NTA (2)

Sol. $f(x) = \frac{5x+3}{6x-\alpha} = y$ ......(i)

$5x + 3 = 6xy - \alpha y$

$x(6y - 5) = \alpha y + 3$

$x = \frac{\alpha y + 3}{6y - 5}$

$f^{-1}(x) = \frac{\alpha x + 3}{6x - 5}$ .....(ii)

fo f(x) = x

$f(x) = f^{-1}(x)$

From eq^n (i) & (ii)

Clearly ($\alpha = 5$)

9. If $f : \mathbf{R} \to \mathbf{R}$ is given by $f(x) = x + 1$, then the value of

$$\lim_{n\to\infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + ... + f\left(\frac{5(n-1)}{n}\right)\right],$$

is :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{1}{2}$ (4) $\frac{7}{2}$

Official Ans. by NTA (4)

Sol. $I = \sum_{r=0}^{n-1} f\left(\frac{5r}{n}\right)\frac{1}{n}$

$I = \int_0^1 f(5x)dx$

$I = \int_0^1 (5x+1)dx$

$I = \left[\frac{5x^2}{2} + x\right]_0^1$

$I = \frac{5}{2} + 1 = \frac{7}{2}$

10. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is :

(1) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

(2) greater than $\frac{1}{2}$

(3) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(4) exactly equal to $\frac{1}{2}$

Official Ans. by NTA (2)

Sol. $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 1 - k$

$P(\bar{A} \cap C) + P(A \cap \bar{C}) = 1 - 2k$

$P(\bar{B} \cap C) + P(B \cap \bar{C}) = 1 - k$

$P(A \cap B \cap C) = k^2$

$P(A) + P(B) - 2\,P(A \cap B) = 1 - k$ .....(i)

$P(B) + P(C) - 2P(B \cap C) = 1 - k$ .....(ii)

$P(C) + P(A) - 2P(A \cap C) = 1 - 2k$ .....(iii)

(1) + (2) + (3)

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$-P(C \cap A) = \frac{-4k+3}{2}$

So

$P(A \cup B \cup C) = \frac{-4k+3}{2} + k^2$

$P(A \cup B \cup C) = \frac{2k^2 - 4k + 3}{2}$

$= \frac{2(k-1)^2 + 1}{2}$

$P(A \cup B \cup C) > \frac{1}{2}$

11. The sum of all the local minimum values of the twice differentiable function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is :

(1) –22 (2) 5 (3) –27 (4) 0

Official Ans. by NTA (3)

Sol. $f(x) = x^3 - 3x^2 - \frac{3}{2}f''(2)\,x + f''(1)$ .....(i)

$f'(x) = 3x^2 - 6x - \frac{3}{2}f''(2)$ .....(ii)

$f''(x) = 6x - 6$ .....(iii)

Now is 3rd equation

$f''(2) = 12 - 6 = 6$

$f''(11 = 0)$

Use (ii)

$f'(x) = 3x^2 - 6x - \frac{3}{2}f''(2)$

$f'(x) = 3x^2 - 6x - \frac{3}{2} \times 6$

$f'(x) = 3x^2 - 6x - 9$

$f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$\Rightarrow x = -1 \ \& \ 3$

Use (iii)

$f''(x) = 6x - 6$

$f''(-1) = -12 < 0$ maxima

$f''(3) = 12 > 0$ minima.

Use (i)

$f(x) = x^3 - 3x^2 - \frac{3}{2}\, f''(2)\,x + f''(1)$

$f(x) = x^3 - 3x^2 - \frac{3}{2} \times 6 \times x + 0$

$f(x) = x^3 - 3x^2 - 9x$

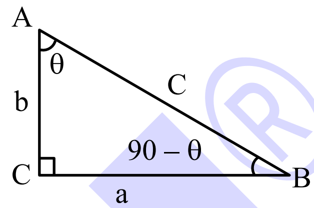
$f(3) = 27 - 27 - 9 \times 3 = -27$

12. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

(1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{5}-1}{2}$ (3) $\frac{\sqrt{2}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

Official Ans. by NTA (2)

Sol.



$\angle A = \theta$

$\angle B = 90 - \theta$

a = smallest side

$c^2 = a^2 + b^2$

$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$

$\frac{b^2c^2}{a^2} = b^2 + c^2$

Use $a = 2R \sin A = 2R \sin\theta$

$b = 2R \sin B = 2R \sin(90 - \theta) = 2R\cos\theta$

$c = 2R \sin C = 2 \sin 90° = 2R$

$\frac{4R^2\cos^2\theta}{4R^2\sin^2\theta} = 4R^2\cos^2\theta + 4R^2$

$\cos^2\theta = \sin^2\theta\cos^2\theta + \sin^2\theta$

$1 - \sin^2\theta = \sin^2\theta(1 - \sin^2\theta) + \sin^2\theta$

$\sin^2\theta = \frac{3-\sqrt{5}}{2}$

$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

13. Let y=y(x) satisfies the equation $\frac{dy}{dx}-|A|=0$, for all $x > 0$, where $A=\begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is :

(1) $\frac{\pi}{2}+\frac{4}{\pi}$ (2) $\frac{\pi}{2}-\frac{1}{\pi}$ (3) $\frac{3\pi}{2}-\frac{1}{\pi}$ (4) $\frac{\pi}{2}-\frac{4}{\pi}$

Official Ans. by NTA (1)

Sol. $|A|=-\frac{y}{x}+2\sin x+2$

$\frac{dy}{dx}=|A|$

$\frac{dy}{dx}=-\frac{y}{x}+2\sin x+2$

$\frac{dy}{dx}+\frac{y}{x}=2\sin x+2$

$\text{I.F.}=e^{\int\frac{1}{x}dx}=x$

$\Rightarrow yx=\int x(2\sin x+2)dx$

$xy = x^2 - 2x\cos x + 2\sin x + c$ .....(i)

Now $x = \pi, y = \pi + 2$

Use in (i)

$c = 0$

Now (i) be comes

$xy = x^2 - 2x\cos x + 2\sin x$

put $x = \pi/2$

$\frac{\pi}{2}y=\left(\frac{\pi}{2}\right)^2-2.\frac{\pi}{2}\cos\frac{\pi}{2}+2\sin\frac{\pi}{2}$

$\frac{\pi}{2}y=\frac{\pi^2}{4}+2$

14. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, –1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ?

(1) (–1, 1, 2) (2) (1, 1, 1)

(3) (1, 1, 2) (4) (1, 2, 2)

Official Ans. by NTA (4)

Sol. Plane p is $\perp^r$ to line

$\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

& passes through pt. (2, 3) equation of plane p

$2(x-2) + 1(y-3) + 1(z+1) = 0$

$2x + y + z - 6 = 0$

pt (1,2,2) satisfies above equation

15. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a-b|$ is equal to :

(1) 9 (2) 11 (3) 7 (4) 1

Official Ans. by NTA (4)

Sol. $10=\frac{7+10+11+15+a+b}{6}$

$\Rightarrow a + b = 17$ .....(i)

$\frac{20}{3}=\frac{7^2+10^2+11^2+15^2+a^2+b^2}{6}-10^2$

$a^2 + b^2 = 145$ ......(ii)

Solve (i) and (ii) a = 9, b = 8 or a = 8, b = 9

$|a-b| = 1$

16. Let $g(t)=\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$, where $f(x)=\log_e\left(x+\sqrt{x^2+1}\right)$, $x \in \mathbf{R}$. Then which one of the following is correct ?

(1) $g(1) = g(0)$ (2) $\sqrt{2}g(1)=g(0)$

(3) $g(1)=\sqrt{2}g(0)$ (4) $g(1)+g(0)=0$

Official Ans. by NTA (2)

Sol. $g(t)=\int_{-\pi/2}^{\pi/2}\left(\cos\frac{\pi}{4}t+f(x)\right)dx$

$g(t)=\pi\cos\frac{\pi}{4}t+\int_{-\pi/2}^{\pi/2}f(x)dx$

$g(t)=\pi\cos\frac{\pi}{4}t$

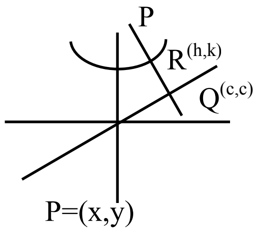
$g(1)=\frac{\pi}{\sqrt{2}},g(0)=\pi$

17. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line y = x is :

(1) $(3x - y)^2 + (x - 3y) + 2 = 0$
(2) $2(3x - y)^2 + (x - 3y) + 2 = 0$
(3) $(3x - y)^2 + 2(x - 3y) + 2 = 0$
(4) $2(x - 3y)^2 + (3x - y) + 2 = 0$

Official Ans. by NTA (2)

Sol.



$$\frac{K-C}{h-C} = -1$$

$$C = \frac{h+K}{2} \qquad P(x,y)$$

$$R = \left(\frac{x+C}{2}, \frac{y+C}{2}\right)$$

$$R = \left(\frac{x}{2} + \frac{h}{4} + \frac{K}{4}, \frac{y}{2} + \frac{h}{4} + \frac{k}{4}\right)$$

$$h = \frac{x}{2} + \frac{h}{4} + \frac{K}{4}$$

$$K = \frac{y}{2} + \frac{h}{4} + \frac{K}{4}$$

$$\Rightarrow x = \frac{3h}{2} - \frac{K}{2}, y = \frac{3K}{2} - \frac{h}{2}$$

$$Y = 4x^2 + 1$$

$$\left(\frac{3k-h}{2}\right) = 4\left(\frac{3h-k}{2}\right)^2 + 1$$

18. The value of $k \in \mathbf{R}$, for which the following system of linear equations

$3x - y + 4z = 3$,

$x + 2y - 3z = -2$,

$6x + 5y + kz = -3$,

has infinitely many solutions, is :

(1) 3 (2) – 5 (3) 5 (4) –3

Official Ans. by NTA (2)

Sol.
$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & K \end{vmatrix} = 0$$

$\Rightarrow 3(2K + 15) + K + 18 - 28 = 0$

$\Rightarrow 7K + 35 = 0 \Rightarrow K = -5$

19. If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + ...$, where x > 0 is 504, then x is equal to

(1) 243 (2) 9 (3) 7 (4) 81

Official Ans. by NTA (4)

Sol. $s = 2\log_9 x + 3 \log_9 x + ... + 22 \log_9 x$

$s = \log_9 x\ (2 + 3 + ... + 22)$

$$s = \log_9 x \left\{\frac{21}{2}(2+22)\right\}$$

Given $252 \log_9 x = 504$

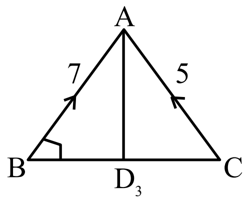
$\Rightarrow \log_9 x = 2 \Rightarrow x = 81$

20. In a triangle ABC, if $|\overrightarrow{BC}| = 3$, $|\overrightarrow{CA}| = 5$ and $|\overrightarrow{BA}| = 7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to

(1) $\frac{19}{2}$ (2) $\frac{13}{2}$ (3) $\frac{11}{2}$ (4) $\frac{15}{2}$

Official Ans. by NTA (3)

Sol.



Projection of $\overrightarrow{BA}$

on $\overrightarrow{BC}$ is equal to

$= |\overrightarrow{BA}| \cos\angle ABC$

$$= 7\left|\frac{7^2 + 3^2 - 5^2}{2 \times 7 \times 3}\right| = \frac{11}{2}$$

SECTION-B

1. Let $A = \{a_{ij}\}$ be a 3×3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then $\det(3\text{Adj}(2A^{-1}))$ is equal to ______.

Official Ans. by NTA (108)

Sol. $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$|A| = 4$

$$\left|3\text{adj}\left(2A^{-1}\right)\right| = \left|3.2^2\text{adj}\left(A^{-1}\right)\right|$$

$$= 12^3\left|\text{adj}\left(A^{-1}\right)\right| = 12^3\left|A^{-1}\right|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = 108$$

2. The number of solutions of the equation

$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$,

$x > 0$, is

Official Ans. by NTA (1)

Sol. $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$

$\log_{(x+1)}(2x+5)(x+1) + 2\log_{(2x+5)}(x+1) = 4$

$\log_{(x+1)}(2x+5) + 1 + 2\log_{(2x+5)}(x+1) = 4$

Put $\log_{(x+1)}(2x+5) = t$

$t + \dfrac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$

$t = 1, 2$

$\log_{(x+1)}(2x+5) = 1$ & $\log_{(x+1)}(2x+5) = 2$

$x + 1 = 2x + 3$ & $2x + 5 = (x+1)^2$

$x = -4$ (rejected) $\quad x^2 = 4 \Rightarrow x = 2, -2$ (rejected)

So, $x = 2$

No. of solution = 1

3. Let a curve $y = y(x)$ be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x} - 1}\,dy$$

If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to ________.

Official Ans. by NTA (2)

Sol. $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x} - 1}\,dy$

Put $\cos^{-1}(e^{-x})\ \theta$, $\theta \in [0, \pi]$

$\cos\theta = e^{-x} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 = e^{-x}$

$\cos\frac{\theta}{2} = \sqrt{\dfrac{e^{-x}+1}{2}} = \sqrt{\dfrac{e^x+1}{2c^x}}$

$\sqrt{\dfrac{e^x+1}{2c^x}}dx = \sqrt{e^{2x}-1}\,dy$

$\dfrac{1}{\sqrt{2}}\displaystyle\int \frac{dx}{\sqrt{e^x}\sqrt{e^x-1}} = \int dy$

Put $e^x = t$, $\dfrac{dt}{dx} = e^x$

$\dfrac{1}{\sqrt{2}}\displaystyle\int \frac{dt}{e^x\sqrt{e^x}\sqrt{e^x-1}} = \int dy$

$\displaystyle\int \frac{dt}{t\sqrt{t^2-t}} = \sqrt{2}y$

Put $t = \dfrac{1}{z}, \dfrac{dt}{dz} = -\dfrac{1}{z^2}$

$\displaystyle\int \frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2} - \frac{1}{z}}} = \sqrt{2}y$

$-\displaystyle\int \frac{dz}{\sqrt{1-z}} = \sqrt{2}y$

$\dfrac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$

$2\left(1 - \dfrac{1}{t}\right)^{1/2} = \sqrt{2}y + c$

$2\left(1 - e^{-x}\right)^{1/2} = \sqrt{2y} + c \xrightarrow{(0,-1)} \Rightarrow c = \sqrt{2}$

$2(1-e^{-x})^{1/2} = \sqrt{2}(y+1)$, passes through $(\alpha, 0)$

$2(1-e^{-\alpha})^{1/2} = \sqrt{2}$

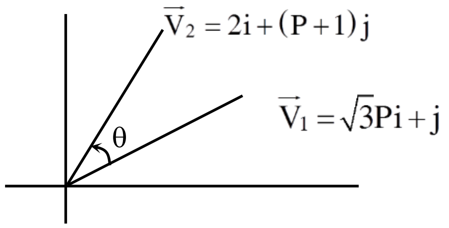
$\sqrt{1-e^{-\alpha}} = \dfrac{1}{\sqrt{2}} \Rightarrow 1 - e^{-\alpha} = \dfrac{1}{2}$

$e^{-\alpha} = \dfrac{1}{2} \Rightarrow e^{\alpha} = 2$

4. For $p > 0$, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta = \frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to ________.

Official Ans. by NTA (6)

Sol.



$|\vec{V}_1| = |\vec{V}_2|$

$3P^2 + 1 = 4 + (P+1)^2$

$2P^2 - 2P - 4 = 0 \Rightarrow P^2 - P - 2 = 0$

$P = 2, -1$ (rejected)

$$\cos\theta = \frac{\vec{V}_1 \cdot \vec{V}_2}{|\vec{V}_1||\vec{V}_2|} = \frac{2\sqrt{3}P + (P+1)}{\sqrt{(P+1)^2 + 4}\sqrt{3P^2+1}}$$

$$\cos\theta = \frac{4\sqrt{3}+3}{\sqrt{13}\sqrt{13}} = \frac{4\sqrt{3}+3}{13}$$

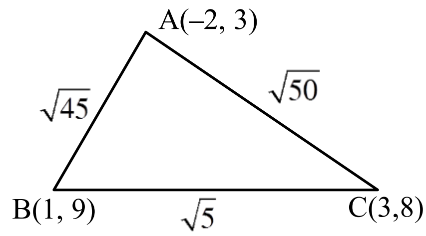
$$\tan\theta = \frac{\sqrt{112 - 24\sqrt{3}}}{4\sqrt{3}+3} = \frac{6\sqrt{3}-2}{4\sqrt{3}+3} = \frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$$

$\Rightarrow \alpha = 6$

5. Consider a triangle having vertices $A(-2, 3)$, $B(1, 9)$ and $C(3, 8)$. If a line L passing through the circum-centre of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is ________.

Official Ans. by NTA (9)

Sol.



$$\left(\sqrt{50}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{5}\right)^2$$

$\angle B = 90°$

Circum-center $= \left(\frac{1}{2}, \frac{11}{2}\right)$

Mid point of BC $= \left(2, \frac{17}{2}\right)$

Line : $\left(y - \frac{11}{2}\right) = 2\left(x - \frac{1}{2}\right) \Rightarrow y = 2x + \frac{9}{2}$

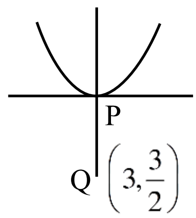
Passing though $\left(0, \frac{\alpha}{2}\right)$

$\frac{\alpha}{2} = \frac{9}{2} \Rightarrow \alpha = 9$

6. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to ________.

Official Ans. by NTA (9)

Sol.



$P \equiv \left(\frac{3}{2}t^2, 3t\right)$

Normal at point P

$tx + y = 3t + \frac{3}{2}t^3$

Passes through $\left(3, \frac{3}{2}\right)$

$\Rightarrow 3t + \frac{3}{2} = 3t + \frac{3}{2}t^3$

$P \equiv \left(\frac{3}{2}, 3\right) = (\alpha, \beta)$

$\Rightarrow t^3 = 1 \Rightarrow t = 1$

$2(\alpha + \beta) = 2\left(\frac{3}{2} + 3\right) = 9$

7. Let a function $g : [0, 4] \to \mathbf{R}$ be defined as

$$g(x)=\begin{cases}\max\limits_{0\le t\le x}\{t^3-6t^2+9t-3\}, & 0\le x\le 3\\ 4-x, & 3<x\le 4\end{cases},$$

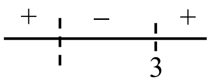
then the number of points in the interval (0, 4) where g(x) is NOT differentiable, is ________.

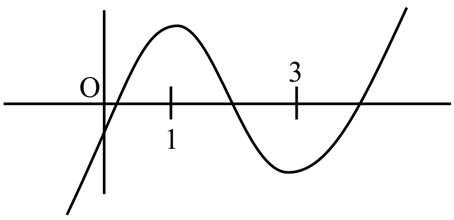
Official Ans. by NTA (1)

Sol. $f(x) = x^3 - 6x^2 + 9x - 3$

$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$

$f(1) = 1 \; f(3) = -3$





$$g(x)=\begin{bmatrix} f(x) & 0\le x\le 1\\ 0 & 1\le x\le 3\\ -1 & 3<x\le 4\end{bmatrix}$$

g(x) is continuous

$$g'(x)=\begin{bmatrix} 3(x-1)(x-3) & 0\le x\le 1\\ 0 & 1\le x\le 3\\ -1 & 3<x\le 4\end{bmatrix}$$

g(x) is non-differentiable at x = 3

8. For $k \in N$, let

$$\frac{1}{\alpha(\alpha+1)(\alpha+2).......(\alpha+20)}=\sum_{K=0}^{20}\frac{A_k}{\alpha+k},$$

where $\alpha > 0$. Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to ________.

Official Ans. by NTA (9)

Sol. $$\frac{1}{\alpha(\alpha+1).....(\alpha+20)}=\sum_{k=0}^{20}\frac{A_k}{\alpha+k}$$

$$A_{14}=\frac{1}{(-14)(-13).....(-1)(1).....(6)}=\frac{1}{14!\cdot 6!}$$

$$A_{15}=\frac{1}{(-15)(-14).....(-1)(1).....(5)}=\frac{1}{15!\cdot 5!}$$

$$A_{13}=\frac{1}{(-13).....(-1)(1).....(7)}=\frac{-1}{13!\cdot 7!}$$

$$\frac{A_{14}}{A_{13}}=\frac{1}{14!\cdot 6!}\times -13!\times 7!=\frac{-7}{14}=-\frac{1}{2}$$

$$\frac{A_{15}}{A_{13}}=-\frac{1}{15!\times 5!}\times -13!\times 7!=\frac{42}{15\times 14}=\frac{1}{5}$$

$$100\left(\frac{A_{14}}{A_{13}}+\frac{A_{15}}{A_{13}}\right)^2=100\left(-\frac{1}{2}+\frac{1}{5}\right)^2=9$$

9. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \ge 1$. Then the value of $47\sum\limits_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to ________.

Official Ans. by NTA (7)

Sol. $a_{n+2}=2a_{n+1}+a_n$, let $\sum\limits_{n=1}^{\infty}\frac{a_n}{8^n}=P$

Divide by 8^n we get

$$\frac{a_{n+2}}{8^n}=\frac{2a_{n+1}}{8^n}+\frac{a_n}{8^n}$$

$$\Rightarrow 64\frac{a_{n+2}}{8^{n+2}}=\frac{16a_{n+1}}{8^{n+1}}+\frac{a_n}{8^n}$$

$$64\sum_{n=1}^{\infty}\frac{a_{n+2}}{8^{n+2}}=16\sum_{n=1}^{\infty}\frac{a_{n+1}}{8^{n+1}}+\sum_{n=1}^{\infty}\frac{a_n}{8^n}$$

$$64\left(P-\frac{a_1}{8}-\frac{a_2}{8^2}\right)=16\left(P-\frac{a_1}{8}\right)+P$$

$$\Rightarrow 64\left(P-\frac{1}{8}-\frac{1}{64}\right)=16\left(P-\frac{1}{8}\right)+P$$

$64P - 8 - 1 = 16P - 2 + P$

$47P = 7$

10. If $\lim\limits_{x\to 0}\frac{\alpha xe^{x}-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2 x}=10$, $\alpha, \beta, \gamma \in \mathbf{R}$,

then the value of $\alpha+\beta+\gamma$ is ________.

Official Ans. by NTA (3)

Sol. $$\lim_{x\to 0}\frac{\alpha x\left(1+x+\frac{x^2}{2}\right)-\beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)+\gamma x^2(1-x)}{x^3}$$

$$\lim_{x\to 0}\frac{x(\alpha-\beta)+x^2\left(\alpha+\frac{\beta}{2}+\gamma\right)+x^3\left(\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right)}{x^3}=10$$

For limit to exist

$\alpha-\beta=0,\ \alpha+\frac{\beta}{2}+\gamma=0$

$\frac{\alpha}{2}-\frac{\beta}{3}-\gamma=10\ \ldots\ldots$(i)

$\beta=\alpha,\ \gamma=-3\frac{\alpha}{2}$

Put in (i)

$\frac{\alpha}{2}-\frac{\alpha}{3}+\frac{3\alpha}{2}=10$

$\frac{\alpha}{6}+\frac{3\alpha}{2}=10 \Rightarrow \frac{\alpha+9\alpha}{6}=10$

$\Rightarrow \alpha=6$

$\alpha=6, \beta=6, \gamma=-9$

$\alpha+\beta+\gamma=3$