EMI & AC

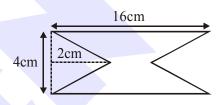
- **1.** A long solenoid of radius R carries a time (t)-dependent current $I(t) = I_0 t(1 - t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced $EMF(V_R)$ in the ring change as:
 - (1) At t = 0.5 direction of I_R reverses and V_R is
 - (2) Direction of $\boldsymbol{I}_{\!\scriptscriptstyle R}$ remains unchanged and $\boldsymbol{V}_{\!\scriptscriptstyle R}$ is zero at t = 0.25
 - (3) Direction of I_R remains unchanged and V_R is maximum at t = 0.5
 - (4) At t = 0.25 direction of I_R reverses and V_R is maximum
- 2. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical springmass damped oscillator having damping constant 'b', the correct equivalence would be:

$$(1) L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$$

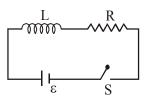
$$(2) \ L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$$

- (3) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$
- (4) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$
- **3.** An emf of 20 V is applied at time t=0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t = \infty$ and at t = 40 s is close to : (Take $e^2 = 7.389$)
 - (1) 1.06
- (2) 1.15
- (3) 1.46
- (4) 0.84

- 4. A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at:
 - (1) 2.5 s and 7.5 s
- (2) 5.0 s and 7.5 s
- (3) 5.0 s and 10.0 s
- (4) 2.5 s and 5.0 s
- At time t = 0 magnetic field of 100 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is:



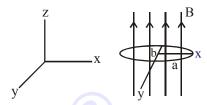
- (1) $36 \mu V$
- (2) $48 \mu V$
- (3) 56 µV
- (4) $28 \mu V$
- 6. A shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and $t = t_c$ (t_c is the time constant of the circuit) is:



- $(1) \frac{\varepsilon L}{R^2} \left(1 \frac{1}{e} \right) \qquad (2) \frac{\varepsilon R}{eL^2}$
- (3) $\frac{\varepsilon L}{R^2}$

- 7. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be _______.
- 8. In LC circuit the inductance L = 40 mH and capacitance C = 100 μ F. If a voltage $V(t) = 10\sin(314\ t)$ is applied to the circuit, the current in the circuit is given as:
 - (1) 0.52 cos 314 t
- (2) 0.52 sin 314 t
- (3) 10 cos 314 t
- (4) 5.2 cos 314 t
- 9. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2s. The maximum value of EMF induced (in μ V) in the coil will be close to the integer ______.
- 10. An inductance coil has a reactance of 100 Ω . When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is :
 - (1) $1.1 \times 10^{-2} \text{ H}$
- (2) $1.1 \times 10^{-1} \text{ H}$
- $(3) 5.5 \times 10^{-5} H$
- $(4) 6.7 \times 10^{-7} H$
- 11. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω , an inductance of 0.1803 H and a capacitance of 10 μ F all in series. The time in which the resistance (heat capacity 2J/ °C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to :
 - (1) 418 s
- (2) 245 s
- (3) 348 s
- (4) 365 s

12. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is:

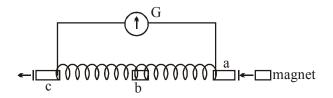


- $(1) \frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$
- (2) Zero
- $(3) \ \frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$
- $(4) \frac{\pi abB\omega}{R}$
- 13. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB/dt = 0.032 Ts⁻¹. The induced current in the loop is close to

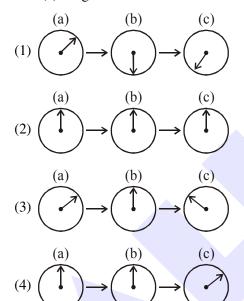
(Resistivity of the metal wire is $1.23 \times 10^{-8} \Omega m$)

- (1) 0.61 A
- (2) 0.34 A
- (3) 0.43 A
- (4) 0.53 A

14. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations wil be seen on the galvanometer G attached across the coil?



Three positions shown describe: (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.



15. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the

inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum

value, is:

$$(1) \ \frac{L}{R} ln \Biggl(\frac{\sqrt{n}-1}{\sqrt{n}} \Biggr) \qquad \quad (2) \ \frac{L}{R} ln \Biggl(\frac{\sqrt{n}}{\sqrt{n}+1} \Biggr)$$

(2)
$$\frac{L}{R} \ln \left(\frac{\sqrt{n}}{\sqrt{n}+1} \right)$$

$$(3) \ \frac{L}{R} ln \left(\frac{\sqrt{n}}{\sqrt{n} - 1} \right) \qquad (4) \ \frac{L}{R} ln \left(\frac{\sqrt{n} + 1}{\sqrt{n} - 1} \right)$$

$$(4) \frac{L}{R} \ln \left(\frac{\sqrt{n} + 1}{\sqrt{n} - 1} \right)$$

16. A circular coil has moment of inertia 0.8 kg m² around any diameter and is carrying current to produce a magnetic moment of 20 Am². The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be:

(1) 10 rad s^{-1}

(2) 20 π rad s⁻¹

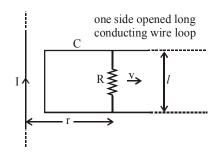
(3) 10 π rad s⁻¹

(4) 20 rad s⁻¹

17. Two concentric circular coils, C_1 and C_2 , are placed in the XY plane. C₁ has 500 turns, and a radius of 1 cm. C₂ has 200 turns and radius of 20 cm. C, carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A where t is in s. The emf induced in C_1 (in mV), at the instant t = 1s

is $\frac{4}{x}$. The value of x is ____.

18. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is:



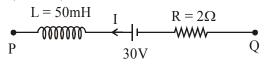
 $(1) \frac{\mu_0}{\pi} \frac{Ivl}{Rr}$

(2) $\frac{\mu_0}{2\pi} \frac{\text{Iv}l}{\text{Rr}}$

(3) $\frac{2\mu_0}{\pi} \frac{\text{Iv}l}{\text{Rr}}$

 $(4) \frac{\mu_0}{4\pi} \frac{Ivl}{Rr}$

- 19. An AC circuit has R = 100 Ω , C = 2 μ F and L = 80 mH, connected in series. The quality factor of the circuit is :
 - (1) 0.5
- (2) 2
- (3) 20
- (4) 400
- 20. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of $10^2 A \ s^{-1}$. The value of the potential difference $V_P V_Q$, (in volts) at that instant, is.



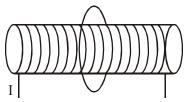
21. In a scries LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking

the value of C as $\left(\frac{n}{3\pi}\right)\mu F$, then value of n is

SOLUTION

1. NTA Ans. (1)

Sol.



Magnetic flux (ϕ) through ring is $\phi = \pi(R)^2$.B $\phi = (\pi R^2)(\mu_0 n I) = (\pi R^2 \mu_0 n I_0)(t - t^2)$

Induced e.m.f. of
$$V_R = \frac{-d\phi}{dt}$$
$$= \left(\pi R^2 \mu_0 n I_0\right) (2t-1)$$

and induced current
$$I_R = \frac{\pi R^2 \mu_0 n I_0 (2t-1)}{R_R}$$

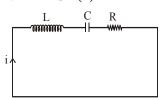
 $(R_R \rightarrow Resistance of Ring)$

Clearly V_R and I_R are zero at $t = \frac{1}{2} = 0.5$ sec.

and their sign also changes at t = 0.5 sec.

2. NTA Ans. (1)

Sol.



By kVL

$$-L\frac{di}{dt} - \frac{q}{C} - iR = 0$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q + R\frac{dq}{dt} = 0$$

for damped oscillator

$$net force = -kx - bv = ma$$

$$\frac{md^2x}{dt^2} + kx + \frac{bdx}{dt} = 0$$

by comparing; Equivalence is

$$L \to m ; C \to \frac{1}{K} ; R \to b.$$

3. NTA Ans. (1)

ALLEN Ans. (2)

Sol.
$$i = i_0 (1 - e^{-Rt/L})$$

$$\frac{\mathbf{i}_0}{\mathbf{i}} = \frac{1}{1 - e^{-2 \times 10^4}}$$

$$\frac{i_0}{i} \simeq 1$$

4. NTA Ans. (4)

Sol. Flux
$$\phi = \vec{B} \cdot \vec{A} = BA \cos \theta = BA \cos \omega t$$

|Induced emf| = |e| =
$$\left| \frac{d\phi}{dt} \right|$$
 = |BA ω sin ωt |

lel will be maximum at $\omega t = \frac{\pi}{2}$

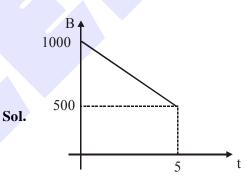
$$\left(\frac{2\pi}{T}\right)t = \frac{\pi}{2}$$

$$\left(\frac{2\pi}{10}\right)$$
t = $\frac{\pi}{2}$ \Rightarrow t = 2.5 sec

lel will be minimum at $\omega t = \pi$

$$\left(\frac{2\pi}{10}\right)$$
t = $\pi \implies$ t = 5 sec

5. NTA Ans. (3)



$$\frac{dB}{dt} = 100$$

$$A = 16 \times 4 - 4 \times 2 = 56 \text{ cm}^2$$

$$\varepsilon = \frac{dB}{dt} A = 100 \times 10^{-4} \times 56 \times 10^{-4}$$

6. NTA Ans. (4)

Sol.
$$i = i_0 (1 - e^{-Rt/L}) = i_0 (1 - e^{-t/T_C})$$

$$q = \int\limits_0^{T_C} i \ dt \qquad \qquad \Rightarrow = \int\limits_0^{T_C} \frac{\epsilon}{R} \big(1 - e^{-t/T_C} \, \big)$$

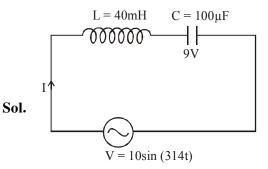
$$= \frac{\varepsilon}{R} \left(t - \frac{e^{-t/T_C}}{-1/T_C} \right) \Big|_{0}^{T_C}$$

$$\begin{split} &= \frac{e}{R} \Big(T_C - T_C e^{-1} \Big) - \frac{e}{R} (0 + T_C) \Rightarrow q = \frac{e}{R} \times T_C \ e^{-1} \\ &= \frac{\epsilon}{R} \times \frac{L}{R} \frac{1}{e} \qquad \Rightarrow = \frac{\epsilon L}{eR^2} \end{split}$$

7. NTA Ans. (10.00)

Sol.
$$V = \left| L \frac{di}{dt} \right|$$

$$\Rightarrow L = \frac{V}{\left| \frac{di}{dt} \right|} = \frac{100}{0.25} = 10 \text{mH}$$

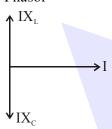


$$X_L = \omega L = 314 \times 40 \times 10^{-3} = 12.56\Omega$$

$$X_{C} = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}}$$

$$=\frac{10^4}{314}=31.84\Omega$$

Phasor



$$V_{\rm m} = I_{\rm m}(X_{\rm C} - X_{\rm L})$$

10 = $I_{\rm m}(31.84 - 12.56)$

$$I_{\rm m} = \frac{10}{19.28} = 0.52A$$

$$I = 0.52 \sin \left(314t + \frac{\pi}{2} \right)$$

:. Correct answer (1)

9. Official Ans. by NTA (15)

Sol.
$$r = 0.1 \text{ m}$$
 $\frac{T}{2} = 0.2 \text{ sec}$
 $B = 3 \times 10^{-5} \text{ m}$ $T = 0.4 \text{ sec}$
At any time

flux
$$\phi = BA \cos \omega t$$

$$|emf| = \left| \frac{d\phi}{dt} \right| = \left| BA\omega \sin \omega t \right|$$

$$(emf)_{max} = BA\omega = BA \frac{2\pi}{T}$$
$$- \frac{3 \times 10^{-5} \times \pi \times (0.1)^2 \times 2\pi}{}$$

$$= \frac{3 \times 10^{-5} \times \pi \times (0.1)^{2} \times 2\pi}{0.4}$$

$$= \frac{6\pi^{2}}{4} \times 10^{-6} \qquad \left(\frac{\pi^{2} \approx 10}{\text{take}}\right)$$

$$= 15 \times 10^{-6}$$

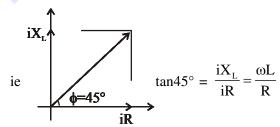
$$= 15 \text{ } \mu\text{V}$$

10. Official Ans. by NTA (1)

• Reactance of inductance coil

$$= \sqrt{R^2 + x_L^2} = 100 \qquad(i)$$

- f = 1000 Hz of applied AC signal
- Voltage leads current by 45°



ie
$$R = X_1 = \omega L$$

Putting in eqn (i) :
$$\sqrt{X_L^2 + X_L^2} = 100$$

$$\sqrt{2}X_x = 100 \Rightarrow X_x = 50\sqrt{2}$$

ie
$$\omega L = 50\sqrt{2}$$

$$L = \frac{50\sqrt{2}}{\omega} = \frac{50\sqrt{2}}{2\pi f} = \frac{25\sqrt{2}}{\pi \times 1000} \quad H$$
$$= 1.125 \times 10^{-2} \text{ H}$$

Sol.
$$f = 750 \text{ Hz}, V_{rms} = 20V,$$

 $R = 100 \Omega, L = 0.1803 \text{ H},$
 $C = 10 \mu\text{F}, S = 2 \text{ J/}^{\circ}\text{C}$

$$= \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

Putting values

$$|Z| = 834\Omega$$

In AC power $P = V_{rms} i_{rms} \cos \phi$

$$\cos\phi = \frac{R}{|Z|}$$
 $i_{rms} = \frac{V_{rms}}{|Z|}$

$$=\frac{V_{\rm rms}^2 R}{\left(\mid Z\mid\right)^2}$$

$$=\left(\frac{20}{834}\right)^2 \times 100 = 0.0575 \text{ J/s}$$

$$H = Pt = S\Delta\theta$$

$$t = \frac{2(10)}{0.0575} = 348 \text{ sec}$$

12. Official Ans. by NTA (1)

Sol. \in = NAB ω cos ω t

$$N = 1$$

$$P_{\text{avg}} = \langle \frac{\epsilon^2}{R} \rangle = \langle \frac{(AB\omega \cos \omega t)^2}{R} \rangle$$

$$= \frac{A^2 B^2 \omega^2}{R} \frac{1}{2} = \frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$$

Official Ans. by NTA (1) 13.

Sol.

$$\longrightarrow \stackrel{r=2 \text{ mm}}{\leftarrow}$$

$$q_i = \frac{d(Ba^2)}{dt} = a^2 \frac{dB}{dt}$$

$$i = \frac{q}{R} = \frac{a^2 dB/dt}{\frac{\rho(40)}{\pi r^2}}$$

Official Ans. by NTA (3)

When bar magnet is entering with constant speed, flux will change and an e.m.f. is induced, so galvanometer will deflect in positive direction.

When magnet is completely inside, flux will not change, so reading of galvanometer will

When bar magnet is making on exit, again flux will change and on e.m.f. is induced in opposite direction to not of (a), so galvanometer will deflect in negative direction.

Looking at options, option (3) is correct.

15. Official Ans. by NTA (3)

Sol.
$$U_{\text{max}} = \frac{1}{2}LI_{\text{max}}^2$$

$$i = I_{max} \left(1 - e^{-Rt/L} \right)$$

For U to be
$$\frac{U_{\text{max}}}{n}$$
; i has to be $\frac{I_{\text{max}}}{\sqrt{n}}$

$$\frac{I_{\text{max}}}{\sqrt{n}} = I_{\text{max}} \left(1 - e^{-Rt/L} \right)$$

$$e^{-Rt/L} = 1 - \frac{1}{\sqrt{n}} = \frac{\sqrt{n} - 1}{\sqrt{n}}$$

$$-\frac{Rt}{L} = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$t = \frac{L}{R} \ln \left(\frac{\sqrt{n}}{\sqrt{n} - 1} \right)$$

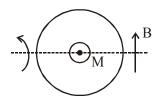
16. Official Ans. by NTA (1) Official Ans. by ALLEN (BONUS)

Sol.
$$I_{dia} = 0.8 \text{ kg/m}^2$$

 $M = 20 \text{ Am}^2$

$$M_{dia} = 0.8 \text{ kg/m}$$

 $M = 20 \text{ Am}^2$



$$U_i + K_i = U_f + K_f$$

$$0 + 0 = -MB \cos 30^\circ + \frac{1}{2}I\omega^2$$

$$20 \times 4 \times \frac{\sqrt{3}}{2} = \frac{1}{2}(0.8) \omega^2$$

$$\omega = \sqrt{100\sqrt{3}} = 10(3)^{1/4}$$

17. Official Ans. by NTA (5.00)

Sol.



$$B = \frac{\mu_0 NI}{2R}$$

$$\phi = \frac{\mu_0 N N' I}{2R} \pi r^2$$

$$\epsilon = \frac{d\phi}{dt} = \frac{2\pi \times 10^{-7} \times 10^5 \times \pi \times 10^{-4}}{0.2}$$

$$= 8 \times 10^{-4} = 0.8 \text{ mV}$$

18. Official Ans. by NTA (2)

Sol.
$$B = \frac{\mu_0 i}{2\pi r}$$

$$\phi = \frac{\mu_0 i}{2\pi r} \ell dr$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{\mu_0 i\ell}{2\pi r} \cdot \frac{dr}{dt}$$

$$\Rightarrow e = \frac{\mu_0}{2\pi} \cdot \frac{iv\ell}{r}$$

$$i = \frac{e}{R} = \frac{\mu_0}{2\pi}.\,\frac{iv\ell}{Rr}$$

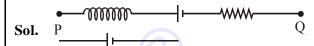
19. Official Ans. by NTA (2)

Sol.
$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{100} \sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}}$$

$$=\frac{1}{100}\sqrt{40\times10^3}$$

$$=\frac{200}{100}=2$$

Official Ans. by NTA (33.00) 20.



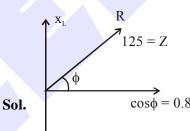
$$\frac{\text{Ldi}}{\text{dt}} = 5$$

$$V_P - 5 - 30 + 2 \times 1 = VQ$$

 $V_P - V_Q = 33 \text{ volt}$
Ans. 33.00

$$V_p - V_Q = 33 \text{ volt}$$

Official Ans. by NTA (400.00) 21.



$$P = \frac{E_{rms}^2}{Z} \cos \phi$$

$$400 = \frac{(250)^2 \times 0.8}{Z}$$

$$Z = 25 \times 5 = 125$$

$$X_L = 125 \sin \phi = 125 \times 0.6 = 75$$