

DIFFERENTIAL EQUATION

- माना अवकल समीकरण $(y^2 - x) \frac{dy}{dx} = 1$, का हल वक्र $y = y(x)$, $y(0) = 1$ को सन्तुष्ट करता है। यह वक्र x -अक्ष को जिस बिन्दु पर काटता है उसका भुज है :
 (1) $2 + e$ (2) 2
 (3) $2 - e$ (4) $-e$
- यदि अवकलन समीकरण, $e^y \left(\frac{dy}{dx} - 1 \right) = e^x$, जबकि $y(0) = 0$, का हल $y = y(x)$ है, तो $y(1)$ बराबर है :
 (1) $2 + \log_e 2$ (2) $2e$
 (3) $\log_e 2$ (4) $1 + \log_e 2$
- वक्रों $x^2 = 4b(y + b)$, $b \in \mathbb{R}$, के कुल का अवकल समीकरण है :
 (1) $x(y')^2 = x + 2yy'$
 (2) $x(y')^2 = 2yy' - x$
 (3) $xy'' = y'$
 (4) $x(y')^2 = x - 2yy'$
- माना $y = y(x)$, अवकल समीकरण $\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$ का एक हल है। यदि $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$ है, तो $y\left(\frac{-1}{\sqrt{2}}\right)$ बराबर है :
 (1) $-\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$
 (3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{\sqrt{2}}$

- यदि $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; $y(1) = 1$ है, तो $y(x) = e$ को सन्तुष्ट करने वाला x का एक मान है :
 (1) $\sqrt{2}e$ (2) $\frac{e}{\sqrt{2}}$
 (3) $\frac{1}{2}\sqrt{3}e$ (4) $\sqrt{3}e$
- यदि $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ है $f(0) = 0$ है, तो $f(1)$ का मान है :
 (1) $\frac{\pi-1}{4}$ (2) $\frac{\pi+2}{4}$
 (3) $\frac{\pi+1}{4}$ (4) $\frac{1}{4}$
- यदि $x \geq 0$ के लिए $y = y(x)$, अवकल समीकरण $(x+1)dy = ((x+1)^2 + y - 3)dx$, $y(2) = 0$ का हल है, तो $y(3)$ का मान है ———।
- माना $y = y(x)$, अवकल समीकरण $\frac{2+\sin x}{y+1} \cdot \frac{dy}{dx} = -\cos x, y > 0, y(0) = 1$, का हल है। यदि $y(\pi) = a$ है तथा $x = \pi$ पर $\frac{dy}{dx} = b$ है, तो क्रमित युग्म (a, b) बराबर है।
 (1) $(2, 1)$ (2) $\left(2, \frac{3}{2}\right)$
 (3) $(1, -1)$ (4) $(1, 1)$
- यदि एक वक्र, $y = f(x)$ बिन्दु $(1, 2)$ से होकर जाता है तथा अवकल समीकरण $2x^2 dy = (2xy + y^2)dx$ का हल है, तो $f\left(\frac{1}{2}\right)$ बराबर है -
 (1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$
 (3) $\frac{-1}{1+\log_e 2}$ (4) $1+\log_e 2$

10. अवकल समीकरण $(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$ का हल वक्र, जो बिन्दु $(0, 1)$ से होकर जाता है, है-

(1) $y^2 = 1 + y \log_e \left(\frac{1+e^x}{2} \right)$

(2) $y^2 + 1 = y \left(\log_e \left(\frac{1+e^x}{2} \right) + 2 \right)$

(3) $y^2 = 1 + y \log_e \left(\frac{1+e^{-x}}{2} \right)$

(4) $y^2 + 1 = y \left(\log_e \left(\frac{1+e^{-x}}{2} \right) + 2 \right)$

11. यदि $x^3 dy + xy dx = x^2 dy + 2y dx$; $y(2) = e$ तथा $x > 1$, तो $y(4)$ बराबर है :

(1) $\frac{3}{2} + \sqrt{e}$

(2) $\frac{3}{2} \sqrt{e}$

(3) $\frac{1}{2} + \sqrt{e}$

(4) $\frac{\sqrt{e}}{2}$

12. माना अवकल समीकरण $xy' - y = x^2(x \cos x + \sin x)$, $x > 0$ का हल $y = y(x)$ हैं। यदि $y(\pi) = \pi$ हो, तो

$y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ होगा

(1) $2 + \frac{\pi}{2}$

(2) $1 + \frac{\pi}{2}$

(3) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$

(4) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

13. अवकल समीकरण $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ का हल है :

(जहाँ C एक समाकलन अचर है।)

(1) $x - 2 \log_e(y+3x) = C$

(2) $x - \log_e(y+3x) = C$

(3) $x - \frac{1}{2} (\log_e(y+3x))^2 = C$

(4) $y + 3x - \frac{1}{2} (\log_e x)^2 = C$

14. यदि अवकल समीकरण $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$, का हल $y = y(x)$ है, जिसके लिए $y(0) = 1$ है, तो $y(\log_e 13)$ का एक मान है :

(1) 1

(2) -1

(3) 2

(4) 0

15. माना $y = y(x)$, अवकल समीकरण $\cos x \frac{dy}{dx} + 2y \sin$

$x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$ का हल है। यदि

$y(\pi/3) = 0$ है तो $y(\pi/4)$ बराबर है :

(1) $\sqrt{2} - 2$

(2) $\frac{1}{\sqrt{2}} - 1$

(3) $2 - \sqrt{2}$

(4) $2 + \sqrt{2}$

16. निम्न में से कौनसा बिन्दु वक्र $x^4 e^y + 2\sqrt{y+1} = 3$ के बिन्दु $(1, 0)$ पर खींची गई स्पर्श रेखा पर स्थित है ?

(1) $(2, 2)$

(2) $(-2, 6)$

(3) $(-2, 4)$

(4) $(2, 6)$

17. अवकल समीकरण $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ का

व्यापक हल है :

(जहाँ C एक समाकलन अचर है)

$$(1) \sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right) + C$$

$$(2) \sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right) + C$$

$$(3) \sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right) + C$$

$$(4) \sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right) + C$$

18. यदि अवकल समीकरण $\frac{dy}{dx} + p(x)y = \frac{2}{\pi} \operatorname{cosec} x, 0$

$< x < \frac{\pi}{2}$, का हल $y = \left(\frac{2}{\pi} x - 1 \right) \operatorname{cosec} x$ है, तो फलन

$p(x)$ बराबर है

(1) $\cot x$

(2) $\tan x$

(3) $\operatorname{cosec} x$

(4) $\sec x$

SOLUTION

1. NTA Ans. (3)

Sol. $(y^2 - x) \frac{dy}{dx} = 1$

$$\Rightarrow \frac{dx}{dy} + x = y^2$$

$$\text{I.F.} = e^{\int dy} = e^y$$

Solution is given by

$$x e^y = \int y^2 e^y dy + C$$

$$\Rightarrow x e^y = (y^2 - 2y + 2)e^y + C$$

$$x = 0, y = 1, \text{ gives } C = -e$$

$$\text{If } y = 0, \text{ then } x = 2 - e$$

2. NTA Ans. (4)

Sol. $e^y \frac{dy}{dx} - e^y = e^x$, Let $e^y = t$

$$\Rightarrow e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - t = e^x$$

$$\text{I.F.} = e^{\int -dx} = e^{-x}$$

$$t e^{-x} = x + c \Rightarrow e^{y-x} = x + c$$

$$y(0) = 0 \Rightarrow c = 1$$

$$e^{y-x} = x + 1 \Rightarrow y(1) = 1 + \log_e 2$$

3. NTA Ans. (1)

Sol. $2x = 4by' \Rightarrow y' = \frac{2x}{4b}$

$$\text{Required D.E. is } x^2 = \frac{2x}{y'} y + \left(\frac{x}{y'} \right)^2$$

$$x(y')^2 = 2yy' + x$$

(1) Option

4. NTA Ans. (2)

ALLEN Ans. (BONUS)

Note: As per the given informaton, x cannot be negative. So, it is invalid to ask $y(x)$ for $x < 0$. Hence, it should be bonus but, NTA retained its answer as option (2).

Sol. $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ so, $\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$

$$\text{Integrating, } \sin^{-1} x + \sin^{-1} y = c$$

$$\text{so, } \frac{\pi}{6} + \frac{\pi}{3} = c$$

$$\text{Hence, } \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

$$\text{Put } x = -\frac{1}{\sqrt{2}}, \sin^{-1} y = \frac{3\pi}{4} \text{ (Not possible)}$$

5. NTA Ans. (4)

Sol. $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

$$\text{Let } y = vx$$

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{xvx}{x^2 + v^2x^2} = \frac{v}{1 + v^2}$$

$$x \frac{dv}{dx} = \frac{v}{1 + v^2} - v = \frac{v - v - v^3}{1 + v^2} = -\frac{v^3}{1 + v^2}$$

$$\int \frac{1 + v^2}{v^3} \cdot dv = \int -\frac{dx}{x}$$

$$\Rightarrow \int v^{-3} \cdot dv + \int \frac{1}{v} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{v^{-2}}{-2} + \ln v = -\ln x + \lambda$$

$$\Rightarrow -\frac{1}{2v^2} + \ln \left(\frac{y}{x} \right) = -\ln x + \lambda$$

$$\Rightarrow -\frac{1}{2} \frac{x^2}{y^2} + \ln y - \ln x = -\ln x + \lambda$$

$$\Rightarrow -\frac{1}{2} + 0 = \lambda \Rightarrow \lambda = -\frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \frac{x^2}{y^2} + \ln y + \frac{1}{2} = 0 \text{ at } y = e$$

$$\Rightarrow -\frac{1}{2} \frac{x^2}{e^2} + 1 + \frac{1}{2} = 0 \Rightarrow \frac{x^2}{2e^2} = \frac{3}{2} \Rightarrow x^2 = 3e^2$$

$$\therefore x = \sqrt{3}e$$

6. NTA Ans. (3)

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x)$

$$f'(x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$$

$$\therefore -\frac{\pi}{2} < x < \frac{\pi}{2} \Rightarrow 0 < \frac{\pi}{4} + \frac{x}{2} < \frac{\pi}{2}$$

$$\Rightarrow f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$$\therefore f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$\therefore f(0) = 0 \Rightarrow c = 0$$

$$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$$

$$\therefore f(1) = \frac{\pi + 1}{4}$$

7. NTA Ans. (3.00)

Sol. $(x+1)dy - ydx = ((x+1)^2 - 3)dx$

$$\Rightarrow \frac{(x+1)dy - ydx}{(x+1)^2} = \left(1 - \frac{3}{(x+1)^2}\right)dx$$

$$\Rightarrow d\left(\frac{y}{(x+1)}\right) = \left(1 - \frac{3}{(x+1)^2}\right)dx$$

integrating both sides

$$\frac{y}{x+1} = x + \frac{3}{(x+1)} + C$$

$$\text{Given } y(2) = 0 \Rightarrow c = -3$$

$$\therefore y = (x+1)\left(x + \frac{3}{(x+1)} - 3\right)$$

$$\therefore y(3) = 3.00$$

8. Official Ans. by NTA (4)

Sol. $\frac{2 + \sin x}{y+1} \frac{dy}{dx} = -\cos x, y > 0$

$$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2 + \sin x} dx$$

By integrating both sides :

$$\ln |y+1| = -\ln |2 + \sin x| + \ln K$$

$$\Rightarrow y+1 = \frac{K}{2 + \sin x} \quad (y+1 > 0)$$

$$\Rightarrow y(x) = \frac{K}{2 + \sin x} - 1$$

$$\text{Given } y(0) = 1 \Rightarrow K = 4$$

$$\text{So, } y(x) = \frac{4}{2 + \sin x} - 1$$

$$a = y(\pi) = 1$$

$$b = \left. \frac{dy}{dx} \right|_{x=\pi} = \left. \frac{-\cos x}{2 + \sin x} (y(x)+1) \right|_{x=\pi} = 1$$

$$\text{So, } (a, b) = (1, 1)$$

9. Official Ans. by NTA (2)

Sol. $2x^2 dy = (2xy + y^2) dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \quad \{\text{Homogeneous D.E.}\}$$

$$\left\{ \begin{array}{l} \text{let } y = xt \\ \Rightarrow \frac{dy}{dx} = t + x \frac{dt}{dx} \end{array} \right\}$$

$$\Rightarrow t + x \frac{dt}{dx} = \frac{2x^2 t + x^2 t^2}{2x^2}$$

$$\Rightarrow t + x \frac{dt}{dx} = t + \frac{t^2}{2}$$

$$\Rightarrow x \frac{dt}{dx} = \frac{t^2}{2}$$

$$\Rightarrow 2 \int \frac{dt}{t^2} = \int \frac{dx}{x}$$

$$\Rightarrow 2 \left(-\frac{1}{t} \right) = \ln(x) + C \quad \left\{ \text{Put } t = \frac{y}{x} \right\}$$

$$\Rightarrow -\frac{2x}{y} = \ln x + C \quad \left\{ \begin{array}{l} \text{Put } x = 1 \text{ \& } y = 2 \\ \text{then we get } C = -1 \end{array} \right\}$$

$$\Rightarrow \frac{-2x}{y} = \ln(x) - 1 \Rightarrow y = \frac{2x}{1 - \ln x}$$

$$\Rightarrow f(x) = \frac{2x}{1 - \log_e x}$$

$$\text{so, } \boxed{f\left(\frac{1}{2}\right) = \frac{1}{1 + \log_e 2}}$$

10. Official Ans. by NTA (1)

$$\text{Sol. } (1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$$

$$\Rightarrow (1 + y^2) dy = \left(\frac{e^x}{1 + e^x} \right) dx$$

$$\Rightarrow \left(y - \frac{1}{y} \right) = \ln(1 + e^x) + c$$

$$\therefore \text{ It passes through } (0, 1) \Rightarrow c = -\ln 2$$

$$\Rightarrow y^2 = 1 + y \ln \left(\frac{1 + e^x}{2} \right)$$

11. Official Ans. by NTA (2)

$$\text{Sol. } x^3 dy + xy dx = x^2 dy + 2y dx$$

$$\Rightarrow dy(x^3 - x^2) = dx(2y - xy)$$

$$\Rightarrow -\int \frac{1}{y} dy = \int \frac{x-2}{x^2(x-1)} dx$$

$$\Rightarrow -\ln y = \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} \right) dx$$

$$\text{Where } A = 1, B = +2, C = -1$$

$$\Rightarrow -\ln y = \ln x - \frac{2}{x} - \ln(x-1) + \lambda$$

$$\Rightarrow y(2) = e$$

$$\Rightarrow -1 = \ln 2 - 1 - 0 + \lambda$$

$$\therefore \lambda = -\ln 2$$

$$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln(x-1) + \ln 2$$

$$\text{Now put } x = 4 \text{ in equation}$$

$$\Rightarrow \ln y = -\ln 4 + \frac{1}{2} + \ln 3 + \ln 2$$

$$\Rightarrow \ln y = \ln \left(\frac{3}{2} \right) + \frac{1}{2} \ln e$$

$$\Rightarrow y = \frac{3}{2} \sqrt{e}$$

12. Official Ans. by NTA (1)

$$\text{Sol. } x \frac{dy}{dx} - y = x^2(x \cos x + \sin x), x > 0$$

$$\frac{dy}{dx} - \frac{y}{x} = x(x \cos x + \sin x) \Rightarrow \frac{dy}{dx} + Py = Q$$

$$\text{so, I.F.} = e^{\int \frac{-1}{x} dx} = \frac{1}{|x|} = \frac{1}{x} \quad (x > 0)$$

$$\text{Thus, } \frac{y}{x} = \int \frac{1}{x} (x(x \cos x + \sin x)) dx$$

$$\Rightarrow \frac{y}{x} = x \sin x + C$$

$$\therefore y(\pi) = \pi \Rightarrow C = 1$$

$$\text{so, } y = x^2 \sin x + x \Rightarrow (y)_{\pi/2} = \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$\text{Also, } \frac{dy}{dx} = x^2 \cos x + 2x \sin x + 1$$

$$\Rightarrow \frac{d^2y}{dx^2} = -x^2 \sin x + 4x \cos x + 2 \sin x$$

$$\Rightarrow \left. \frac{d^2y}{dx^2} \right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$$

$$\text{Thus, } y\left(\frac{\pi}{2}\right) + \frac{d^2y}{dx^2}\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 2$$

13. Official Ans. by NTA (3)

Sol. $\ln(y + 3x) = z$ (let)

$$\frac{1}{y + 3x} \left(\frac{dy}{dx} + 3 \right) = \frac{dz}{dx}$$

..(1)

$$\frac{dy}{dx} + 3 = \frac{y + 3x}{\ln(y + 3x)} \quad (\text{given})$$

$$\frac{dz}{dx} = \frac{1}{z}$$

$$\Rightarrow z \, dz = dx \Rightarrow \frac{z^2}{2} = x + C$$

$$\Rightarrow \frac{1}{2} \ln^2(y + 3x) = x + C$$

$$\Rightarrow x - \frac{1}{2} (\ln(y + 3x))^2 = C$$

14. Official Ans. by NTA (2)

Sol. $\frac{(5 + e^x)}{2 + y} \frac{dy}{dx} = -e^x$

$$\int \frac{dy}{2 + y} = \int \frac{-e^x}{e^x + 5} \, dx$$

$$\ln(y + 2) = -\ln(e^x + 5) + k$$

$$(y + 2)(e^x + 5) = C$$

$$\because y(0) = 1$$

$$\Rightarrow C = 18$$

$$y + 2 = \frac{18}{e^x + 5}$$

$$\text{at } x = \ln 13$$

$$y + 2 = \frac{18}{13 + 5} = 1$$

$$\boxed{y = -1}$$

15. Official Ans. by NTA (1)

Sol. $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$

$$\frac{dy}{dx} + \frac{2 \sin x}{\cos x} y = 2 \sin x$$

$$\text{I.F.} = e^{\int 2 \frac{\sin x}{\cos x} dx}$$

$$= e^{2 \ln \sec x} = \sec^2 x$$

$$y \cdot \sec^2 x = \int 2 \sin x \cdot \sec^2 x \, dx$$

$$y \sec^2 x = 2 \int \tan x \sec x \, dx$$

$$y \sec^2 x = 2 \sec x + C$$

$$\text{At } x = \frac{\pi}{3}, y = 0$$

$$\Rightarrow 0 = 2 \sec \frac{\pi}{3} + C \Rightarrow C = -4$$

$$\boxed{y \sec^2 x = 2 \sec x - 4}$$

$$\text{Put } x = \frac{\pi}{4}$$

$$y \cdot 2 = 2\sqrt{2} - 4$$

$$y = \sqrt{2} - 2$$

16. Official Ans. by NTA (2)

Sol. $x^4 e^y + 2\sqrt{y+1} = 3$

d.w.r. to x

$$x^4 e^y y' + e^y 4x^3 + \frac{2y'}{2\sqrt{y+1}} = 0$$

at $P(1, 0)$

$$y'_P + 4 + y'_P = 0$$

$$\Rightarrow y'_P = -2$$

Tangent at $P(1, 0)$ is

$$y - 0 = -2(x - 1)$$

$$2x + y = 2$$

$(-2, 6)$ lies on it

17. Official Ans. by NTA (1)

Sol. $\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0$

$$\Rightarrow \sqrt{(1+x)^2(1+y^2)} + xy \frac{dy}{dx} = 0$$

$$\Rightarrow \sqrt{1+x^2} \sqrt{1+y^2} = -xy \frac{dy}{dx}$$

$$\Rightarrow \int \frac{y dy}{\sqrt{1+y^2}} = - \int \frac{\sqrt{1+x^2}}{x} dx \quad \dots(1)$$

Now put $1+x^2 = u^2$ and $1+y^2 = v^2$

$$2x dx = 2u du \text{ and } 2y dy = 2v dv$$

$$\Rightarrow x dx = u du \text{ and } y dy = v dv$$

substitute these values in equation (1)

$$\int \frac{v dv}{v} = - \int \frac{u^2 \cdot du}{u^2 - 1}$$

$$\Rightarrow \int dv = - \int \frac{u^2 - 1 + 1}{u^2 - 1} du$$

$$\Rightarrow v = - \int \left(1 + \frac{1}{u^2 - 1} \right) du$$

$$\Rightarrow v = -u - \frac{1}{2} \log_e \left| \frac{u-1}{u+1} \right| + c$$

$$\Rightarrow \sqrt{1+y^2} = -\sqrt{1+x^2} + \frac{1}{2} \log_e \left| \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right| + c$$

$$\Rightarrow \sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left| \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right| + c$$

18. Official Ans. by NTA (1)

Sol. $y = \left(\frac{2x}{\pi} - 1 \right) \operatorname{cosec} x$

...(1)

$$\frac{dy}{dx} = \frac{2}{\pi} \operatorname{cosec} x - \left(\frac{2x}{\pi} - 1 \right) \operatorname{cosec} x \cot x$$

$$\frac{dy}{dx} = \frac{2 \operatorname{cosec} x}{\pi} - y \cot x$$

using equation (1)

$$\frac{dy}{dx} + y \cot x = \frac{2 \operatorname{cosec} x}{\pi}$$

$$\frac{dy}{dx} + p(x) \cdot y = \frac{2 \operatorname{cosec} x}{\pi} \quad x \in \left(0, \frac{\pi}{2} \right)$$

Compare : $p(x) = \cot x$