SET

1. Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n \text{ is a multiple of } 2\}$ and $B = \{n \in X : n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is_____

2. Consider the two sets :

$A = \{m \in R : \text{both the roots of } x^2 - (m+1)x + m + 4 = 0 \text{ are real}\}$ and $B = [-3, 5)$.

Which of the following is not true ?

(1) $A - B = (-\infty, -3) \cup (5, \infty)$

(2) $A \cap B = \{-3\}$

(3) $B - A = (-3, 5)$

(4) $A \cup B = R$

3. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to _____.

4. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

(1) 65 (2) 37

(3) 29 (4) 55

5. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to :

(1) 45 (2) 15

(3) 50 (4) 30

6. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

(1) 63 (2) 38

(3) 54 (4) 36

7. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _.

SOLUTION

1. NTA Ans. (29.00)

Sol. $n(A) = 25$

$n(B) = 7$

$n(A \cap B) = 3$

$n(A \cup B) = 25 + 7 - 3 = 29$

2. Official Ans. by NTA (1)

Sol. $A : D \geq 0$

$\Rightarrow (m + 1)^2 - 4(m + 4) \geq 0$

$\Rightarrow m^2 + 2m + 1 - 4m - 16 \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$

$\Rightarrow (m - 5)(m + 3) \geq 0$

$\Rightarrow m \in (-\infty, -3] \cup [5, \infty)$

$\therefore A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

$A \cap B = \{-3\}$

$B - A = (-3, 5)$

$A \cup B = R$

3. Official Ans. by NTA (8)

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, \; y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k, \; y = \frac{k}{2}, \; z = 0$ put in condition

$$15 \leq k^2 + \left(\frac{k}{2}\right)^2 + 0 \leq 150$$

$$12 \leq k^2 \leq 120$$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

4. Official Ans. by NTA (4)

Sol. $n(B) \leq n(A \cup B) \leq n(U)$

$\Rightarrow 76 \leq 76 + 63 - x \leq 100$

$\Rightarrow -63 \leq -x \leq -39$

$\Rightarrow 63 \geq x \geq 39$

5. Official Ans. by NTA (4)

Sol. $n(X_i) = 10. \; \overset{50}{\underset{i=1}{\cup}} X_i = T, \Rightarrow n(T) = 500$

each element of T belongs to exactly 20 elements of $X_i \Rightarrow \frac{500}{20} = 25$ distinct elements

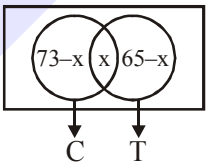
so $\frac{5n}{6} = 25 \Rightarrow n = 30$

6. Official Ans. by NTA (4)

Sol. $C \rightarrow$ person like coffee

$T \rightarrow$ person like Tea

$n(C) = 73$



$n(T) = 65$

$n(C \cup T) \leq 100$

$n(C) + n(T) - n(C \cap T) \leq 100$

$73 + 65 - x \leq 100$

$x \geq 38$

$73 - x \geq 0 \Rightarrow x \leq 73$

$65 - x \geq 0 \Rightarrow x \leq 65$

$\boxed{38 \leq x \leq 65}$

7. Official Ans. by NTA (28.00)

Sol. $2^m - 2^n = 112$

$m = 7, \; n = 4$

$(2^7 - 2^4 = 112)$

$m \times n = 7 \times 4 = 28$