PERMUTATION & COMBINATION

1. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is:

(1) $\frac{5}{2}(6!)$ (2) 5^6

(3) $\frac{1}{2}(6!)$ (4) $6!$

2. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _______.

3. If a,b and c are the greatest value of ${}^{19}C_p, {}^{20}C_q$ and ${}^{21}C_r$ respectively, then

(1) $\frac{a}{11}=\frac{b}{22}=\frac{c}{21}$ (2) $\frac{a}{10}=\frac{b}{11}=\frac{c}{21}$

(3) $\frac{a}{10}=\frac{b}{11}=\frac{c}{42}$ (4) $\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$

4. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is ....

5. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to :

(1) 8 (2) 6

(3) 4 (4) 7

6. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ______.

7. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :-

(1) 199 (2) 101

(3) 201 (4) 200

8. The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - \ldots$ up to 51^{th} term$) + (1! - 2! + 3! - \ldots$ up to 51^{th} term$)$ is equal to :

(1) $1 + (51)!$ (2) $1 - 51(51)!$

(3) $1 + (52)!$ (4) 1

9. The total number of 3-digit numbers, whose sum of digits is 10, is _______.

10. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ____

11. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ________.

12. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is :

(1) 1500 (2) 2255

(3) 3000 (4) 2250

13. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ___________.

SOLUTION

1. NTA Ans. (1)

Sol. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 is

$^5C_1 \times \frac{6!}{2!}$

2. NTA Ans. (2454)

Sol. N $\to$ 2, A $\to$ 2, I $\to$ 2, E, X, M, T, O $\to$ 1

Category	Selection	Arrangement
2 alike of one kind and 2 alike of other kind	$^3C_2 = 3$	$3 \times \frac{4!}{2!\,2!} = 18$
2 alike and 2 different	$^3C_1 \times {}^7C_2$	$^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$
All 4 different	$^8C_4$	$^8C_4 \times 4! = 1680$

Total = 2454

Ans. 2454.00

3. NTA Ans. (4)

Sol. $a = {}^{19}C_{10}$, $b = {}^{20}C_{10}$ and $c = {}^{21}C_{10}$

$\Rightarrow a = {}^{19}C_9$, $b = 2({}^{19}C_9)$ and $c = \frac{21}{11}\left({}^{20}C_{10}\right)$

$\Rightarrow b = 2a$ and $c = \frac{21}{11}b = \frac{42a}{11}$

$\Rightarrow a : b : c = a : 2a : \frac{42a}{11} = 11 : 22 : 42$

4. NTA Ans. (490.00)

ALLEN Ans. (490.00 OR 13.00)

Note: If same coloured marbles are identical then, answer is 13.00. However, NTA took them as distinct and kept only one answer as 490.00

Sol. The question does not mention that whether same coloured marbles are distinct or identical. So, assuming they are distinct our required answer = $^{12}C_4 - {}^5C_4 = 490$

And, if same coloured marbles are identical then required answer = (2 + 3 + 4 + 4) = 13

5. NTA Ans. (1)

Sol. _ _ _ $\underline{2}$ _

No. of five digits numbers =

No. of ways of filling remaining 4 places

= 8 × 8 × 7 × 6

$\therefore\ k = \frac{8 \times 8 \times 7 \times 6}{336} = 8$

6. Official Ans. by NTA (309.00)

Sol. MOTHER

1 $\to$ E

2 $\to$ H

3 $\to$ M

4 $\to$ O

5 $\to$ R

6 $\to$ T

So position of word MOTHER in dictionary

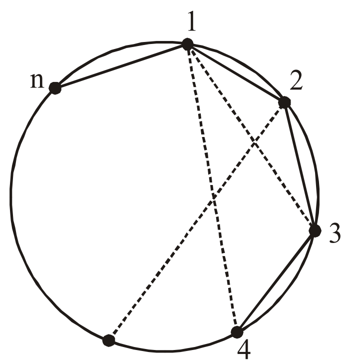
2 × 5! + 2 × 4! + 3 × 3! + 2! + 1

= 240 + 48 + 18 + 2 + 1

$= \boxed{309}$

7. Official Ans. by NTA (3)

Sol.



Number of blue lines = Number of sides = n

Number of red lines = number of diagonals

$= {}^nC_2 - n$

$^nC_2 - n = 99n \Rightarrow \frac{n(n-1)}{2} - n = 99n$

$\frac{n-1}{2} - 1 = 99 \Rightarrow n = 201$

8. Official Ans. by NTA (3)

Sol. $S = (2.{}^1p_0 - 3.{}^2p_1 + 4.{}^3p_2$ .... upto 51 terms) + (1! + 2! + 3! .... upto 51 terms) [$\because {}^np_{n-1} = n!$]

$\therefore$ S = (2 × 1! – 3 × 2! + 4 × 3! .... + 52.51!) + (1! – 2! + 3! .... (51)!)

= (2! – 3! + 4! .... + 52!) + (1! – 2! + 3! – 4! + ... + (51)!)

= 1! + 52!.

9. Official Ans. by NTA (54)

Sol. Let three digit number is xyz

$x + y + z = 10$; $x \geq 1, y \geq 0$ $z \geq 0$ ..... (1)

Let $T = x - 1 \Rightarrow x = T + 1$ where $T \geq 0$

Put in (1)

$T + y + z = 9$; $0 \leq T \leq 8, 0 \leq y, z \leq 9$

No. of non negative integral solution

$= {}^{9+3-1}C_{3-1} - 1$ (when T = 9)

= 55 – 1 = 54

10. Official Ans. by NTA (135)

Sol. Ways $= {}^6C_4 \cdot 1^4 \cdot 3^2$

= 15 × 9

= 135

11. Official Ans. by NTA (240)

Sol. S_2YL_2ABU

ABCC type words

$$= \underbrace{{}^2C_1}_{\substack{\text{selection of}\\ \text{two alike}\\ \text{letters}}} \times \underbrace{{}^5C_2}_{\substack{\text{selection of}\\ \text{of two}\\ \text{distinct letters}}} \times \underbrace{\frac{\lfloor 4}{\lfloor 2}}_{\substack{\text{arrangement of}\\ \text{selected letters}}}$$

= 240

12. Official Ans. by NTA (4)

Sol.

A	B	C
$\boxed{5}$	$\boxed{5}$	$\boxed{5}$
1	2	2
2	1	2
2	2	1
1	1	3
1	3	1
3	1	1

Total number of selection

$= ({}^5C_1\ {}^5C_2\ {}^5C_2)\cdot 3 + ({}^5C_1\ {}^5C_1\ {}^5C_3)\cdot 3$

$= 5 \cdot 10 \cdot 10 \cdot 3 + 5 \cdot 5 \cdot 10 \cdot 3$

= 2250

13. Official Ans. by NTA (120.00)

Sol. LETTER

vowels = EE, consonant = LTTR

_ L _ T _ T _ R _

$$\frac{4!}{2!} \times {}^5C_2 \times \frac{2!}{2!} = 12 \times 10 = 120$$