MAXIMA & MINIMA

1. Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$, then which one of the following is not true?

(1) f is an odd function

(2) x = 1 is a point of minima and x = –1 is a point of maxima of f.

(3) x = 1 is a point of maxima and x = –1 is a point of minimum of f.

(4) f(1) – 4f(–1) = 4

2. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at x = –1 and $f'(x)$ has a critical point at x = 1. Then $f(x)$ has a local minima at x = ________.

3. Let a function $f : [0, 5] \to \mathbf{R}$ be continuous, $f(1) = 3$ and F be defined as : $F(x)=\int_1^x t^2 g(t)\,dt$, where $g(t)=\int_1^t f(u)\,du$.

Then for the function F, the point x = 1 is :

(1) a point of local minima.

(2) not a critical point.

(3) a point of inflection.

(4) a point of local maxima.

4. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is :

(1) $\frac{1}{36\pi}$ (2) $\frac{5}{6\pi}$

(3) $\frac{1}{18\pi}$ (4) $\frac{1}{54\pi}$

5. Let P(h, k) be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, y = 3x – 3. Then the equation of the normal to the curve at P is :

(1) x + 3y – 62 = 0 (2) x – 3y – 11 = 0

(3) x – 3y + 22 = 0 (4) x + 3y + 26 = 0

6. If p(x) be a polynomial of degree three that has a local maximum value 8 at x = 1 and a local minimum value 4 at x = 2; then p(0) is equal to:

(1) 12 (2) –24

(3) 6 (4) –12

7. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is :

(1) 6 (2) 8

(3) 4 (4) 2

8. If x = 1 is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)\,e^x$, then :

(1) x = 1 is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

(2) x = 1 is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.

(3) x = 1 and $x = -\frac{2}{3}$ are local minima of f.

(4) x = 1 and $x = -\frac{2}{3}$ are local maxima of f.

9. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is_.

10. The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x).(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is :

(1) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

SOLUTION

1. NTA Ans. (2)

Sol. $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$

$\Rightarrow f(x) = 2x^3 + ax^4 + bx^5$

$f'(x) = 6x^2 + 4ax^3 + 5bx^4$

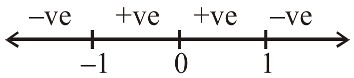
$f'(1) = 0,\ f'(-1) = 0$

$a = 0,\ b = \frac{-6}{5} \Rightarrow f(x) = 2x^3 - \frac{6}{5}x^5$

$f'(x) = 6x^2 - 6x^4$

$= 6x^2(1 - x)(1 + x)$

Sign scheme for f'(x)



Minima at $x = -1$

Maxima at $x = 1$

2. NTA Ans. (3)

Sol. $f''(x) = \lambda(x - 1)$

$f'(x)=\frac{\lambda x^2}{2}-\lambda x+C \Rightarrow f'(-1)=0 \Rightarrow c=\frac{-3\lambda}{2}$

$f(x)=\frac{\lambda x^3}{6}-\frac{\lambda x^2}{2}-\frac{3\lambda}{2}x+d$

$f(1) = -6 \Rightarrow -11\lambda + 6d = -36$ ...(i)

$f(-1) = 10 \Rightarrow 5\lambda + 6d = 60$ ...(ii)

from (i) & (ii) $\lambda = 6$ & $d = 5$

$f(x) = x^3 - 3x^2 - 9x + 5$

Which has minima at $x = 3$

Ans. 3.00

3. NTA Ans. (1)

Sol. $F'(x) = x^2 g(x) = x^2\int_1^x f(u)\,du \Rightarrow F'(1) = 0$

$F''(x) = x^2 f(x) - 2x\int_1^x f(u)\,du$

$F''(1) = 1.f(1) - 2 \times 0$

$F''(1) = 3$

$F'(1) = 0$ and $F''(1) = 3 > 0$ So, Minima

4. NTA Ans. (3)

Sol. Let thickness of ice be 'h'.

Vol. of ice $= v=\frac{4\pi}{3}\left((10+h)^3-10^3\right)$

$\frac{dv}{dt}=\frac{4\pi}{3}\left(3(10+h)^2\right).\frac{dh}{dt}$

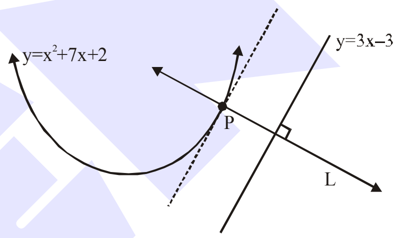
Given $\frac{dv}{dt}=50\text{cm}^3/\text{min}$ and $h = 5\text{cm}$

$\Rightarrow 50=\frac{4\pi}{3}\left(3(10+5)^2\right)\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt}=\frac{50}{4\pi\times 15^2}=\frac{1}{18\pi}\text{cm}/\text{min}$

5. Official Ans. by NTA (4)

Sol.



Let L be the common normal to parabola $y = x^2 + 7x + 2$ and line $y = 3x - 3$

$\Rightarrow$ slope of tangent of $y = x^2 + 7x + 2$ at P = 3

$\Rightarrow \left.\frac{dy}{dx}\right]_{\text{For P}}=3$

$\Rightarrow 2x + 7 = 3 \Rightarrow x = -2 \Rightarrow y = -8$

So P(–2, –8)

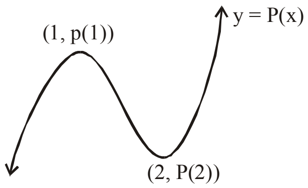
Normal at P : $x + 3y + C = 0$

$\Rightarrow C = 26$ (P satisfies the line)

$\boxed{\text{Normal}: x+3y+26=0}$

6. Official Ans. by NTA (4)

Sol. Since p(x) has realtive extreme at



x = 1 & 2

so p'(x) = 0 at x = 1 & 2

$\Rightarrow$ p'(x) = A(x – 1) (x – 2)

$$\Rightarrow p(x) = \int A(x^2 - 3x + 2)dx$$

$$p(x) = A\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + C \quad \ldots(1)$$

P(1) = 8

From (1)

$$8 = A\left(\frac{1}{3} - \frac{3}{2} + 2\right) + C$$

$$\Rightarrow 8 = \frac{5A}{6} + C \Rightarrow \boxed{48 = 5A + 6C} \quad \ldots(3)$$

P(2) = 4

$$\Rightarrow 4 = A\left(\frac{8}{3} - 6 + 4\right) + C$$

$$\Rightarrow 4 = \frac{2A}{3} + C \Rightarrow \boxed{12 = 2A + 3C} \quad \ldots(4)$$

From 3 & 4, C = –12

So P(0) = C = $\boxed{-12}$

7. Official Ans. by NTA (3)

Sol. f'(x) = x(x + 1) (x – 1) = x^3 – x

$$\int df(x) = \int x^3 - x\, dx$$

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$$

f(x) = f(0)

$$\frac{x^4}{4} - \frac{x^2}{2} = 0$$

x^2 (x^2 – 2) = 0

x = 0, 0, $\sqrt{2}$, $-\sqrt{2}$

$x_1^2 + x_2^2 + x_3^2 = 0 + 2 + 2 = 4$

8. Official Ans. by NTA (1)

Sol. f(x) = (3x² + ax – 2 – a)e^x

f'(x) = ($3x^2$ + ax – 2 – a)e^x + e^x (6x + a)

= e^x($3x^2$ + x(6 + a) – 2)

f'(x) = 0 at x = 1

$\Rightarrow$ 3 + (6 + a) – 2 = 0

a = –7

f'(x) = e^x($3x^2$ – x – 2)

= e^x (x – 1) (3x + 2)

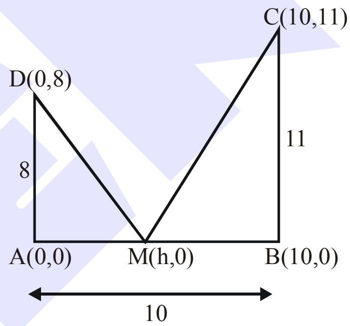
+ – +
–2/3 1

x = 1 is point of local minima

$x = \frac{-2}{3}$ is point of local maxima

9. Official Ans. by NTA (5.00)

Sol.



$(MD)^2 + (MC)^2 = h^2 + 64 + (h - 10)^2 + 121$

$= 2h^2 - 20h + 64 + 100 + 121$

$= 2(h^2 - 10h) + 285$

$= 2(h - 5)^2 + 235$

it is minimum if h = 5

10. Official Ans. by NTA (4)

Sol. $f(x) = (1 - \cos^2 x)(\lambda + \sin x) \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \lambda\sin^2 x + \sin^3 x$

$f'(x) = 2\lambda\sin x\cos x + 3\sin^2 x\cos x$

$f'(x) = \sin x\cos x(2\lambda + 3\sin x)$

$$\sin x = 0, \frac{-2\lambda}{3}, (\lambda \neq 0)$$

for exactly one maxima & minima

$$\frac{-2\lambda}{3} \in (-1, 1) \Rightarrow \lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$$

$$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$$