LIMIT

1. $\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to ____________.

2. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

(1) $\frac{1}{e}$ (2) e^2

(3) e (4) $\frac{1}{e^2}$

3. If $\lim_{x\to 1}\frac{x+x^2+x^3+...+x^n-n}{x-1}=820,(n\in N)$ then the value of n is equal to_______.

4. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to :

(1) 2 (2) e

(3) 1 (4) e^2

5. Let [t] denote the greatest integer $\le$ t. If for some $\lambda\in R-\{0,1\}$, $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right|=L$, then L is equal to :

(1) 1 (2) 2

(3) $\frac{1}{2}$ (4) 0

6. If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is _______ .

7. $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}(a\ne 0)$ is equal to :

(1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

8. Let f : (0, ∞) → (0, ∞) be a differentiable function such that f(1) = e and $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$. If f(x) = 1, then x is equal to :

(1) 2e (2) $\frac{1}{2e}$

(3) e (4) $\frac{1}{e}$

9. If α is the positive root of the equation, $p(x)=x^2-x-2=0$, then $\lim_{x\to\alpha^+}\frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{3}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

10. $\lim_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

(1) does not exist. (2) is equal to $\sqrt{e}$.

(3) is equal to 0. (4) is equal to 1.

11. $\lim_{x\to 1}\left(\frac{\int_0^{(x-1)^2}t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$

(1) does not exist (2) is equal to $\frac{1}{2}$

(3) is equal to 1 (4) is equal to $-\frac{1}{2}$

SOLUTION

1. NTA Ans. (36)

Sol. $\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}} \Rightarrow \lim_{x\to 2}\frac{3^{2x}-12.3^x+27}{3^{x/2}-3}$

$= \lim_{x\to 2}\frac{(3^x-9)(3^x-3)}{(3^{x/2}-3)}$

$= \lim_{x\to 2}\frac{(3^{x/2}+3)(3^{x/2}-3)(3^x-3)}{(3^{x/2}-3)}$

$= 36$

2. NTA Ans. (4)

Sol. Required limit $= e^{\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}-1\right)\frac{1}{x^2}}$

$= e^{\lim_{x\to 0}\left(\frac{-4}{7x^2+2}\right)} = \frac{1}{e^2}$

3. Official Ans. by NTA (40.00)

Sol. $\lim_{x\to 1}\frac{x+x^2+.......+x^2-n}{x-1} = 820$

$\Rightarrow \lim_{x\to 1}\left(\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+......\frac{x^n-1}{x-1}\right) = 820$

$\Rightarrow 1 + 2 + ... + n = 820$

$\Rightarrow n(n + 1) = 2 \times 820$

$\Rightarrow n(n + 1) = 40 \times 41$

Since $n \in N$, so $\boxed{n = 40}$

4. Official Ans. by NTA (4)

Sol. $\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$

$= e^{\lim_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right)-1\right\}}$

$= e^{\lim_{x\to 0}\left(\frac{1+\tan x-1+\tan x}{x(1-\tan x)}\right)}$

$= e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$

$= e^2$

5. Official Ans. by NTA (2)

Sol. LHL : $\lim_{x\to 0^-}\left|\frac{1-x-x}{\lambda-x-1}\right| = \left|\frac{1}{\lambda-1}\right|$

RHL : $\lim_{x\to 0^+}\left|\frac{1-x+x}{\lambda-x+1}\right| = \left|\frac{1}{\lambda}\right|$

For existence of limit

LHL = RHL

$\Rightarrow \frac{1}{|\lambda-1|} = \frac{1}{|\lambda|} \Rightarrow \lambda = \frac{1}{2}$

$\therefore\ L = \frac{1}{|\lambda|} = 2$

6. Official Ans. by NTA (8)

Sol. $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$

$\Rightarrow \lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{4\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{16\left(\frac{x^2}{4}\right)^2} = \frac{1}{8}\times\frac{1}{32} = 2^{-k}$

$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8.$

7. Official Ans. by NTA (1)

Sol. Required limit

$$L = \lim_{h\to 0}\frac{(a+2(a+h))^{1/3}-(3(a+h))^{1/3}}{(3a+a+h)^{1/3}-(4(a+h))^{1/3}}$$

$$=\lim_{h\to 0}\frac{(3a)^{1/3}\left(1+\frac{2h}{3a}\right)^{1/3}-(3a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}{(4a)^{1/3}\left(1+\frac{h}{4a}\right)^{1/3}-(4a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}$$

$$=\lim_{h\to 0}\left(\frac{3^{1/3}}{4^{1/3}}\right)\left[\frac{\left(1+\frac{2h}{9a}\right)-\left(1+\frac{h}{3a}\right)}{\left(1+\frac{h}{12a}\right)-\left(1+\frac{h}{3a}\right)}\right]$$

$$=\left(\frac{3}{4}\right)^{1/3}\frac{\left(\frac{2}{9}-\frac{1}{3}\right)}{\left(\frac{1}{12}-\frac{1}{3}\right)}=\left(\frac{3}{4}\right)^{1/3}\left(\frac{8-12}{3-12}\right)$$

$$=\left(\frac{3}{4}\right)^{1/3}\left(\frac{-4}{-9}\right)=\frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}}=\frac{4^{2/3}}{3^{5/3}}$$

$$=\frac{(8\times 2)^{1/3}}{(27\times 9)^{1/3}}=\frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

8. Official Ans. by NTA (4)

Sol. $L = \lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}$

using L.H. rule

$L = \lim_{t\to x}\frac{2tf^2(x)-x^2\cdot 2f'(t).f(t)}{1}$

$\Rightarrow$ L = 2xf(x) (f(x) – x f'(x)) = 0 (given)

$\Rightarrow$ f(x) = xf '(x) $\Rightarrow \int\frac{f'(x)dx}{f(x)}=\int\frac{dx}{x}$

$\Rightarrow \ell n\,|f(x)| = \ell n\,|x| + C$

$\because$ f(1) = e, x > 0, f(x) > 0

$\Rightarrow$ f(x) = ex, if f(x) = 1 $\Rightarrow x = \frac{1}{e}$

9. Official Ans. by NTA (1)

Sol. $x^2 - x - 2 = 0$

roots are 2 & –1

$$\Rightarrow \lim_{x\to 2^+}\frac{\sqrt{1-\cos(x^2-x-2)}}{(x-2)}$$

$$=\lim_{x\to 2^+}\frac{\sqrt{2\sin^2\frac{(x^2-x-2)}{2}}}{(x-2)}$$

$$=\lim_{x\to 2^+}\frac{\sqrt{2}\sin\left(\frac{(x-2)(x+1)}{2}\right)}{(x-2)}=\frac{3}{\sqrt{2}}$$

10. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$$

$$\because \lim_{x\to 0}\frac{\sqrt{1+x^2+x^4}-1}{x}\ \left(\frac{0}{0}\text{ from}\right)$$

$$\lim_{x\to 0}\frac{(1+x^2+x^4)-1}{x(\sqrt{1+x^2+x^4}+1}$$

$$\lim_{x\to 0}\frac{x(1+x^2)}{\left(\sqrt{1+x^2+x^4}+1\right)}=0$$

$$\text{So }\lim_{x\to 0}\frac{x\left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)}-1\right)}{\sqrt{1+x^2+x^4}-1}\ \left(\frac{0}{0}\text{ from}\right)$$

$$\lim_{x\to 0}\frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)}=1$$

11. Official Ans. by NTA (1)
Official Ans. by ALLEN
(Bonus-Answers musbe zero)

Sol. $$\lim_{x\to 1}\frac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)}\left(\frac{0}{0}\right)$$

Apply L Hopital Rule

$$=\lim_{x\to 1}\frac{2(x-1).(x-1)^2\cos(x-1)^4-0}{(x-1).\cos(x-1)+\sin(x-1)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1}\frac{2(x-1)^3.\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\frac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\frac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\frac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$=\frac{0}{1+1}=0$$