

## INVERSE TRIGONOMETRY FUNCTION

**1.** The domain of the function

$$f(x) = \sin^{-1} \left( \frac{|x|+5}{x^2+1} \right)$$

is  $(-\infty, -a] \cup [a, \infty)$ . Then

a is equal to :

(1)  $\frac{1+\sqrt{17}}{2}$

(2)  $\frac{\sqrt{17}-1}{2}$

(3)  $\frac{\sqrt{17}}{2} + 1$

(4)  $\frac{\sqrt{17}}{2}$

**2.**  $2\pi - \left( \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} \right)$  is equal to:

(1)  $\frac{7\pi}{4}$

(2)  $\frac{5\pi}{4}$

(3)  $\frac{3\pi}{2}$

(4)  $\frac{\pi}{2}$

**3.** If S is the sum of the first 10 terms of the series

$$\tan^{-1} \left( \frac{1}{3} \right) + \tan^{-1} \left( \frac{1}{7} \right) + \tan^{-1} \left( \frac{1}{13} \right) + \tan^{-1} \left( \frac{1}{21} \right) + \dots,$$

then  $\tan(S)$  is equal to :

(1)  $\frac{5}{11}$

(2)  $-\frac{6}{5}$

(3)  $\frac{10}{11}$

(4)  $\frac{5}{6}$

**SOLUTION****1. Official Ans. by NTA (1)**

$$\text{Sol. } f(x) = \sin\left(\frac{|x|+5}{x^2+1}\right)$$

For domain :

$$-1 \leq \frac{|x|+5}{x^2+1} \leq 1$$

Since  $|x| + 5$  &  $x^2 + 1$  is always positive

$$\text{So } \frac{|x|+5}{x^2+1} \geq 0 \quad \forall x \in \mathbb{R}$$

So for domain :

$$\frac{|x|+5}{x^2+1} \leq 1$$

$$\Rightarrow |x| + 5 \leq x^2 + 1$$

$$\Rightarrow 0 \leq x^2 - |x| - 4$$

$$\Rightarrow 0 \leq \left(|x| - \frac{1+\sqrt{17}}{2}\right) \left(|x| - \frac{1-\sqrt{17}}{2}\right)$$

$$\Rightarrow |x| \geq \frac{1+\sqrt{17}}{2} \text{ or } |x| \leq \frac{1-\sqrt{17}}{2} \quad (\text{Rejected})$$

$$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$$

$$\text{So, } a = \frac{1+\sqrt{17}}{2}$$

**2. Official Ans. by NTA (3)**

$$\text{Sol. } 2\pi - \left( \sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) \right)$$

$$= 2\pi - \left( \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{16}{63}\right) \right)$$

$$= 2\pi - \left( \tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{16}{63}\right) \right)$$

$$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

**3. Official Ans. by NTA (4)**

$$\text{Sol. } S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \dots$$

$$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2 \times 3}\right) + \tan^{-1}$$

$$\left(\frac{4-3}{1+3 \times 4}\right) + \dots + \tan^{-1}\left(\frac{11-10}{1+10 \times 11}\right)$$

$$S = (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + (\tan^{-1} 4 - \tan^{-1} 3) + \dots + (\tan^{-1} 11 - \tan^{-1} 10))$$

$$S = \tan^{-1} 11 - \tan^{-1} 1 = \tan^{-1}\left(\frac{11-1}{1+11}\right)$$

$$\tan(S) = \frac{11-1}{1+11 \times 1} = \frac{10}{12} = \frac{5}{6}$$