INDEFINITE INTEGRATION

1. If $\int \frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = f(x)\left(1+\sin^6 x\right)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

(1) –2 (2) $-\frac{9}{8}$

(3) 2 (4) $\frac{9}{8}$

2. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e|f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to :

(1) $(-1, 1 + \tan\theta)$ (2) $(-1, 1 - \tan\theta)$

(3) $(1, 1 - \tan\theta)$ (4) $(1, 1 + \tan\theta)$

3. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to :

(where C is a constant of integration)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$ (2) $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}} + C$

(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$ (4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-\frac{13}{7}} + C$

4. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C$,

where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(1) $\left(x-1, \sqrt{x}\right)$ (2) $\left(x+1, \sqrt{x}\right)$

(3) $\left(x+1, -\sqrt{x}\right)$ (4) $\left(x-1, -\sqrt{x}\right)$

5. The integral $\int\left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to:

(where C is a constant of integration)

(1) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

(2) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(3) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

(4) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

6. If

$\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})}dx = g(x)e^{(e^x + e^{-x})} + c$,

where c is a constant of integration, then g(0) is equal to :

(1) 2 (2) e^2

(3) e (4) 1

7. If $\int \frac{\cos\theta}{5 + 7\sin\theta - 2\cos^2\theta}\,d\theta = A\log_e|B(\theta)| + C$,

where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

(1) $\frac{2\sin\theta + 1}{5(\sin\theta + 3)}$ (2) $\frac{2\sin\theta + 1}{\sin\theta + 3}$

(3) $\frac{5(\sin\theta + 3)}{2\sin\theta + 1}$ (4) $\frac{5(2\sin\theta + 1)}{\sin\theta + 3}$

SOLUTION

1. NTA Ans. (1)

Sol. $\int \frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = \frac{-6}{-6}\int \frac{\cos x\,dx}{\sin^7 x\left(\frac{1}{\sin^6 x}+1\right)^{2/3}}$

$= -\frac{1}{6}\times 3\left(\frac{1}{\sin^6 x}+1\right)^{\frac{1}{3}}+c$

$= -\frac{1}{2}\frac{\left(1+\sin^6 x\right)^{\frac{1}{3}}}{\sin^2 x}+c$

Hence, $\lambda = 3$ and $f(x) = -\frac{1}{2\sin^2 x}$

so, $\lambda f\left(\frac{\pi}{3}\right) = -2$

REMARK : Technically, this question should be marked as bonus. Because $f(x)$ and λ cannot be found uniquely.

For example, another such $f(x)$ and λ can be

$-\frac{\left(1+\sin^6 x\right)^{\frac{1}{6}}}{2\sin^2 x}$ and 6 respectively.

2. NTA Ans. (1)

Sol. $I = \int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)}$

$= \int \frac{\sec^2\theta\,d\theta}{\frac{2\tan\theta}{1-\tan^2\theta}+\frac{1+\tan^2\theta}{1-\tan^2\theta}} = \int \frac{\left(1-\tan^2\theta\right)\sec^2\theta\,d\theta}{(1+\tan\theta)^2}$

$\tan\theta = t \Rightarrow \sec^2\theta\,d\theta = dt$

$I = \int \frac{1-t^2}{(1+t)^2}dt = \int \frac{(1-t)(1+t)}{(1+t)^2}dt$

$= \int \frac{1}{1+t} - \frac{t}{1+t}\,dt$

$= \ell n|1+t| - \int\left(\frac{1+t}{1+t}-\frac{1}{1+t}\right)dt$

$= \ell n|1+t| - t + \ell n|1+t| = 2\ell n|1+t| - t + C$

$= 2\ell n|1+\tan\theta| - \tan\theta + C$

$\lambda = -1,\ f(\theta) = 1+\tan\theta$

3. NTA Ans. (1)

Sol. $I = \int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}} = \int \frac{dx}{\left(\frac{x+4}{x-3}\right)^{\frac{8}{7}}(x-3)^2}$

Let $\frac{x+4}{x-3} = t \Rightarrow \frac{dx}{(x-3)^2} = -\frac{1}{7}dt$

$\Rightarrow\ I = -\frac{1}{7}\int\frac{dt}{t^{8/7}} = -\frac{1}{7}\int t^{-8/7}dt$

$= t^{-1/7}+C = +\left(\frac{x+4}{x-3}\right)^{-1/7}+C = \left(\frac{x-3}{x+4}\right)^{1/7}+C$

4. Official Ans. by NTA (3)

Sol. Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\,d\theta$

$\int \theta.(2\tan\theta\cdot\sec^2\theta)\,d\theta$

$\downarrow\quad\downarrow$

I II (By parts)

$= \theta.\tan^2\theta - \int \tan^2\theta\,d\theta$

$= \theta.\tan^2\theta - \int(\sec^2\theta - 1)\,d\theta$

$= \theta(1+\tan^2\theta) - \tan\theta + C$

$= \tan^{-1}\left(\sqrt{x}\right)(1+x) - \sqrt{x} + C$

5. Official Ans. by NTA (4)

Sol. $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx=\int\left(\frac{x}{\cos x}\right).\frac{x\cos x\,dx}{(x\sin x+\cos x)^2}$

$$=\frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)$$

$$+\int\left(\frac{\cos x+x\sin x}{\cos^2 x}\right)\left(\frac{1}{x\sin x+\cos x}\right)dx$$

$$=-\frac{x\sec x}{x\sin x+\cos x}+\int\sec^2 x\,dx$$

$$=-\frac{x\sec x}{x\sin x+\cos x}+\tan x+C$$

6. Official Ans. by NTA (1)

Sol. $e^{2x}+2e^x-e^{-x}-1$

$=e^x(e^x+1)-e^{-x}(e^x+1)+e^x=[(e^x+1)(e^x-e^{-x})+e^x]$

$$\text{so } I=\int(e^x+1)(e^x-e^{-x})e^{e^x+e^{-x}}+\int e^x\cdot e^{e^x+e^{-x}}dx$$

$$=(e^x+1)e^{e^x+e^{-x}}-\int e^x.e^{e^x+e^{-x}}dx+\int e^x.e^{e^x+e^{-x}}dx$$

$$=(e^x+1)e^{e^x+e^{-x}}+C \qquad \therefore g(x)=e^x+1$$

$\Rightarrow g(0)=2$

7. Official Ans. by NTA (4)

Sol. $\int\frac{\cos\theta\,d\theta}{5+7\sin\theta-2\cos^2\theta}$

$$\int\frac{\cos\theta\,d\theta}{3+7\sin\theta+2\sin^2\theta} \qquad \boxed{\begin{array}{l}\sin\theta=t\\ \cos\theta d\theta=dt\end{array}}$$

$$\int\frac{dt}{2t^2+7t+3}=\int\frac{dt}{(2t+1)(t+3)}$$

$$=\frac{1}{5}\int\left(\frac{2}{2t+1}-\frac{1}{t+3}\right)dt$$

$$=\frac{1}{5}ln\left|\frac{2t+1}{t+3}\right|+C=\frac{1}{5}ln\left|\frac{2\sin\theta+1}{\sin\theta+3}\right|+C$$

$$A=\frac{1}{5}\text{ and }B(\theta)=\frac{2\sin\theta+1}{\sin\theta+3}$$