DETERMINANT

1. If the system of linear equations,

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$3x + 2y + \lambda z = \mu$$

has more two solutions, then $\mu - \lambda^2$ is equal to

____

2. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0,$

where a, b, c $\in$ R are non-zero and distinct; has a non-zero solution, then :

(1) a, b, c are in A.P.

(2) a + b + c = 0

(3) a, b, c are in G.P.

(4) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

3. The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has

(1) infinitely many solutions when $\lambda = 2$

(2) a unique solution when $\lambda = -8$

(3) no solution when $\lambda = 8$

(4) no solution when $\lambda = 2$

4. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent ?

(1) (1,0) (2) (4,6)

(3) (3,4) (4) (4,3)

5. Let a – 2b + c = 1. If

$$f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}, \text{ then :}$$

(1) $f(-50) = 501$ (2) $f(-50) = -1$

(3) $f(50) = 1$ (4) $f(50) = -501$

6. The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has

(1) infinitely many solutions, (x, y, z) satisfying $x = 2z$

(2) no solution

(3) only the trivial solution

(4) infinitely many solutions, (x, y, z) satisfying $y = 2z$

7. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S

(1) contains more than two elements.

(2) is a singleton.

(3) contains exactly two elements.

(4) is an empty set.

8. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to _______.

9. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$, then B + C is equal to :

(1) –1 (2) 1

(3) –3 (4) 9

10. If the system of equations

$x - 2y + 3z = 9$

$2x + y + z = b$

$x - 7y + az = 24,$

has infinitely many solutions, then a – b is equal to ________ .

11. If the system of equations

$x + y + z = 2$

$2x + 4y - z = 6$

$3x + 2y + \lambda z = \mu$

has infinitely many solutions, then :

(1) $\lambda - 2\mu = -5$ (2) $2\lambda - \mu = 5$

(3) $2\lambda + \mu = 14$ (4) $\lambda + 2\mu = 14$

12. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to R$, defined by :

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$ are m and M

respectively, then the ordered pair (m, M) is equal to:

(1) (0, 4) (2) (–4, 4)

(3) $(0, 2\sqrt{2})$ (4) (–4, 0)

13. Let $\lambda \in R$. The system of linear equations

$2x_1 - 4x_2 + \lambda x_3 = 1$

$x_1 - 6x_2 + x_3 = 2$

$\lambda x_1 - 10x_2 + 4x_3 = 3$

is inconsistent for :

(1) exactly one negative value of λ.

(2) exactly one positive value of λ.

(3) every value of λ.

(4) exactly two values of λ.

14. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to :

(1) 9 (2) –3

(3) –9 (4) 3

15. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$$ is equal to :

(1) 0 (2) y(a – b)

(3) y (b – a) (4) y(a – c)

16. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively

(1) 5 and 7 (2) 6 and 8

(3) 4 and 9 (4) 5 and 8

17. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}.$$ Then the ordered pair (m,M) is equal to

(1) (–3,–1) (2) (–4,–1)

(3) (1,3) (4) (–3,3)

18. The sum of distinct values of λ for which the system of equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$

has non-zero solutions, is ____________.

SOLUTION

1. NTA Ans. (13.00)

Sol. System has intfinitely many solution

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 1$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 1 \end{vmatrix} = 0$$

$\mu = 14$

$\mu - \lambda^2 = 13$

2. NTA Ans. (4)

Sol. For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0, \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow (3b - 2a)(c - a) - (b - a)(4c - 2a) = 0$

$\Rightarrow 2ac = bc + ab$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ Hence $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

3. NTA Ans. (4)

Sol. $D = \begin{vmatrix} \lambda & 3 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = (\lambda + 8)(2 - \lambda)$

for $\lambda = 2$; $D_1 \neq 0$

Hence, no solution for $\lambda = 2$

(4) Option

4. NTA Ans. (4)

Sol. $2 \times (ii) - 2 \times (i) - (iii)$: -

$0 = 2\mu - 2 - \delta$

$\Rightarrow \delta = 2(\mu - 1)$

5. NTA Ans. (3)

Sol. $R_1 \to R_1 + R_3 - 2R_2$

$$f(x) = \begin{vmatrix} a+c-2b & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$= (a + c - 2b)((x + 3)^2 - (x + 2)(x + 4))$

$= x^2 + 6x + 9 - x^2 - 6x - 8 = 1$

$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$

6. NTA Ans. (1)

Sol. $7x + 6y - 2z = 0$ .... (1)

$3x + 4y + 2z = 0$ .... (2)

$x - 2y - 6z = 0$ .... (3)

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0 \Rightarrow \text{infinite solutions}$$

Now (1) + (2) $\Rightarrow y = -x$ put in (1), (2) & (3) all will lead to $x = 2z$

7. Official Ans. by NTA (3)

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution :

$$D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$

$\Rightarrow -2\lambda^2 + \lambda + 1 = 0$

$\Rightarrow \lambda = 1, -\frac{1}{2}$

$$D_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$$

$= 2(1 + \lambda)$

whichis not equal to zero for

$\lambda = 1, -\frac{1}{2}$

8. Official Ans. by NTA (8)

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let x = k

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, \ y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k$, $y = \frac{k}{2}$, $z = 0$ put in condition

$15 \le k^2 + \left(\frac{k}{2}\right)^2 + 0 \le 150$

$12 \le k^2 \le 120$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

9. Official Ans. by NTA (3)

Sol. $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D.$

$R_2 \to R_2 - R_1$ $\quad\quad$ $R_3 \to R_3 - R_2$

$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$

$= (x-1)(x-2) \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$

$= -3(x-1)^2(x-2) = -3x^3 + 12x^2 - 15x + 6$

$\therefore$ B + C = 12 – 15 = –3

10. Official Ans. by NTA (5)

Sol. $D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0 \Rightarrow a = 8$

also, $D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0 \Rightarrow b = 3$

hence, a – b = 8 – 3 = 5

11. Official Ans. by NTA (3)

Sol. For infinite solutions

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

Now $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = \frac{9}{2}$

$\Delta_{x=0} \Rightarrow \begin{vmatrix} 2 & 1 & 1 \\ 6 & 4 & -1 \\ \mu & 2 & -\frac{9}{2} \end{vmatrix} = 0$

$\Rightarrow \mu = 5$

For $\lambda = \frac{9}{2}$ & $\mu = 5$, $\Delta_y = \Delta_z = 0$

Now check option $2\lambda + \mu = 14$

12. Official Ans. by NTA (4)

Sol. $C_3 \to C_3 - (C_1 - C_2)$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 0 \\ -\cos^2\theta & -1-\cos^2\theta & 0 \\ 12 & 10 & -4 \end{vmatrix}$$

$= -4[(1 + \cos^2\theta) \sin^2\theta - \cos^2\theta (1 + \sin^2\theta)]$

$= -4[\sin^2\theta + \sin^2\theta \cos^2\theta - \cos^2\theta - \cos^2\theta \sin^2\theta]$

$f(\theta) = 4 \cos 2\theta$

$\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$2\theta \in \left[\frac{\pi}{2}, \pi\right]$

$f(\theta) \in [-4, 0]$

$(m, M) = (-4, 0)$

13. Official Ans. by NTA (1)

Sol. $D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix}$

$= 2(3\lambda + 2)(\lambda - 3)$

$D_1 = -2(\lambda - 3)$

$D_2 = -2(\lambda + 1)(\lambda - 3)$

$D_3 = -2(\lambda - 3)$

When $\boxed{\lambda = 3}$, then

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ Infinite many solution

when $\boxed{\lambda = -\frac{2}{3}}$ then D_1, D_2, D_3 none of them

is zero so equations are inconsistant

$\boxed{\therefore \lambda = -\frac{2}{3}}$

14. Official Ans. by NTA (2)

Sol. $x + y + 3z = 0$ .....(i)

$x + 3y + k^2z = 0$ .....(ii)

$3x + y + 3z = 0$ .....(iii)

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 9 + 3 + 3k^2 - 27 - k^2 - 3 = 0$

$\Rightarrow k^2 = 9$

(i) – (iii) $\Rightarrow -2x = 0 \Rightarrow x = 0$

Now from (i) $\Rightarrow y + 3z = 0$

$\Rightarrow \frac{y}{z} = -3$

$x + \frac{y}{z} = -3$

15. Official Ans. by NTA (2)

Sol. $a + x = b + y = c + z + 1$

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \quad C_3 \to C_3 - C_1$$

$$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \quad R_3 \to R_3 - R_1,\ R_2 \to R_2 - R_1$$

$$\begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$= (-y)[(y - x)(c - a) - (b - a)(z - x)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c - 1)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c) + b - a)$

$= -y(b - a) = y(a - b)$

16. Official Ans. by NTA (4)

Sol. For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

Now $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

Now $D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$

$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

$\mu = 8$

17. Official Ans. by NTA (1)

Sol. $\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$\begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

$= -1\left(\sin^2 x\right) - 1\left(1 + \sin 2x + \cos^2 x\right)$

$= -\sin 2x - 2$

$m = -3,\ M = -1$

18. Official Ans. by NTA (3.00)

Sol. $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + (3\lambda - 3)z = 0$

$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} 0 & 3-\lambda & \lambda-3 \\ \lambda-3 & \lambda-3 & -2(\lambda-3) \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$

$(\lambda-3)^2 \begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$

$(\lambda - 3)^2\ [(3\lambda + 1) + (3\lambda - 1)] = 0$

$6\lambda(\lambda - 3)^2 = 0 \Rightarrow \lambda = 0, 3$

Sum = 3