COMPOUND ANGLE

1. Let α and β be two real roots of the equation $(k + 1)\tan^2 x - \sqrt{2}\,.\,\lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is ;

(1) 5 (2) 10 (3) $5\sqrt{2}$ (4) $10\sqrt{2}$

2. If $\dfrac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \dfrac{1}{7}$ and $\sqrt{\dfrac{1-\cos 2\beta}{2}} = \dfrac{1}{\sqrt{10}}$, α, $\beta \in \left(0, \dfrac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to ______.

3. The value of

$$\cos^3\left(\frac{\pi}{8}\right)\cdot\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right)\cdot\sin\left(\frac{3\pi}{8}\right)$$ is :

(1) $\dfrac{1}{4}$ (2) $\dfrac{1}{\sqrt{2}}$ (3) $\dfrac{1}{2\sqrt{2}}$ (4) $\dfrac{1}{2}$

4. If $L = \sin^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$ and $M = \cos^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$, then :

(1) $M = \dfrac{1}{2\sqrt{2}} + \dfrac{1}{2}\cos\dfrac{\pi}{8}$

(2) $L = \dfrac{1}{4\sqrt{2}} - \dfrac{1}{4}\cos\dfrac{\pi}{8}$

(3) $M = \dfrac{1}{4\sqrt{2}} + \dfrac{1}{4}\cos\dfrac{\pi}{8}$

(4) $L = -\dfrac{1}{2\sqrt{2}} + \dfrac{1}{2}\cos\dfrac{\pi}{8}$

SOLUTION

1. NTA Ans. (2)

Sol. $\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$

$\tan\alpha.\ \tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\lambda\sqrt{2}}{2} = \frac{\lambda}{\sqrt{2}}$

$\Rightarrow \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10\ \&\ -10$

2. NTA Ans. (1)

Sol. $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7} \Rightarrow \tan\alpha = \frac{1}{7}$

$\sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3} \Rightarrow \tan 2\beta = \frac{3}{4}$

$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan 2\beta}{1-\tan\alpha\tan 2\beta} = 1$

Ans. 1.00

3. NTA Ans. (3)

Sol. $\cos^3\frac{\pi}{8}\cdot\sin\frac{\pi}{8} + \sin^3\frac{\pi}{8}\cdot\cos\frac{\pi}{8}$

$= \sin\frac{\pi}{8}.\cos\frac{\pi}{8} = \frac{1}{2}\sin\frac{\pi}{4} = \frac{1}{2\sqrt{2}}$

4. Official Ans. by NTA (1)

Sol. $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$\left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$

$\Rightarrow L = \left(\frac{1-\cos(\pi/8)}{2}\right) - \left(\frac{1-\cos(\pi/4)}{2}\right)$

$L = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{8}\right)\right]$

$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$M = \frac{1+\cos(\pi/8)}{2} - \frac{1-\cos(\pi/4)}{2}$

$M = \frac{1}{2}\cos\left(\frac{\pi}{8}\right) + \frac{1}{2\sqrt{2}}$