

## AREA UNDER THE CURVE

- 1.** The area (in sq. units) of the region  $\{(x, y) \in R^2 : 4x^2 \leq y \leq 8x + 12\}$  is :
- (1)  $\frac{127}{3}$       (2)  $\frac{125}{3}$   
 (3)  $\frac{124}{3}$       (4)  $\frac{128}{3}$
- 2.** The area of the region, enclosed by the circle  $x^2 + y^2 = 2$  which is not common to the region bounded by the parabola  $y^2 = x$  and the straight line  $y = x$ , is :
- (1)  $\frac{1}{3}(12\pi - 1)$       (2)  $\frac{1}{6}(12\pi - 1)$   
 (3)  $\frac{1}{6}(24\pi - 1)$       (4)  $\frac{1}{3}(6\pi - 1)$
- 3.** The area (in sq. units) of the region  $\{(x, y) \in R^2 : x^2 \leq y \leq 3 - 2x\}$ , is
- (1)  $\frac{29}{3}$       (2)  $\frac{31}{3}$   
 (3)  $\frac{34}{3}$       (4)  $\frac{32}{3}$
- 4.** For  $a > 0$ , let the curves  $C_1 : y^2 = ax$  and  $C_2 : x^2 = ay$  intersect at origin O and a point P. Let the line  $x = b$  ( $0 < b < a$ ) intersect the chord OP and the x-axis at points Q and R, respectively. If the line  $x = b$  bisects the area bounded by the curves,  $C_1$  and  $C_2$ , and the area of  $\triangle OQR = \frac{1}{2}$ , then 'a' satisfies the equation
- (1)  $x^6 - 12x^3 + 4 = 0$   
 (2)  $x^6 - 12x^3 - 4 = 0$   
 (3)  $x^6 + 6x^3 - 4 = 0$   
 (4)  $x^6 - 6x^3 + 4 = 0$

- 5.** Given :  $f(x) = \begin{cases} x & , 0 \leq x < \frac{1}{2} \\ \frac{1}{2} & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x \leq 1 \end{cases}$  and  
 $g(x) = \left(x - \frac{1}{2}\right)^2$ ,  $x \in R$ . Then the area (in sq. units) of the region bounded by the curves,  $y = f(x)$  and  $y = g(x)$  between the lines,  $2x = 1$  and  $2x = \sqrt{3}$ , is :
- (1)  $\frac{1}{3} + \frac{\sqrt{3}}{4}$       (2)  $\frac{\sqrt{3}}{4} - \frac{1}{3}$   
 (3)  $\frac{1}{2} + \frac{\sqrt{3}}{4}$       (4)  $\frac{1}{2} - \frac{\sqrt{3}}{4}$
- 6.** Area (in sq. units) of the region outside  $\frac{|x|}{2} + \frac{|y|}{3} = 1$  and inside the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$  is :
- (1)  $3(4 - \pi)$       (2)  $6(\pi - 2)$   
 (3)  $3(\pi - 2)$       (4)  $6(4 - \pi)$
- 7.** Consider a region  $R = \{(x, y) \in R^2 : x^2 \leq y \leq 2x\}$ . If a line  $y = \alpha$  divides the area of region R into two equal parts, then which of the following is true?
- (1)  $\alpha^3 - 6\alpha^2 + 16 = 0$   
 (2)  $3\alpha^2 - 8\alpha + 8 = 0$   
 (3)  $\alpha^3 - 6\alpha^{3/2} - 16 = 0$   
 (4)  $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$
- 8.** The area (in sq. units) of the region  $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2\}$  is:
- (1)  $\frac{79}{16}$       (2)  $\frac{23}{6}$   
 (3)  $\frac{79}{24}$       (4)  $\frac{23}{16}$

9. The area (in sq. units) of the region  $A = \{(x, y) : (x - 1)[x] \leq y \leq 2\sqrt{x}, 0 \leq x \leq 2\}$ , where  $[t]$  denotes the greatest integer function, is :

(1)  $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

(2)  $\frac{8}{3}\sqrt{2} - 1$

(3)  $\frac{4}{3}\sqrt{2} - \frac{1}{2}$

(4)  $\frac{4}{3}\sqrt{2} + 1$

10. The area (in sq. units) of the region  $A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$  is :

(1)  $\frac{1}{6}$

(2)  $\frac{1}{3}$

(3)  $\frac{7}{6}$

(4)  $\frac{5}{6}$

11. The area (in sq. units) of the region enclosed by the curves  $y = x^2 - 1$  and  $y = 1 - x^2$  is equal to:

(1)  $\frac{4}{3}$

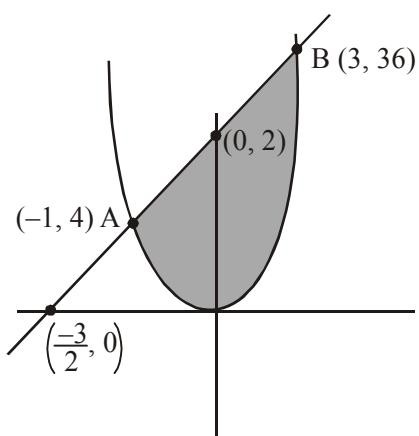
(2)  $\frac{8}{3}$

(3)  $\frac{16}{3}$

(4)  $\frac{7}{2}$

**SOLUTION****1. NTA Ans. (4)**

**Sol.**  $4x^2 - y \leq 0$  and  $8x - y + 12 \geq 0$



On solving  $y = 4x^2$

and  $y = 8x + 12$

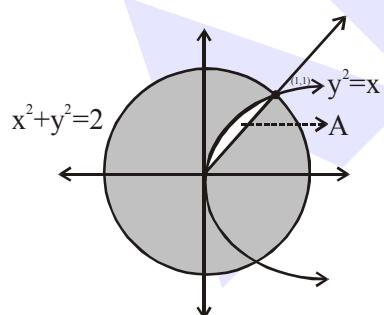
We get A (-1, 4) & B(3, 36)

Required area = area of the shaded region

$$= \int_{-1}^3 (8x + 12 - 4x^2) dx = \frac{128}{3}$$

**2. NTA Ans. (2)**

**Sol.**  $A = \int_0^1 (\sqrt{x} - x) dx$

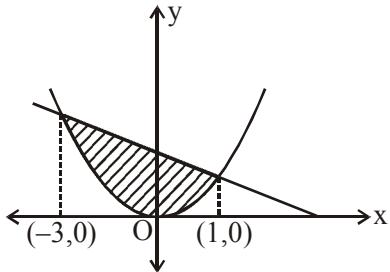


$$= \left[ \frac{2}{3}x^{3/2} - \frac{x^2}{2} \right]_0^1 = \frac{1}{6}$$

$$\text{Required Area : } \pi r^2 - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$$

**3. NTA Ans. (4)**

**Sol.**

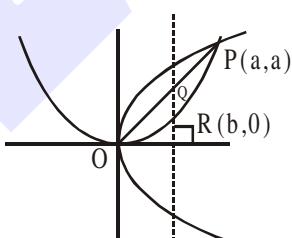


$$\text{Area} = \int_{-3}^1 (3 - 2x - x^2) dx = \frac{32}{3}$$

(4) option

**4. NTA Ans. (1)**

**Sol.**  $\int_0^b \left( \sqrt{ax} - \frac{x^2}{a} \right) dx = \frac{1}{2} \times \frac{16 \left( \frac{a}{4} \right) \left( \frac{a}{4} \right)}{3}$



$$\Rightarrow \left[ \frac{2\sqrt{a}}{3}x^{3/2} - \frac{x^3}{3a} \right]_0^b = \frac{a^2}{6}$$

$$\Rightarrow \frac{2\sqrt{a}}{3}b^{3/2} - \frac{b^3}{3a} = \frac{a^2}{6}$$

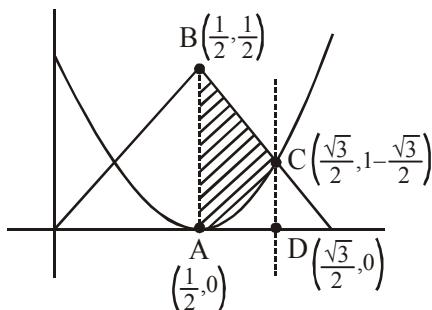
...(i)

$$\text{Also, } \frac{1}{2} \times b^2 = \frac{1}{2} \Rightarrow b = 1$$

$$\text{so, } \frac{2\sqrt{a}}{3} - \frac{1}{3a} = \frac{a^2}{6} \Rightarrow a^3 - 4a^{3/2} + 2 = 0$$

$$\Rightarrow a^6 + 4a^3 + 4 = 16a^3 \Rightarrow a^6 - 12a^3 + 4 = 0$$

## 5. NTA Ans. (2)

**Sol.**

Required area = Area of trapezium ABCD -

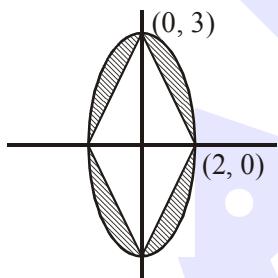
$$\text{Area of parabola between } x = \frac{1}{2} \text{ & } x = \frac{\sqrt{3}}{2}$$

$$A = \frac{1}{2} \left( \frac{\sqrt{3}}{2} - \frac{1}{2} \right) \left( \frac{1}{2} + 1 - \frac{\sqrt{3}}{2} \right) - \int_{1/2}^{\sqrt{3}/2} \left( x - \frac{1}{2} \right)^2 dx = \frac{\sqrt{3}}{4} - \frac{1}{3}$$

## 6. Official Ans. by NTA (2)

$$\text{Sol. } \frac{|x|}{2} + \frac{|y|}{3} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



$$\text{Area of Ellipse} = \pi ab = 6\pi$$

Required area,

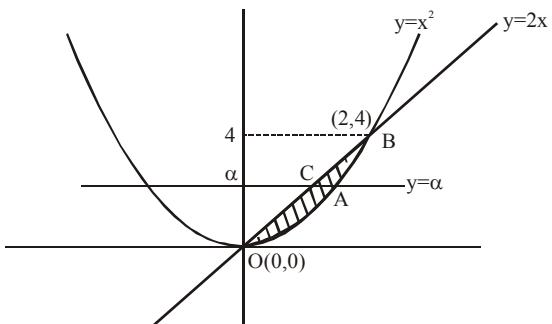
$$= \pi \times 2 \times 3 - (\text{Area of quadrilateral})$$

$$= 6\pi - \frac{1}{2} \cdot 6 \times 4$$

$$= 6\pi - 12$$

$$= 6(\pi - 2)$$

## 7. Official Ans. by NTA (4)

**Sol.**

\*  $y \geq x^2 \Rightarrow$  upper region of  $y = x^2$

$y \leq 2x \Rightarrow$  lower region of  $y = 2x$

According to ques, area of OABC = 2 area of OAC

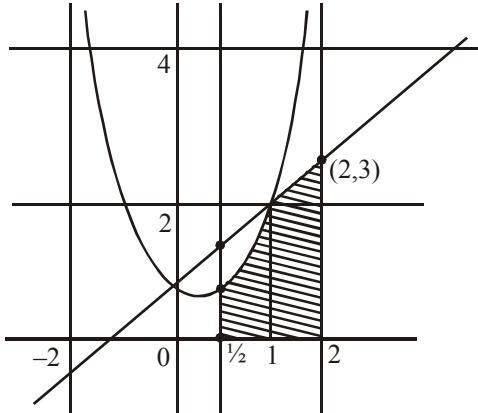
$$\Rightarrow \int_0^4 \left( \sqrt{y} - \frac{y}{2} \right) dy = 2 \int_0^{\alpha} \left( \sqrt{y} - \frac{y}{2} \right) dy$$

$$\Rightarrow \frac{4}{3} = 2 \left[ \frac{2}{3} \alpha^{3/2} - \frac{1}{4} \cdot \alpha^2 \right]$$

$$\Rightarrow 3\alpha^2 - 8\alpha^{3/2} + 8 = 0$$

## 8. Official Ans. by NTA (3)

$$\text{Sol. } 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2$$



$$\text{Required area} = \int_{1/2}^1 (x^2 + 1) dx + \frac{1}{2}(2 + 3) \times 1$$

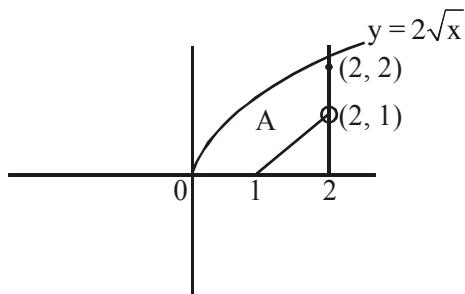
$$= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}$$

## 9. Official Ans. by NTA (1)

Sol.  $(x - 1)[x] \leq y \leq 2\sqrt{x}$ ,  $[0 \leq x \leq 2]$

Draw  $y = 2\sqrt{x} \Rightarrow y^2 = 4x$   $[x \geq 0]$

$$y = (x - 1)[x] = \begin{cases} 0 & , 0 \leq x < 1 \\ x - 1 & , 1 \leq x < 2 \\ 2 & , x = 2 \end{cases}$$



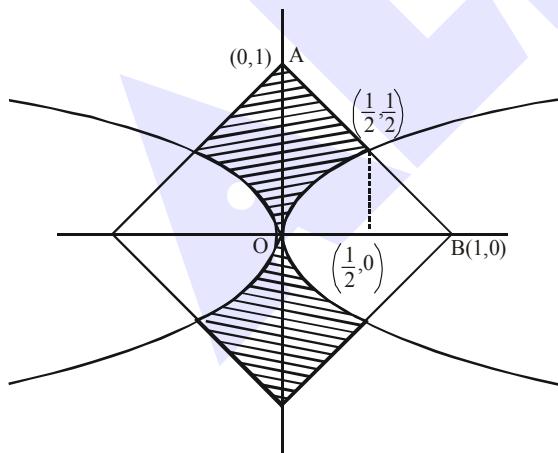
$$A = \int_0^2 2\sqrt{x} dx - \frac{1}{2} \cdot 1 \cdot 1$$

$$A = 2 \left[ \frac{x^{3/2}}{(3/2)} \right]_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

## 10. Official Ans. by NTA (4)

Sol.  $|x| + |y| \leq 1$

$$2y^2 \geq |x|$$



For point of intersection

$$x + y = 1 \Rightarrow x = 1 - y$$

$$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$$

$$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$$

$$(2y - 1)(y + 1) = 0$$

$$y = \frac{1}{2} \text{ or } -1$$

$$\text{Now Area of } \Delta OAB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

$$\text{Area of Region } R_1 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$\text{Area of Region } R_2 = \frac{1}{\sqrt{2}} \int_0^{\frac{1}{2}} \sqrt{x} dx = \frac{1}{6}$$

$$\begin{aligned} \text{Now area of shaded region in first quadrant} \\ = \text{Area of } \Delta OAB - R_1 - R_2 \end{aligned}$$

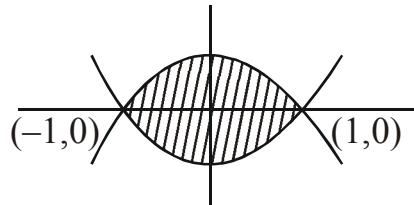
$$= \frac{1}{2} - \left( \frac{1}{6} \right) - \left( \frac{1}{8} \right) = \frac{5}{24}$$

$$\text{So required area} = 4 \left( \frac{5}{24} \right) = \frac{5}{6}$$

so option (4) is correct.

## 11. Official Ans. by NTA (2)

Sol.  $y = x^2 - 1$  and  $y = 1 - x^2$



$$A = \int_{-1}^1 ((1 - x^2) - (x^2 - 1)) dx$$

$$A = \int_{-1}^1 (2 - 2x^2) dx = 4 \int_0^1 (1 - x^2) dx$$

$$A = 4 \left( x - \frac{x^3}{3} \right)_0^1 = 4 \left( \frac{2}{3} \right) = \frac{8}{3}$$