## WORK, POWER & ENERGY

**1.** A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed $v_0$ and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity) **[JEE(Advanced) 2020]**
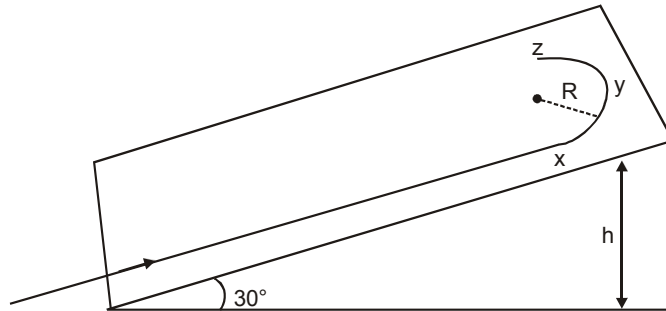


(A) $v_0^2 - 2gh = \dfrac{1}{2}gR$

(B) $v_0^2 - 2gh = \dfrac{\sqrt{3}}{2}gR$

(C) the centripetal force required at points x and z is zero

(D) the centripetal force required is maximum at points x and z

**2.** A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y \hat{i} + 2\alpha x \hat{j})N$, where x and y are in meter and $\alpha = -1$ N/m$^{-1}$. The work done on the particle by this force $\vec{F}$ will be _____ Joule. **[JEE(Advanced) 2019]**
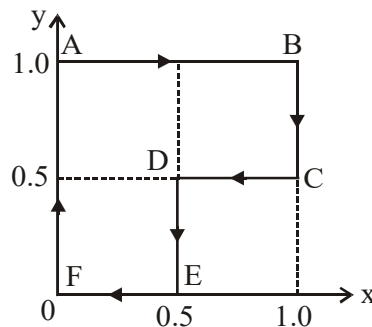


**3.** A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as dK/dt = γt, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ? **[JEE(Advanced) 2018]**

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

**4.** Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms$^{-2}$) **[JEE(Advanced) 2014]**
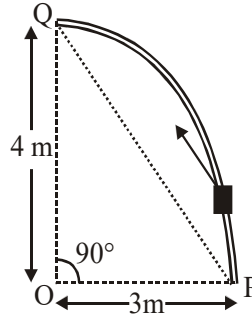


**5.** A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

**[JEE(Advanced) 2014]**
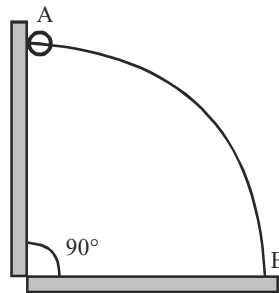


(A) Always radially outwards

(B) Always radially inwards

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

**SOLUTIONS**

**1.**      **Ans. (A, D)**

**Sol.** By the energy conservation (ME) between bottom point and point Y

$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv_1^2$$

$$\therefore v_1^2 = v_0^2 - 2gh \qquad \ldots(i)$$

Now at point Y the centripetal force provided by the component of mg

$$\therefore mg\sin 30° = \frac{mv_1^2}{R}$$

$$\therefore v_1^2 = \frac{gR}{2}$$

$$\therefore \text{from (i)}$$

$$\frac{gR}{2} = v_0^2 - 2gh$$

At point x and z of circular path, the points are at same height but less then h. So the velocity more than a point y.

So required centripetal $= \dfrac{mv^2}{r}$ is more.

**2.**      **Ans. (0.75)**

**Sol.** $F = \left(\alpha y\,\hat{i} + 2\alpha x\,\hat{j}\right)$

$$W_{AB} = \left(-1\hat{i}\right)\cdot\left(1\hat{i}\right) = -1J$$

$$\begin{bmatrix} \vec{F} = -1\hat{i} + 2\alpha\, x\, \hat{j} \\ \vec{S} = 1\hat{i} \end{bmatrix}$$

Similarly,

$W_{BC} = 1J$

$W_{CD} = 0.25J$

$W_{DE} = 0.5\ J$

$W_{EF} = W_{FA} = 0\ J$

$\therefore$ New work in cycle $= 0.75\ J$

**3.**      **Ans. (A, B, D)**

**Sol.** $\dfrac{dk}{dt} = \gamma t$ as $k = \dfrac{1}{2}mv^2$

$$\therefore \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$$

$$\therefore m\int_0^v v\,dv = \gamma\int_0^t t\,dt$$

$$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$$

$$v = \sqrt{\frac{\gamma}{m}}\,t \qquad \ldots(i)$$

$$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}} = \text{constant}$$

since F = ma

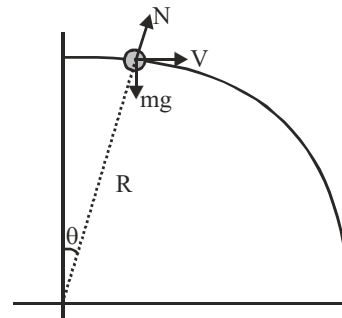$$\therefore F = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$$

**4.**      **Ans. (5)**

**Sol.** $K_f - K_i = W_{all}$

$K_f = W_{ext} + W_{gr}$

$= 18 \times 5 - 1 \times 10 \times 4 = 50 = 5 \times 10$

**5.**      **Ans. (D)**

**Sol.**



By circular motion equation

$$mg\cos\theta - \frac{mv^2}{R} = N$$

by conservation of energy

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta)$$

$$\frac{mv^2}{R} = 2mg(1-\cos\theta)$$

So, mg cos θ – 2mg (1 – cos θ) = N

$\Rightarrow$ N = mg (3 cosθ – 2)

For $0 \le \theta < \cos^{-1}\left(\dfrac{2}{3}\right)$, N is positive

so, N on ball is outward & its reaction force on wire is inward.

for $\theta > \cos^{-1}\left(\dfrac{2}{3}\right)$, N is negative,

So, N on ball is inward, & its reaction force on wire is outward.