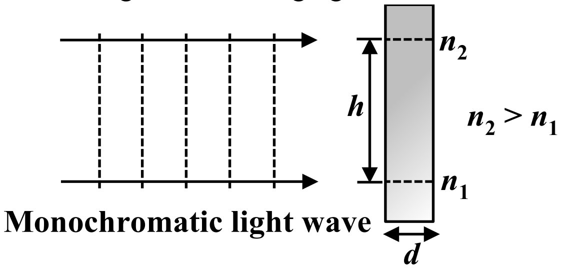
WAVE OPTICS

1. A monochromatic light wave is incident normally on a glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from n_1 to n_2 over the height h. Which of the following statement(s) is (are) true about the light wave emerging out of the slab ? **[JEE(Advanced) 2023]**



(A) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2^2 - n_1^2)d}{2h}\right]$

(B) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2 - n_1)d}{h}\right]$

(C) It will not deflect.

(D) The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of n_1 and n_2.

2. The electric field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ V m^{-1}. Which of the following option(s) is(are) correct ?

[Given: The speed of light in vacuum, $c = 3 \times 10^8$ ms^{-1}] **[JEE(Advanced) 2023]**

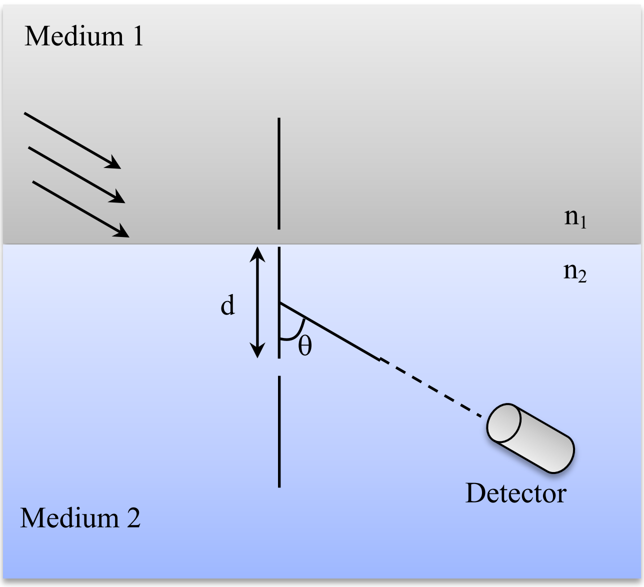
(A) $B_x = -2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm^{-2}.

(B) $B_y = 2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm^{-2}

(C) The wave is polarized in the xy-plane with polarization angle 30° with respect to the x-axis.

(D) The refractive index of the medium is 2.

3. A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index n_2. The other slit is at the interface of this medium with another medium 1 of refractive index $n_1 (\neq n_2)$. The line joining the slits is perpendicular to the interface and the distance between the slits is d. The slit widths are much smaller than d. A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle θ from the line joining them, so that θ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.



[JEE(Advanced) 2022]

Which of the following statement(s) is (are) correct?

(A) The phase difference between the two rays is independent of *d*.

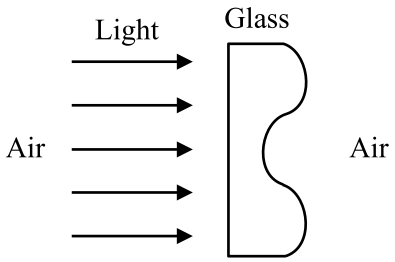
(B) The two rays interfere constructively at the detector.

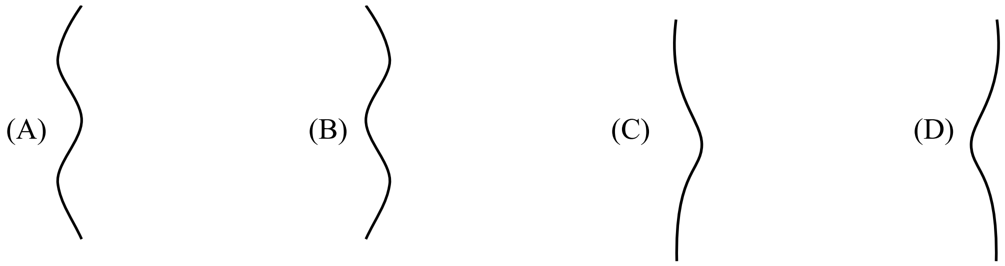
(C) The phase difference between the two rays depends on n_1 but is independent of n_2.

(D) The phase difference between the two rays vanishes only for certain values of *d* and the angle of incidence of the beam, with θ being the corresponding angle of refraction.

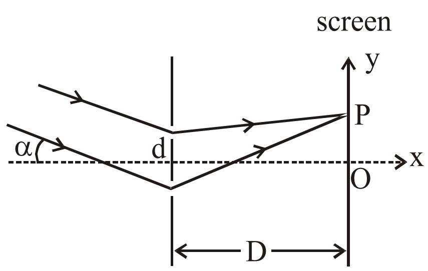
4. A parallel beam of light strikes a piece of transparent glass having cross section as shown in the figure below. Correct shape of the emergent wavefront will be (figures are schematic and not drawn to scale)-

[JEE(Advanced) 2020]





5. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1m. A parallel beam of light of wavelength 600nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct? **[JEE(Advanced) 2019]**



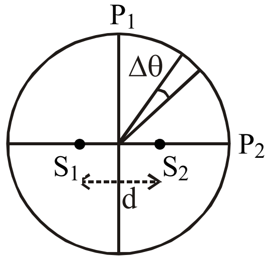
(A) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

(B) Fringe spacing depends on α

(C) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P

(D) For $\alpha = 0$, there will be constructive interference at point P.

6. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct ? **[JEE(Advanced) 2017]**



(A) A dark spot will be formed at the point P_2

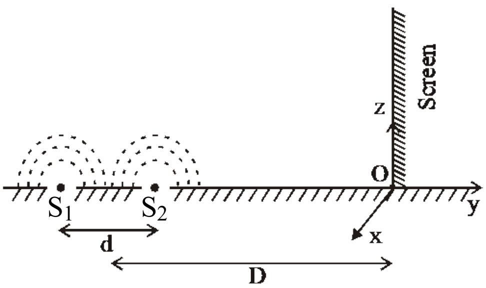
(B) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

(C) At P_2 the order of the fringe will be maximum

(D) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

7. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for z > 0) at a distance D = 3m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is (are) true of the intensity pattern on the screen ?

[JEE(Advanced) 2016]



(A) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

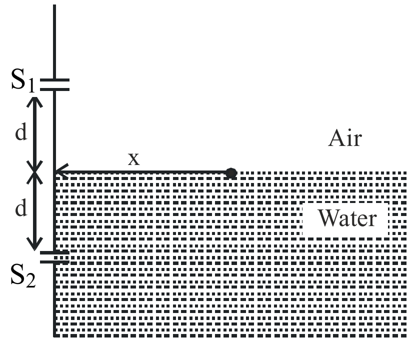
(B) Semi circular bright and dark bands centered at point O

(C) The region very close to the point O will be dark

(D) Straight bright and dark bands parallel to the x-axis

8. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is.

[JEE(Advanced) 2015]



9. A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then :-

[JEE(Advanced) 2014]

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

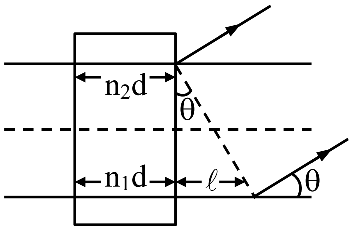
(C) From the central maximum, 3^{rd} maximum of λ_2 overlaps with 5^{th} minimum of λ_1

(D) The angular separation of fringes of λ_1 is greater than λ_2

SOLUTIONS

1. Ans. (B, D)

Sol.

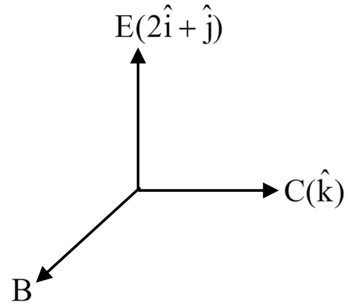


$n_1 d + \ell = n_2 d$

$$\tan\theta = \frac{\ell}{h} = \frac{(n_2 - n_1)d}{h}$$

2. Ans. (A, D)

Sol.



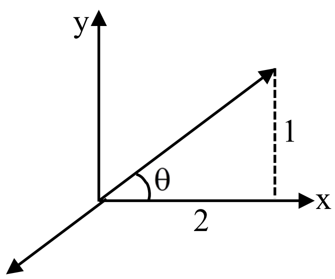
$$C_{medium} = \frac{5\times10^{14}}{10^7/3} = 1.5 \times 10^8 \text{ m/s} \quad (\therefore\ \mu = 2)$$

$$C_{medium} = \frac{E}{B}$$

$$\Rightarrow B = \frac{E}{C_m} = \frac{30\sqrt{5}}{1.5\times10^8} = 2\sqrt{5}\times10^{-7}$$

$$\vec{B}_{direction} \equiv \hat{k}\times(2\hat{i}+\hat{j}) \equiv \frac{2\hat{j}-\hat{i}}{\sqrt{5}}$$

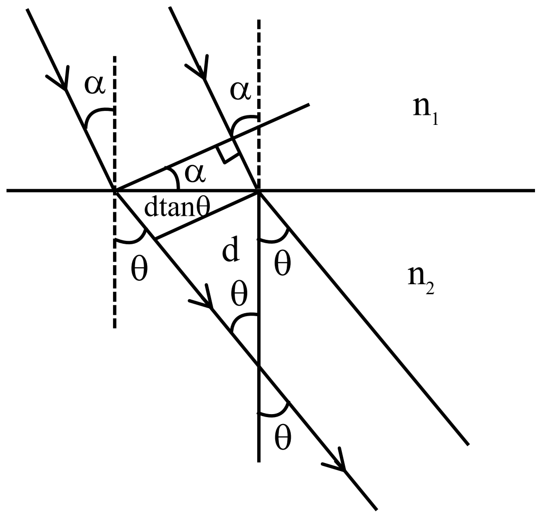
$$\therefore \vec{B} = 2\times10^{-7}(-\hat{i}+2\hat{j})\sin\left[27\left(5\times10^{17}t - \frac{10^7}{3}z\right)\right]$$



$$\tan\theta = \frac{1}{2}$$

3. Ans. (A, B)

Sol.



Optical path difference →

$\Delta x = n_1(d\tan\theta)\sin\alpha - n_2(d\tan\theta)\sin\theta$

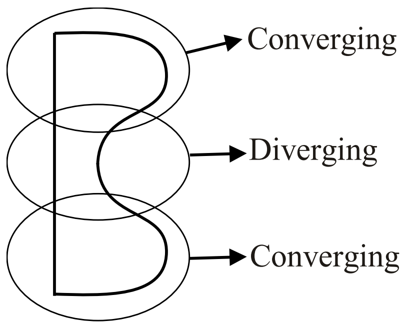
$= (n_1\sin\alpha - n_2\sin\theta)\, d\tan\theta$

$= 0$

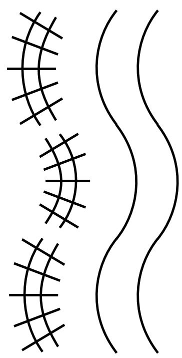
$\Rightarrow \Delta\phi = 0$

4. Ans. (A)

Sol.



(A)



5. Ans. (C)

Sol. (A) $\Delta x = d\sin\alpha = d\alpha$ (as α is very small)

$$\alpha = \frac{.36}{180} = (2\times10^{-3})\text{ rad}$$

$$\frac{\Delta x}{\lambda} = \frac{(3\times10^{-4})\,(2\times10^{-3})}{6\times10^{-7}} = 1$$

so constructive interference

(B) $\beta = \frac{D\lambda}{d}$

(C) $\Delta x_p = d\alpha + \frac{dy}{D}$

$= 3 \times 10^{-4} (2 \times 10^{-3} + 11 \times 10^{-3})$

$= 39 \times 10^{-7}$

$\frac{\Delta x_p}{\lambda} = \frac{39 \times 10^{-7}}{6 \times 10^{-7}} = 6.5$ so destructive

(D) $\Delta x_p = \frac{dy}{D} = (3 \times 10^{-4}) \times 11 \times 10^{-3}$

$= 33 \times 10^{-7}$

$\frac{\Delta x_p}{\lambda} = \frac{33 \times 10^{-7}}{6 \times 10^{-7}} = 5.5 \Rightarrow$ destructive

6. Ans. (C, D)

Sol. At point P_2; $\Delta x = d = 1.8$ mm $= 3000\ \lambda$

hence a (bright fringe) will be formed at P_2

Now, $\Delta x = d \cos\theta = n\lambda$

$\cos\theta = \frac{n\lambda}{d}$

$-\sin\theta\ \Delta\theta = (\Delta n)\ \frac{\lambda}{d}$

$\Delta\theta = -(\Delta n)\frac{\lambda}{d\sin\theta}$

$\Delta\theta$ increases as θ decreases

At P_2, the order of fringe will be maximum.

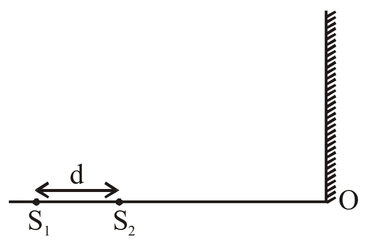
For total no. of bright fringes

$d = n\lambda \Rightarrow n = 3000$

$\therefore$ total no. of fringes = 3000

7. Ans. (B, C)

Sol.



Path difference at point O = d = .6003 mm

= 600300 nm

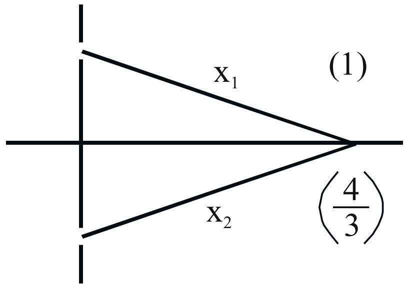
$= \frac{2001}{2}(600\text{nm}) = 1000\lambda + \frac{\lambda}{2}$

$\Rightarrow$ minima form at point O

Line S_1S_2 and screen are $\perp$ to each other so fringe pattern is circular (semi-circular because only half of screen is available)

8. Ans. (3)

Sol.



$x_1 = \sqrt{x^2 + d^2}$

$x_2 = \frac{4}{3}\sqrt{x^2 + d^2}$

$\Delta x = \left(\frac{4}{3} - 1\right)\sqrt{x^2 + d^2} = n\lambda$

$\frac{1}{3}\sqrt{x^2 + d^2} = n\lambda$

$(x^2 + d^2) = 9n^2\lambda^2$

$\therefore$ (P = 3)

9. Ans. (A, B, C)

Sol. $\beta = \frac{\lambda D}{d}$

$\lambda_1 < \lambda_2$

$\beta_2 > \beta_1$

$m_1\beta_1 = m_2\beta_2 = y$

$\Rightarrow m_1 > m_2$

$y_1 = 3\frac{\lambda_2 D}{d}$

$y_2 = 4.5\frac{\lambda_1 D}{d}$

Here $y_1 = y_2$

$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

Hence A, B & C are correct choices