WAVE ON STRING

1. A string of length 1 m and mass 2×10^{-5} kg is under tension T. when the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is _______ Newton. **[JEE(Advanced) 2023]**

2. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II the magnitude of some quantity.

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be, **[JEE(Advanced) 2019]**

(A) I→P, II→R, III→S, IV→Q
(B) I→P, II→Q, III→T, IV→S
(C) I→Q, II→S, III→R, IV→P
(D) I→Q, II→P, III→R, IV→T

3. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

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	(T) 3/16
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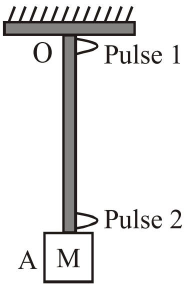
The length of the string 1, 2, 3 and 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3 and 4 are vibrated at their 1^{st}, 3^{rd}, 5^{th} and 14^{th} harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be.

[JEE(Advanced) 2019]

(A) I→P, II→Q, III→T, IV→U (B) I→T, II→Q, III→R, IV→U

(C) I→P, II→Q, III→R, IV→T (D) I→P, II→R, III→T, IV→U

4. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are **correct** ? **[JEE(Advanced) 2017]**



(A) The time $T_{AO} = T_{OA}$

(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

(C) The wavelength of Pulse 1 becomes longer when it reaches point A

(D) The velocity of any pulse along the rope is independent of its frequency and wavelength.

5. One end of a taut string of length 3m along the x–axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) :-

[JEE(Advanced) 2014]

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

SOLUTIONS

1. Ans. (5)

Sol. $f = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$

$750 = \frac{P}{2}\sqrt{\frac{T}{\mu}}$ ....(1)

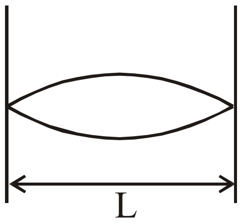
$1000 = \frac{P+1}{2}\sqrt{\frac{T}{\mu}}$ ....(2)

$\frac{4}{3} = \frac{P+1}{P}$ $\therefore$ P = 3

$\Rightarrow 1000 = \frac{4}{2}\sqrt{\frac{T}{2\times10^{-5}}}$ $\therefore$ T = 5N

2. Ans. (A)

Sol. For fundamental mode



$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$

$f = \frac{V}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

For string (1)

$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \Rightarrow (P)$

For string (2)

$f = \frac{1}{2L}\sqrt{\frac{T}{2\mu}} = \frac{f_0}{\sqrt{2}} \Rightarrow (R)$

For string (3)

$f = \frac{1}{2L}\sqrt{\frac{T}{3\mu}} = \frac{f_0}{\sqrt{3}} \Rightarrow (S)$

For string (4)

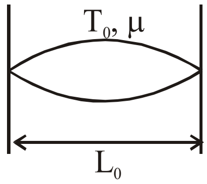
$f = \frac{1}{2L}\sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \Rightarrow (Q)$

3. Ans. (A)

Sol. For string (1)

Length of string = L_0

It is vibrating in I^{st} harmonic i.e. fundamental mode.



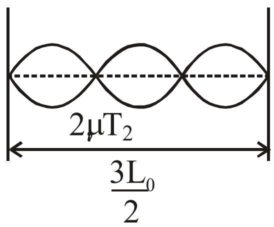
$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow (P)$

For string (2)

Length of string = $\frac{3L_0}{2}$

It is vibrating in III^{rd} harmonic but frequency is still f_0.

$f_0 = \frac{3v}{2L}$



$f_0 = \frac{3}{2\left(\frac{3L_0}{2}\right)}\sqrt{\frac{T_2}{2\mu}}$

$\Rightarrow f_0 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

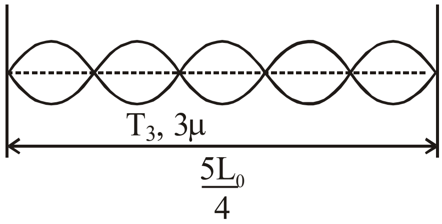
$\Rightarrow \boxed{T_2 = \frac{T_0}{2}} \Rightarrow (Q)$

For string (3)

Length of string = $\frac{5L_0}{4}$

It is vibrating in 5^{th} harmonic but frequency is still f_0.

$f_0 = \frac{5V}{2L}$



$$\Rightarrow f_0 = \frac{5}{2\left(\frac{5L_0}{4}\right)}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

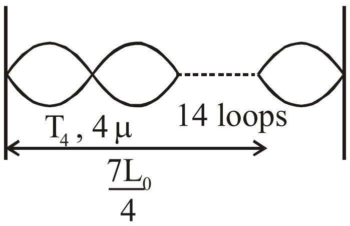
$$\Rightarrow \frac{2}{L_0}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$T_3 = \frac{3T_0}{16} \Rightarrow (T)$$

For string (4)

Length of string = $\frac{7L_0}{4}$

It is vibrating in 14^{th} harmonic but frequency is still f_0.



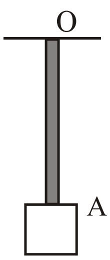
$$f_0 = \frac{14v}{2L}$$

$$\Rightarrow f_0 = \frac{14}{2\left(\frac{7L_0}{4}\right)}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \frac{4}{L_0}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow \boxed{T_4 = \frac{T_0}{16}} \Rightarrow (U)$$

4. Ans. (A, D)

Sol.



(A) Speed of wave is property of medium so time taken to cross the string will be equal

(B) Speeds are same but velocity is vector, has opposite directions

(C) Wavelength $\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$ and $T_O > T_A$

(D) Velocity of any pulse is $v = \sqrt{\frac{T}{\mu}}$ and it is property of medium.

5. Ans. (A, C, D)

Sol. $\left.\begin{array}{l} \text{At } x = 0 \quad y = 0 \\ x = 3 \quad y \neq 0 \\ \frac{\omega}{k} = 100 \text{ m/s} \end{array}\right]$ The equation satisfying all three conditions is correct. Hence answer ACD