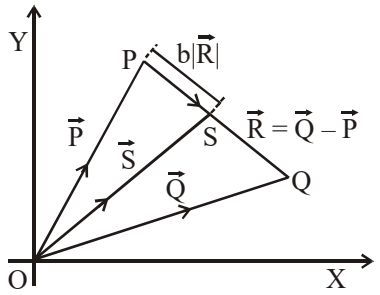
VECTOR

1. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a\left(\cos\omega t\hat{i} + \sin\omega t\hat{j}\right)$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A} + \vec{B}| = \sqrt{3}|\vec{A} - \vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is _____.

[JEE(Advanced) 2018]

2. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is :

[JEE(Advanced) 2017]



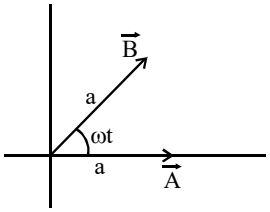
(A) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$ (C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (D) $\vec{S} = \left(1-b^2\right)\vec{P} + b\vec{Q}$

SOLUTIONS

1. Ans. (2.00)

Sol. $|\vec{A}+\vec{B}| = 2a\cos\frac{\omega t}{2}$

$|\vec{A}-\vec{B}| = 2a\sin\frac{\omega t}{2}$



So

$2a\cos\frac{\omega t}{2} = \sqrt{3}\ \left(2a\sin\frac{\omega t}{2}\right)$

$\tan\frac{\omega t}{2} = \frac{1}{\sqrt{3}}$

$\frac{\omega t}{2} = \frac{\pi}{6} \Rightarrow \omega t = \frac{\pi}{3}$

$\frac{\pi}{6}t = \frac{\pi}{3}$ $\quad t = 2.00$ sec

2. Ans. (C)

Sol. Let vector from point P to point S be $\vec{C}$

$\Rightarrow \vec{C} = b|\vec{R}|\hat{R} = b|\vec{R}|\left(\frac{\vec{R}}{|\vec{R}|}\right)$

$= b\vec{R} = b\left(\vec{Q}-\vec{P}\right)$

from triangle rule of vector addition

$\vec{P}+\vec{C} = \vec{S}$

$\vec{P}+b\left(\vec{Q}-\vec{P}\right) = \vec{S} \Rightarrow \vec{S} = (1-b)\vec{P}+b\vec{Q}$