UNIT & DIMENSION

1. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^{\alpha} h^{\beta} G^{\gamma}$. Which of the following is the correct option ? **[JEE(Advanced) 2023]**

(A) $\alpha = 7, \beta = -1, \gamma = -2$
(B) $\alpha = -7, \beta = -1, \gamma = -2$
(C) $\alpha = 7, \beta = -1, \gamma = 2$
(D) $\alpha = -7, \beta = 1, \gamma = -2$

2. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and Coulomb's constant $k = \frac{1}{4\pi \in_0}$, where $\in_0$ is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^{\alpha} [m_e]^{\beta} [h]^{\gamma} [k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ is ________. **[JEE(Advanced) 2022]**

3. A physical quantity $\vec{S}$ is defined as $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity (ies) ? **[JEE(Advanced) 2021]**

(A) $\frac{\text{Energy}}{\text{charge} \times \text{current}}$
(B) $\frac{\text{Force}}{\text{Length} \times \text{Time}}$
(C) $\frac{\text{Energy}}{\text{Volume}}$
(D) $\frac{\text{Power}}{\text{Area}}$

4. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity X as follows: [position] = $[X^{\alpha}]$; [speed] = $[X^{\beta}]$; [acceleration] = $[X^{p}]$; [linear momentum] = $[X^{q}]$; [force] = $[X^{r}]$. Then - **[JEE(Advanced) 2020]**

(A) $\alpha + p = 2\beta$
(B) $p + q - r = \beta$
(C) $p - q + r = \alpha$
(D) $p + q + r = \beta$

5. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct ? **[JEE(Advanced) 2019]**

(A) The dimension of force is L^{-3}
(B) The dimension of energy is L^{-2}
(C) The dimension of power is L^{-5}
(D) The dimension of linear momentum is L^{-1}

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\in_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on Paragraph "X", the question given below is one of them)

6. The relation between [E] and [B] is :- **[JEE(Advanced) 2018]**

(A) $[E] = [B][L][T]$
(B) $[E] = [B][L]^{-1}[T]$
(C) $[E] = [B][L][T]^{-1}$
(D) $[E] = [B][L]^{-1}[T]^{-1}$

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\in_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on Paragraph "X", the question given below is one of them)

7. The relation between $[\in_0]$ and $[\mu_0]$ is :- **[JEE(Advanced) 2018]**

(A) $[\mu_0] = [\in_0][L]^2[T]^{-2}$
(B) $[\mu_0] = [\in_0][L]^{-2}[T]^2$
(C) $[\mu_0] = [\in_0]^{-1}[L]^2[T]^{-2}$
(D) $[\mu_0] = [\in_0]^{-1}[L]^{-2}[T]^2$

8. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant k_B, the absolute temperature T, the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles, Which of the following expressions(s) for ℓ is(are) dimensionally correct? **[JEE(Advanced) 2016]**

(A) $\ell = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$
(B) $\ell = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$
(C) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$
(D) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

9. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are) **[JEE(Advanced) 2015]**

(A) $\mu_0 I^2 = \varepsilon_0 V^2$
(B) $\varepsilon_0 I = \mu_0 V$
(C) $I = \varepsilon_0 c V$
(D) $\mu_0 c I = \varepsilon_0 V$

10. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are) :- **[JEE(Advanced) 2015]**

(A) $M \propto \sqrt{c}$
(B) $M \propto \sqrt{G}$
(C) $L \propto \sqrt{h}$
(D) $L \propto \sqrt{G}$

11. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is. **[JEE(Advanced) 2014]**

SOLUTIONS

1. **Ans. (A)**

Sol. $Y = c^{\alpha}h^{\beta}G^{\gamma}$

$$ML^{-1}T^{-2} = (LT^{-1})^{\alpha}(ML^{2}T^{-1})^{\beta}(M^{-1}L^{3}T^{-2})^{\gamma}$$

$1 = \beta - \gamma$ ...(1)

$-1 = \alpha + 2\beta + 3\gamma$ ...(2)

$-2 = -\alpha - \beta - 2\gamma$ ...(3)

$-3 = \beta + \gamma$

$1 = \beta - \gamma$

$-2 = 2\beta$ $\Rightarrow \beta = -1, \gamma = -2$

$-1 = \alpha - 2 - 6$ $\therefore\ \alpha = 7$

2. **Ans. (4)**

Sol. $B = e^{\alpha}\left(m_e\right)^{\beta} h^{\gamma}k^{\delta}$

$$[B] = \left[e^{\alpha}\right]\left[m_e\right]^{\beta}\left[h\right]^{\gamma}\left[k^{\delta}\right]$$

$$\left[M^{1}T^{-2}A^{-1}\right] = [AT]^{\alpha}[m]^{\beta}\left[ML^{2}T^{-1}\right]^{\gamma}\left[ML^{3}A^{-2}T^{-4}\right]^{\delta}$$

$$M^{1}T^{-2}A^{-1} = m^{\beta+\gamma+\delta}L^{2r+3\delta}T^{\alpha-\gamma-4\delta}A^{\alpha-2\delta}$$

Compare : $\beta + \gamma + \delta = 1$; $2\gamma + 3\delta = 0$,

$\alpha - \gamma - 4\delta = -2$, $\alpha - 2\delta = -1$

On solving $\alpha = 3$, $\beta = 2$, $\gamma = -3$, $\delta = 2$

$\alpha + \beta + \gamma + \delta = 4$

3. **Ans. (B, D)**

Sol. $\vec{S} = [\vec{E} \times \vec{B}]\dfrac{1}{\mu_0}$

S is poynting vector denotes flow of energy per unit area per unit time

$\vec{S} = \dfrac{\text{watt}}{m^2}$

Hence B, D are correct

4. **Ans. (A, B)**

Sol. Given,

$L = x^{\alpha}$ ....(1)

$LT^{-1} = x^{\beta}$ ....(2)

$LT^{-2} = x^{p}$ ....(3)

$MLT^{-1} = x^{q}$ ....(4)

$MLT^{-2} = x^{r}$ ....(5)

$\dfrac{(1)}{(2)} \Rightarrow\ T = x^{\alpha-\beta}$

From (3)

$$\frac{x^{\alpha}}{x^{2(\alpha-\beta)}} = x^{p}$$

$\Rightarrow\quad \alpha + p = 2\beta$ (A)

From (4)

$M = x^{q-\beta}$

From (5) $\Rightarrow$ $x^{q} = x^{r}\ x^{\alpha-\beta}$

$\Rightarrow\quad \alpha + r - q = \beta$ ....(6)

Replacing value 'α' in equation (6) from (A)

$2\beta - p + r - q = \beta$

$\Rightarrow\quad p + q - r = \beta$(B)

Replacing value of 'β' in equation (6) from (A)

$2\alpha + 2r - 2q = \alpha + p$

$\alpha = p + 2q - 2r$

5. **Ans. (A, B, D)**

Sol. Mass $= M^{0}L^{0}T^{0}$

$MVr = M^{0}L^{0}T^{0}$

$M^{0}\dfrac{L^{1}}{T^{1}}\ .\ L^{1} = M^{0}L^{0}T^{0}$

$L^{2} = T^{1}$ ....(1)

Force $= M^{1}L^{1}T^{-2}$ (in SI)

$= M^{0}\ L^{1}\ L^{-4}$ (In new system from equation (1))

$= L^{-3}$

Energy $= M^{1}L^{2}T^{-2}$ (In SI)

$= M^{0}L^{2}L^{-4}$ (In new system from equation (1))

$= L^{-2}$

Power $= \dfrac{\text{Energy}}{\text{Time}}$

$= M^{1}L^{2}T^{-3}$ (in SI)

$= M^{0}L^{2}L^{-6}$ (In new system from equation (1))

$= L^{-4}$

Linear momentum $= M^{1}L^{1}T^{-1}$ (in SI)

$= M^{0}L^{1}L^{-2}$ (In new system from equation (1))

$= L^{-1}$

6. **Ans. (C)**

Sol. We have $\dfrac{E}{C} = B$

$\therefore\ [B] = \dfrac{[E]}{[C]} = [E]L^{-1}T^{1}$

$\Rightarrow [E] = [B]\ [L][T^{-1}]$

7. Ans. (D)

Sol. We have,

$$C = \frac{1}{\sqrt{\mu_0 \in_0}}$$

$$\therefore [C^2] = \left[\frac{1}{\mu_0 \in_0}\right]$$

$$\Rightarrow L^2T^{-2} = \frac{1}{[\mu_0][\in_0]} \Rightarrow [\mu_0] = [\in_0]^{-1}[L]^{-2}[T]^2$$

8. Ans. (B, D)

Sol. We know,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \qquad \Rightarrow \frac{q^2}{\varepsilon_0} = (Fr^2)4\pi$$

So, dimension

$$\frac{q^2}{\varepsilon_0} = \dim(Fr^2) = MLT^{-2} \times L^2 = ML^3T^{-2}$$

Similarly; $E = \frac{3}{2}K_BT \Rightarrow$

dim (K_BT) = dim(Energy) = ML^2T^{-2}

(A) $\sqrt{\frac{nq^2}{\varepsilon k_BT}} = \sqrt{\frac{L^{-3} \times ML^3T^{-2}}{ML^2T^{-2}}} = \frac{1}{L}$

(B) $\sqrt{\frac{(E) \times vol}{Fr^2}} = \sqrt{\frac{ML^2T^{-2} \times L^3}{MLT^{-2} \times L^2}} = L$

(C) $\sqrt{\frac{Fr^2(vol)^{2/3}}{(K\varepsilon)}} = \sqrt{\frac{MLT^{-2} \times L^2 \times L^2}{ML^2T^{-2}}} = \sqrt{L^3} = L^{3/2}$

(D) $\sqrt{\frac{Fr^2(vol)^{1/3}}{Energy}} = \sqrt{\frac{MLT^{-2}L^2 \times L}{ML^2T^{-2}}} = L$

[$\therefore$ dimension n = $\dim\left(\frac{1}{vol}\right) = L^{-3}$]

9. Ans. (A, C)

Sol. Using $C = \frac{1}{\sqrt{\mu_0 \in_0}}$ &

$R = \sqrt{\frac{\mu_0}{\in_0}}$ we can check the correctness.

(A) $\mu_0 I^2 = \in_0 V^2$

$$\frac{\mu_0}{\in_0} = \frac{V^2}{I^2} = R^2$$

$\therefore R^2 = R^2$ correct

(B) $\in_0 I = \mu_0 V$

$$\frac{\in_0}{\mu_0} = \frac{V}{I}$$

$\frac{1}{R^2} = R$ not correct

(C) $I = \in_0 cV$

$$\frac{I}{V} = \in_0 c = \frac{\in_0}{\sqrt{\mu_0 \in_0}}$$

$\frac{1}{R} = \sqrt{\frac{\in_0}{\mu_0}} = \frac{1}{R}$ correct

(D) $\mu_0 CI = \in_0 V$

$$\frac{\mu_0 C}{\in_0} = \frac{V}{I}$$

$$\frac{\mu_0}{\in_0}\frac{1}{\sqrt{\mu_0 \in_0}} = R$$

$\therefore \frac{1}{\in_0}\sqrt{\frac{\mu_0}{\in_0}} = R \Rightarrow \frac{R}{\in_0} = R$ incorrect

10. Ans. (A, C, D)

Sol. $[h] = [ML^2T^{-1}]$

$[c] = [LT^{-1}]$

$[G] = [M^{-1}L^3T^{-2}]$

For unit of length

Let $L \propto h^x c^y G^z$

$\Rightarrow$ From principle of homogeneity

[LHS] = [RHS]

$\Rightarrow [M^0LT^0] = [ML^2T^{-1}]^x [LT^{-1}]^y [M^{-1}L^3T^{-2}]^z$

$M^0\,L\,T^0 = M^{x-z}\,L^{2x+y+3z}\,T^{-x-y-2z}$

comparing we get

$x - z = 0$

$2x + y + 3z = 1$

$-x - y - 2z = 0$

Solving we get, $x = \frac{1}{2}, y = \frac{-3}{2}, z = \frac{1}{2}$

$$L = \sqrt{\frac{hG}{c^3}}$$

<u>For unit of mass</u>

Let $M \propto h^x c^y G^z$

Solving in similar manner as above

We get $x = \frac{1}{2}, y = \frac{1}{2}, z = \frac{-1}{2}$

$$M = \sqrt{\frac{hc}{G}}$$

11. Ans. (3)

Sol. $L = (I)^n(d)^y(f)^z$

$L = (M^1L^0T^{-3})^x(M^1L^{-3})^y(T^{-1})^2$

$L = M^{x+y}L^{-3y}T^{-3x-2}$

$-3y = 1 \qquad x + y = 0$

$y = -\dfrac{1}{3} \qquad x - \dfrac{1}{3} = 0$

$x = \dfrac{1}{3}$

$L = (I)^{1/3}(d)^{-1/3}(f)^2$

$n = 3$