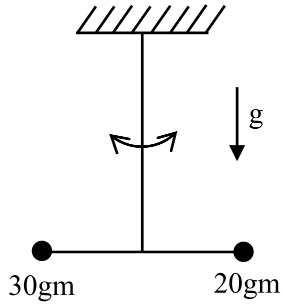
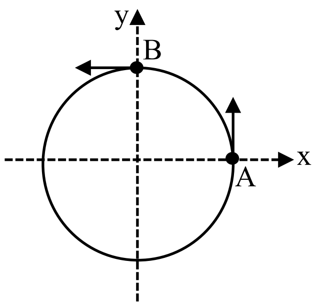
SIMPLE HARMONIC MOTION

1. Two point-like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm. This system is suspended vertically from a rigid ceiling using a thin wire attached to its center of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is 1.2×10^{-8} N m rad^{-1}. The angular frequency of the oscillations is $n \times 10^{-3}$ rad s^{-1}. The value of n is _____. **[JEE(Advanced) 2023]**

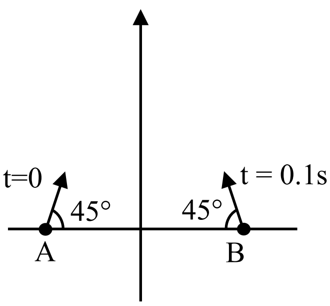


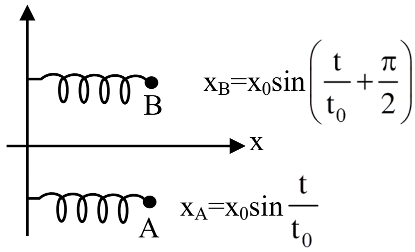
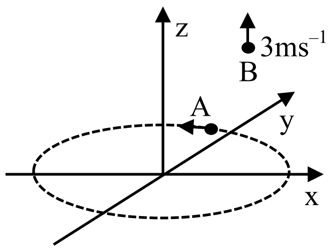
2. List I describes four systems, each with two particles *A* and *B* in relative motion as shown in figure. List II gives possible magnitudes of then relative velocities (in ms^{-1}) at time $t = \frac{\pi}{3}$ s.

	List-I		List-II
(I)	A and B are moving on a horizontal circle of radius 1 m with uniform angular speed $\omega = 1$ rad s^{-1}. The initial angular positions of A and B at time t = 0 are $\theta = 0$ and $\theta = \frac{\pi}{2}$ respectively.	(P)	$\frac{\sqrt{3}+1}{2}$



(II)	Projectiles A and B are fired (in the same vertical plane) at t = 0 and t = 0.1 s respectively, with the same speed $v = \frac{5\pi}{\sqrt{2}}$ m s^{-1} and at 45° from the horizontal plane. The initial separation between A and B is large enough so that they do not collide, (g = 10 m s^{-2}).	(Q)	$\frac{(\sqrt{3}-1)}{\sqrt{2}}$



(III)	Two harmonic oscillators A and B moving in the x direction according to $x_A = x_0 \ \sin\frac{t}{t_0}$ sin and $x_B = x_0 \sin\left(\frac{t}{t_0}+\frac{\pi}{2}\right)$ respectively, starting from $t = 0$. Take $x_0 = 1$ m, $t_0 = 1$ s. 	(R)	$\sqrt{10}$
(IV)	Particle A is rotating in a horizontal circular path of radius 1 m on the xy plane, with constant angular speed $\omega = 1$ rad s^{-1}. Particle B is moving up at a constant speed 3 m s^{-1} in the vertical direction as shown in the figure. (Ignore gravity.) 	(S)	$\sqrt{2}$
		(T)	$\sqrt{25\pi^2+1}$

Which one of the following options is correct? **[JEE(Advanced) 2022]**

(A) I → R, II → T, III → P, IV → S

(B) I → S, II → P, III → Q, IV → R

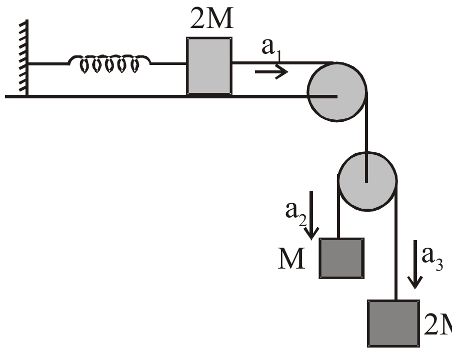
(C) I → S, II → T, III → P, IV → R

(D) I → T, II → P, III → R, IV → S

3. On a frictionless horizontal plane, a bob of mass m = 0.1 kg is attached to a spring with natural length l_0 = 0.1 m. The spring constant is k_1 = 0.009 Nm^{-1} when the length of the spring $l > l_0$ and is k_2 = 0.016 Nm^{-1} when $l < l_0$. Initially the bob is released from l = 0.15 m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is T = (nπ) s, then the integer closest to n is ________.

[JEE(Advanced) 2022]

4. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct?[g is the acceleration due to gravity. Neglect friction]



[JEE(Advanced) 2019]

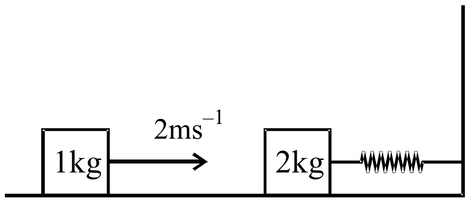
(A) $x_0 = \frac{4Mg}{k}$

(B) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(C) $a_2 - a_1 = a_1 - a_3$

(D) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

5. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _____. **[JEE(Advanced) 2018]**



6. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? **[JEE(Advanced) 2016]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

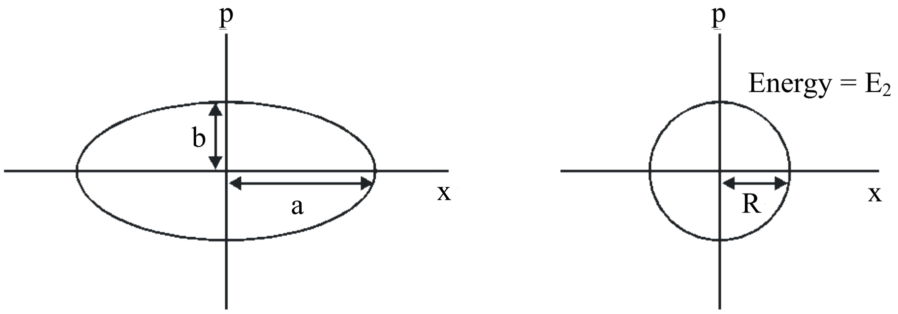
(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

7. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{\alpha}{R} = n$, then the correct equation (s) is (are)

[JEE(Advanced) 2015]



(A) $E_1\omega_1 = E_2\omega_2$

(B) $\frac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\omega_2 = n^2$

(D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

8. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column-II .

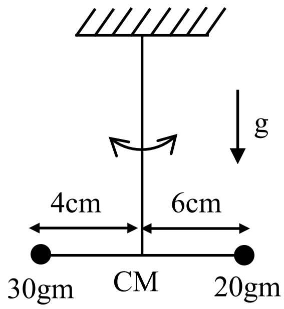
[JEE(Advanced) 2015]

Column-I	Column-II
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at x = a.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at x = 0.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at x = –a.
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards x = 0 in the region $\lvert x\rvert < a$.
	(T) The particle with total enegy $\frac{U_0}{4}$ can oscillate about the point x = –a.

SOLUTIONS

1. Ans. (10)

Sol.



$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}$

$\Rightarrow \omega = \sqrt{\frac{C}{I}}$

Where I = moment of inertia

$I = (30)(4)^2 + (20)(6)^2$

$= 1200 \text{ gm-cm}^2$

$= 1.2 \times 10^{-4} \text{ kg-m}^2$

$\Rightarrow \omega = \sqrt{\frac{1.2\times10^{-8}}{1.2\times10^{-4}}}$

$\Rightarrow \omega = \sqrt{10^{-4}}$

$\omega = (10^{-2})$

$n \times 10^{-3} = 10^{-2} \Rightarrow n = 10$

2. Ans. (C)

Sol. (I) $v_{BA}^2 = v_A^2 + v_B^2 - 2v_{AB}\cos\theta$

As $\omega_A = \omega_B, \theta = 90°$ remains constant.

Also, $v_A = v_B = 1$ m/s

So, $v_{BA} = \sqrt{2}\text{m/s}$

(II) $\vec{u}_A = \frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$

$\vec{v}_A = \frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{2} - 10\cdot\frac{\pi}{3}\right)\hat{j} = \frac{5\pi}{2}\hat{i} - \frac{5\pi}{6}\hat{j}$

$\vec{u}_B = -\frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$

$\vec{u}_B = -\frac{5\pi}{2}\hat{i} - \left(\frac{5\pi}{6} + 1\right)\hat{j}$

$\vec{v}_{B,A} = -5\pi\hat{i} - \hat{j}$

$v_{BA} = \sqrt{25\pi^2 + 1}$

(III) $x_A = \sin t$

$v_A = \cos t = \frac{1}{2}\text{m/s}$

$x_B = \cos t$

$v_B = -\sin t = -\frac{\sqrt{3}}{2}\text{m/s}$

$v_{BA} = -\frac{\sqrt{3}}{2} - \frac{1}{2}$

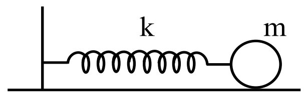
(IV) $\vec{v}_A$ & $\vec{v}_B$ are always perpendicular

So, $|\vec{v}_{BA}| = \sqrt{v_A^2 + v_B^2} = \sqrt{10}\text{m/s}$

Ans. (C), I-S, II-T, III-P, IV-R

3. Ans. (6)

Sol.



$\ell > \ell_0 \rightarrow k = k_1$

$\ell < \ell_0 \rightarrow k = k_2$

Time period of oscillation,

$T = \pi\sqrt{\frac{m}{k_1}} + \pi\sqrt{\frac{m}{k_2}}$

$T = \pi\sqrt{\frac{0.1}{0.009}} + \pi\sqrt{\frac{0.1}{0.016}}$

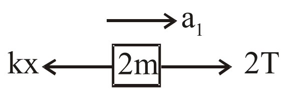
$T = \frac{\pi}{0.3} + \frac{\pi}{0.4} \Rightarrow T = \frac{0.7}{0.12}\pi \Rightarrow T = 5.83\pi$

$T \approx 6\pi$

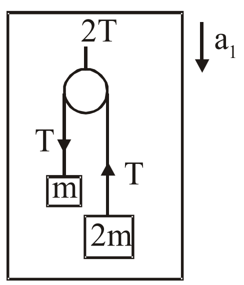
So, n = 6

4. Ans. (C)

Sol.



$2T - kx = 2ma_1$



$T = \frac{2(2m)(m)}{3m}(g - a_1)$

$= \frac{4m}{3}(g - a_1)$

$$\frac{8m}{3}(g-a_1)-kx=2ma_1$$

$$\frac{8Mg}{3}-\frac{8ma_1}{3}-kx=2ma_1$$

$$\frac{8Mg}{3}-kx=\frac{14ma_1}{3}$$

$$\frac{8Mg-3kx}{14m}=a_1$$

$$a_1=\frac{8Mg-3kx}{14m}$$

$$\frac{vdv}{dx}=\left(\frac{8Mg}{14m}-\frac{3kx}{14m}\right)$$

$$\int vdv=\frac{1}{14m}\int(8Mg-3kx)dx$$

for max elongation

$$0=\frac{1}{14m}\int_0^{x_0}(8Mg-3kx)dx$$

$$=\frac{1}{14m}\left(8Mgx_0-\frac{3kx_0^2}{2}\right)$$

$$8Mgx_0=\frac{3kx_0^2}{2}$$

$$\boxed{x_0=\frac{16Mg}{3k}}$$

at $x=\frac{x_0}{2}$

$$\int_0^v vdv=\frac{1}{14m}\int_0^{x_0/2}(8Mg-3kx)dx$$

$$\frac{v^2}{2}=\frac{1}{14m}\left(\frac{8Mgx_0}{2}-\frac{3kx_0^2}{2\times4}\right)$$

$$v^2=\frac{1}{7m}\left(\frac{8Mg}{2}\times\frac{16Mg}{3x}-\frac{3x}{8}\times\frac{16M^2g^2}{3x\times3x}\right)$$

$$=\frac{1}{7m}\left(\frac{64M^2g^2}{3x}-\frac{2M^2g^2}{3x}\right)$$

$$v^2=\frac{62Mg^2}{21k}$$

For acc. $\boxed{2a_1=a_2+a_3}$ therefore

$a_2-a_1=a_1-a_3$

$$a_1=\frac{8Mg-3k\,x_0/4}{14m}=\frac{8g}{14}-\frac{3kx_0}{14m\times4}$$

$$=\frac{8g}{14}-\frac{3x}{14m\times4}\times\frac{16Mg}{3x}=\frac{8g}{14}-\frac{4g}{14}$$

$$=\frac{4g}{14}=\frac{2g}{7}$$

OR

$$\frac{8mg}{3}-\frac{8m}{3}a_1-kx=2ma_1$$

$$\frac{14m}{3}a_1=-k\left[x-\frac{8mg}{3k}\right]$$

$$a_1=-\frac{3k}{14m}\left[x-\frac{8mg}{3k}\right] \qquad ....(i)$$

that means, block 2m (connected with the spring) will perform SHM about $x_1=\frac{8mg}{3k}$ therefore.

maximum elongation in the spring $x_0=2x_1$

$$=\frac{16mg}{3k}$$

on comparing equation (1) with

$a=-\omega^2(x-x_0)$

$$\omega=\sqrt{\frac{3k}{14m}}$$

at $\left(\frac{x_0}{2}\right)$, block will be passing through its mean position therefore at mean position

$$v_0=A\omega=\frac{8mg}{3k}.\sqrt{\frac{3k}{14m}}$$

At, $\frac{x_0}{4}\Rightarrow x=\frac{A}{2}$

$$\therefore a_{cc}=-\frac{A}{2}\omega^2$$

$$=-\frac{4mg}{3k}.\frac{3h}{14m}=-\frac{2g}{7}$$

5. Ans. (2.09)

Sol. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \sec$

block returns to original position in $\frac{T}{2} = \pi \sec$

$d = \frac{2}{3}(\pi) = \frac{2}{3}(3.14) = 2.0933\text{m}$

d = 2.09 m

6. Ans. (A, B, D)

Sol. $T_i = 2\pi\sqrt{\frac{M}{K}}, T_f = 2\pi\sqrt{\frac{M+m}{K}}$

case (i) :

M (Aω) = (M + m)V

$\therefore$ Velocity decreases at equilibrium position.

By energy conservation

$A_f = A_i\sqrt{\frac{M}{M+m}}$

case (ii) :

No energy loss, amplitude remains same

At equilibrium (x_0) velocity = Aω.

In both cases ω decreases so velocity decreases in both cases

7. Ans. (B, D)

Sol. $P_{1\,max} = ma\omega_1 = b$

$P_{2\,max} = mR\omega_2 = R$

$\frac{\omega_1}{\omega_2} = \frac{1}{n^2}$

$\frac{\omega_2}{\omega_1} = n^2$

$E_1 = \frac{1}{2}m\omega_1^2 a^2$

$E_2 = \frac{1}{2}m\omega_2^2 R^2$

$\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2}\frac{a^2}{R^2} = \frac{\omega_1^2}{\omega_2^2}n^2 = \frac{\omega_1^2}{\omega_2^2}\frac{\omega_2}{\omega_1}$

$\frac{E_1}{E_2} = \frac{\omega_1}{\omega_2}$

$\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

8. Ans. (A)-P,Q,R,T; (B)-Q,S; (C)-P,Q,R,S; (D)-P,R,T

Sol. $U_1(x) = \frac{U_0}{2}\left[1-\frac{x^2}{a^2}\right]^2$

$F = -\frac{U_0}{2}2\left[1-\frac{x^2}{a^2}\right]\left[-\frac{2x}{a^2}\right] = \frac{2U_0}{a^4}\left[a^2-x^2\right]x$

$F = \frac{2U_0}{a^4}x(a-x)(a+x)$

F = 0, at x = 0, a, –a

$\left.\begin{array}{l} x=-a, U=0, \\ x=0, U=\frac{U_0}{2} \end{array}\right\} \Rightarrow$ Oscillate when total ME is less then $\frac{U_0}{2}$

A $\rightarrow$ P,Q,R,T

(B) $U_0(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$

$F = -\frac{U_0 x}{a^2}$

F = 0, x = 0

B $\rightarrow$ Q,S

(C) $U_3(x) = \frac{U_0}{2}\left[\left(\frac{x}{a}\right)^2 \exp\left(-\frac{x^2}{a^2}\right)\right]$

$F = -\frac{U_0}{2}\left[\frac{2x}{a^2}\exp\left(-\frac{x^2}{a^2}\right) + \frac{x^2}{a^2}\exp\left(-\frac{x^2}{a^2}\right)\left(-\frac{2x}{a^2}\right)\right]$

$= -\frac{U_0}{2}\frac{2x}{a^4}\exp\left(-\frac{x^2}{a^2}\right)\left[a^2-x^2\right]$

$= \frac{U_0}{a^4}e^{-\frac{x^2}{a^2}}\left[x(x+a)(x-a)\right]$

x = 0, x = –a, x = +a, F = 0

x = 0, U = 0 even function hence minima

C $\rightarrow$ P,Q,R,S

(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{x^3}{3a^3}\right]$

$F = -\frac{U_0}{a}\left[\frac{1}{a} - \frac{x^2}{a^3}\right] = \frac{U_0}{2a^3}\left[x^2-a^2\right]$

x = +a, x = –a, F = 0

x = –a, $U = -\frac{U_0}{3}$, x = +a, $U = +\frac{U_0}{3}$

Hence oscillate about x = –a if T.M.E < $\frac{U_0}{3}$

D $\rightarrow$ P,R,T