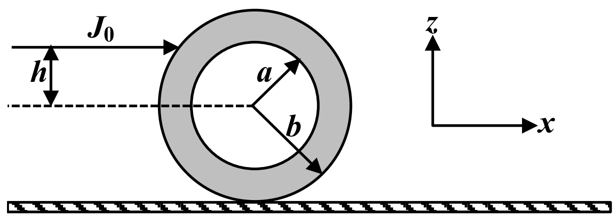
ROTATIONAL MOTION

1. A bar of mas M = 1.00 kg and length L = 0.20 m is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass m = 0.10 kg is moving on the same horizontal surface with 5.00 m s^{-1} speed on a path perpendicular to the bar. It hits the bar at a distance L/2 from the pivoted end and returns back on the same path with speed v. After this elastic collision, the bar rotates with an angular velocity ω. Which of the following statement is correct ? **[JEE(Advanced) 2023]**

(A) ω = 6.98 rad s^{-1} and v = 4.30 m s^{-1} (B) ω = 3.75 rad s^{-1} and v = 4.30 m s^{-1}

(C) ω = 3.75 rad s^{-1} and v = 10.0 m s^{-1} (D) ω = 6.80 rad s^{-1} and v = 4.10 m s^{-1}

2. An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $J_0\hat{x}$ is applied at a height h above the center of the disk. If h = h_m then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct ? **[JEE(Advanced) 2023]**



(A) For μ ≠ 0 and a → 0, h_m = b/2

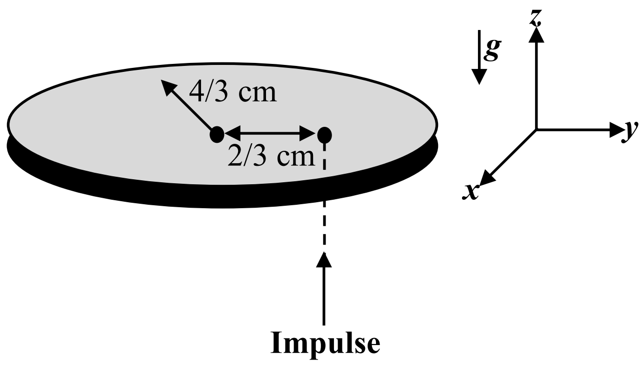
(B) For μ ≠ 0 and a → b, h_m = b

(C) For h = h_m, the initial angular velocity does **not** depend on the inner radius a.

(D) For μ = 0 and h = 0, the wheel always slides without rolling.

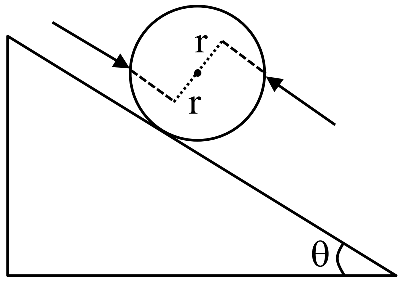
3. A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal xy-plane. The coin is tossed vertically up (+z direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2}$ N-s at a distance 2/3 cm from its center. The coin spins about its diameter and moves along the +z direction. By the time the coin reaches back to its initial position, it completes n rotations. The value of n is ____.

[Given: The acceleration due to gravity g = 10 ms^{-2}] **[JEE(Advanced) 2023]**

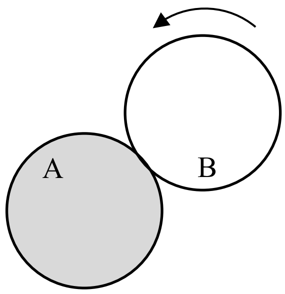


4. At time t = 0, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha = \frac{2}{3}$ rad s^{-2}. A small stone is stuck to the disk. At t = 0, it is at the contact point of the disk and the plane. Later, at time $t = \sqrt{\pi}s$, the stone detaches itself and flies off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is $\frac{1}{2} + \frac{x}{10}$. The value of x is ______. [Take g = 10 m s^{-2}.] **[JEE(Advanced) 2022]**

5. A solid sphere of mass 1 kg and radius 1 m rolls without slipping on a fixed inclined plane with an angle of inclination θ = 30° from the horizontal. Two forces of magnitude 1 N each, parallel to the incline, act on the sphere, both at distance r = 0.5 m from the center of the sphere, as shown in the figure. The acceleration of the sphere down the plane is______ms^{-2}. (Take g = 10 ms^{-2} .) **[JEE(Advanced) 2022]**



6. A flat surface of a thin uniform disk *A* of radius *R* is glued to a horizontal table. Another thin uniform disk *B* of mass *M* and with the same radius *R* rolls without slipping on the circumference of *A*, as shown in the figure. A flat surface of *B* also lies on the plane of the table. The center of mass of *B* has fixed angular speed *ω* about the vertical axis passing through the center of *A*. The angular momentum of *B* is $nM\omega R^2$ with respect to the center of *A*. Which of the following is the value of *n*? **[JEE(Advanced) 2022]**

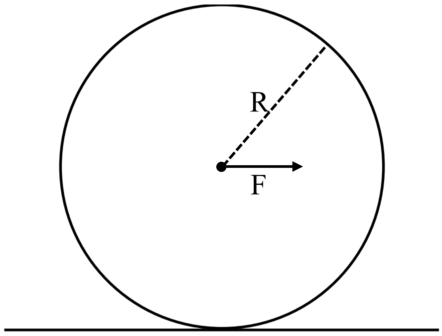


(A) 2 (B) 5 (C) $\frac{7}{2}$ (D) $\frac{9}{2}$

7. A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j})$ kgms^{-2} with k = 1 kgs^{-2}. At time t = 0, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right)$m and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right)$ms^{-1}. Let v_x and v_y denote the x and the y components of the particle's velocity, respectively. **Ignore gravity.** When z = 0.5 m, the value of $(x\,v_y - y\,v_x)$ is_________ m^2s^{-1}.
[JEE(Advanced) 2022]

8. A horizontal force F is applied at the center of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The center of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct ?

[JEE(Advanced) 2021]



(A) For the same F, the value of a does not depend on whether the cylinder is solid or hollow

(B) For a solid cylinder, the maximum possible value of a is 2μg

(C) The magnitude of the frictional force on the object due to the ground is always μmg

(D) For a thin-walled hollow cylinder, $a = \frac{F}{2m}$

9. A particle of mass M = 0.2 kg is initially at rest in the xy-plane at a point (x = –l, y = –h), where l = 10 m and h = 1 m. The particle is accelerated at time t = 0 with a constant acceleration a = 10 m/s^2 along the positive x-direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$, respectively. $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z-directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct ?

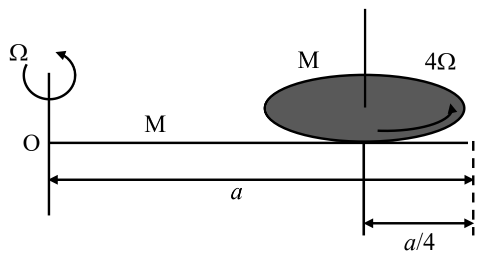
(A) The particle arrives at the point (x = l, y = –h) at time t = 2s. **[JEE(Advanced) 2021]**

(B) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point (x = l, y = –h)

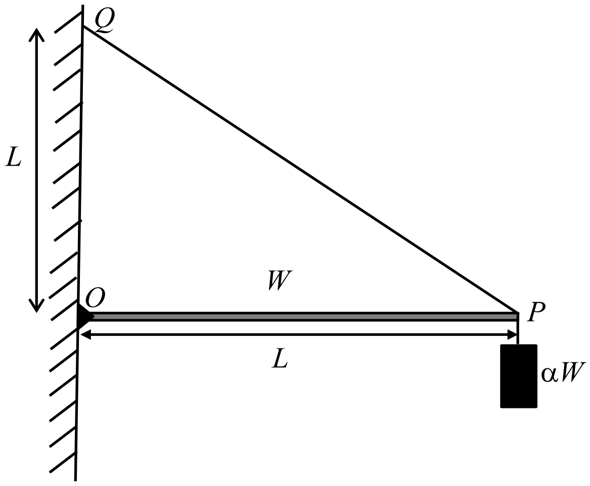
(C) $\vec{L} = 4\hat{k}$ when the particle passes through the point (x = l, y = –h)

(D) $\vec{\tau} = \hat{k}$ when the particle passes through the point (x = 0, y = –h)

10. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius $a/4$ is pivoted on this rod with its center at a distance $a/4$ from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)n$. The value of n is______. **[JEE(Advanced) 2021]**



11. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $(2\sqrt{2})$ W. Which of the following statement(s) is(are) correct ? **[JEE(Advanced) 2021]**



(A) The vertical component of reaction force at O does **not** depend on α

(B) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$

(C) The tension in the rope is 2W for $\alpha = 0.5$

(D) The rope breaks if $\alpha > 1.5$

Question Stem for Question Nos. 12 and 13

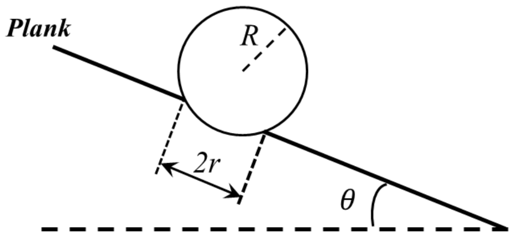
Question Stem

A pendulum consists of a bob of mass m = 0.1 kg and a massless inextensible string of length L = 1.0 m. It is suspended from a fixed point at height H = 0.9 m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse P = 0.2 kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg-m^2/s. The kinetic energy of the pendulum just after the lift-off is K Joules.

12. The value of J is _______. **[JEE(Advanced) 2021]**

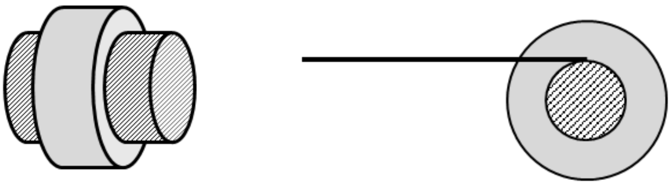
13. The value of K is _______. **[JEE(Advanced) 2021]**

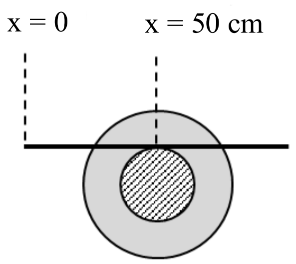
14. A football of radius R is kept on a hole of radius r ($r < R$) made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in the figure below. The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale) - **[JEE(Advanced) 2020]**

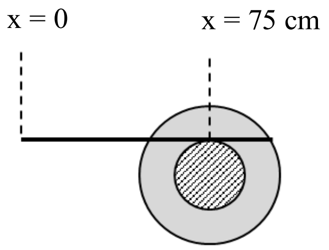


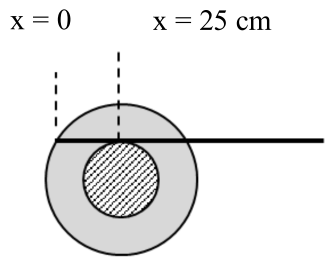
(A) $\sin\theta = \frac{r}{R}$ (B) $\tan\theta = \frac{r}{R}$ (C) $\sin\theta = \frac{r}{2R}$ (D) $\cos\theta = \frac{r}{2R}$

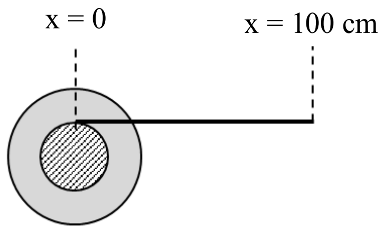
15. A small roller of diameter 20 cm has an axle of diameter 10 cm (see figure below on the left). It is on a horizontal floor and a meter scale is positioned horizontally on its axle with one edge of the scale on top of the axle (see figure on the right). The scale is now pushed slowly on the axle so that it moves without slipping on the axle, and the roller starts rolling without slipping. After the roller has moved 50 cm, the position of the scale will look like (figures are schematic and not drawn to scale)- **[JEE(Advanced) 2020]**



(A) 

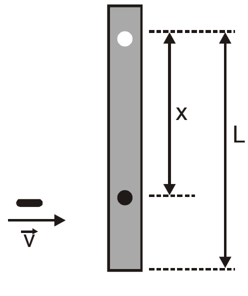
(B) 

(C) 

(D) 

16. Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the center (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is ______. **[JEE(Advanced) 2020]**

17. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_M is achieved for $x = x_M$. Then **[JEE(Advanced) 2020]**



(A) $\omega = \frac{3vx}{L^2 + 3x^2}$ (B) $\omega = \frac{12vx}{L^2 + 12x^2}$ (C) $x_M = \frac{L}{\sqrt{3}}$ (D) $\omega_M = \frac{v}{2L}\sqrt{3}$

18. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60º with vertical ?
[g is the acceleration due to gravity] **[JEE(Advanced) 2019]**

(A) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(B) The angular acceleration of the rod will be $\frac{2g}{L}$

(C) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(D) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

19. The potential energy of a particle of mass m at a distance r from a fixed point O is given by V (r) = $kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ? **[JEE(Advanced) 2018]**

(A) $v=\sqrt{\frac{k}{2m}}R$ (B) $v=\sqrt{\frac{k}{m}}R$ (C) $L=\sqrt{mk}R^2$ (D) $L=\sqrt{\frac{mk}{2}}R^2$

20. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F}=\left(\alpha t\hat{i}+\beta\hat{j}\right)$ is applied on the body, where α = 1.0 Ns^{-1} and β = 1.0 N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true? **[JEE(Advanced) 2018]**

(A) $|\vec{\tau}|=\frac{1}{3}Nm$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1s is $\vec{v}=\frac{1}{2}\left(\hat{i}+2\hat{j}\right)ms^{-1}$

(D) The magnitude of displacement of the body at t = 1s is $\frac{1}{6}m$

21. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $\left(2-\sqrt{3}\right)/\sqrt{10}s$, then the height of the top of the inclined plane, in meters, is _______. (Take g = 10 ms^{-2}) **[JEE(Advanced) 2018]**

22. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq B$. In each case, the force acting on the particle is either zero or conservative. In List–II, five physical quantities of the particle are mentioned; $\vec{p}$ is the linear momentum $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List–I with those quantities in List–II, which are conserved for that path. **[JEE(Advanced) 2018]**

List–I		**List–II**	
P.	$\vec{r}(t) = \alpha t\,\hat{i} + \beta t\,\hat{j}$	1.	$\vec{p}$
Q.	$\vec{r}(t) = \alpha\cos\omega t\,\hat{i} + \beta\sin\omega t\,\hat{j}$	2.	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$	3.	K
S	$\vec{r}(t) = \alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$	4.	U
		5.	E

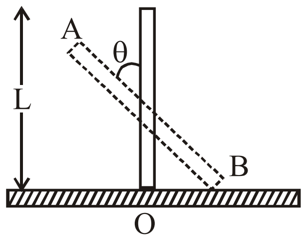
(A) P → 1,2,3,4,5; Q → 2,5; R → 2,3,4,5; S → 5

(B) P → 1,2,3,4,5; Q → 3,5; R → 2,3,4,5; S → 2,5

(C) P → 2,3,4; Q → 5; R → 1,2,4; S → 2,5

(D) P → 1,2,3,5; Q → 2,5; R → 2,3,4,5; S → 2,5

23. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct? **[JEE(Advanced) 2017]**



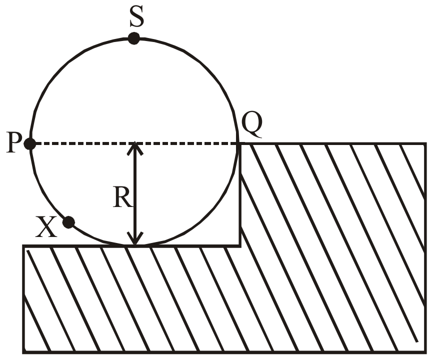
(A) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cosθ)

(B) The midpoint of the bar will fall vertically downward

(C) Instantaneous torque about the point in contact with the floor is proportional to sinθ

(D) The trajectory of the point A is a parabola

24. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct ? **[JEE(Advanced) 2017]**



(A) If the force is applied normal to the circumference at point X then τ is constant

(B) If the force is applied tangentially at point S then τ ≠ 0 but the wheel never climbs the step

(C) If the force is applied normal to the circumference at point P then τ is zero

(D) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

25. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$, where $\alpha = \frac{10}{3} ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle? **[JEE(Advanced) 2016]**

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j}) ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\left(\frac{5}{3}\right)\hat{k}$ Nms

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})N$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\left(\frac{20}{3}\right)\hat{k}$ Nm

Paragraph for Question No. 26 and 27

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

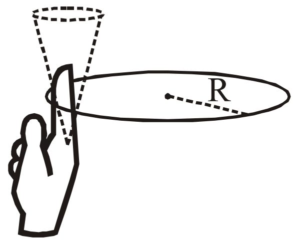


Figure 1

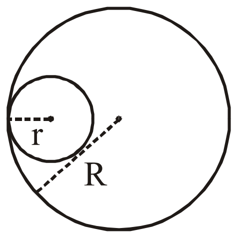


Figure 2

26. The total kinetic energy of the ring is :- **[JEE(Advanced) 2017]**

(A) $M\omega_0^2 R^2$

(B) $M\omega_0^2 (R-r)^2$

(C) $\frac{1}{2} M\omega_0^2 (R-r)^2$

(D) $\frac{3}{2} M\omega_0^2 (R-r)^2$

27. The minimum value of ω_0 below which the ring will drop down is :- **[JEE(Advanced) 2017]**

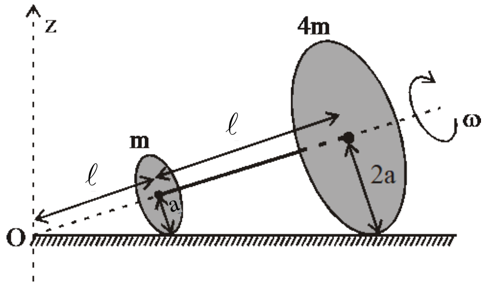
(A) $\sqrt{\frac{3g}{2\mu(R-r)}}$

(B) $\sqrt{\frac{g}{\mu(R-r)}}$

(C) $\sqrt{\frac{2g}{\mu(R-r)}}$

(D) $\sqrt{\frac{g}{2\mu(R-r)}}$

28. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, right rod of length $\ell = \sqrt{24}a$ throught their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true ? **[JEE(Advanced) 2016]**



(A) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(B) The magnitude of the z-component of $\vec{L}$ is $55\ ma^2\omega$

(C) The magnitude of angular momentum of center of mass of the assembly about the point O is $81\ ma^2\omega$

(D) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5

29. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h ($<\ell$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are: ($g = 10\ ms^{-2}$) **[JEE(Advanced) 2016]**

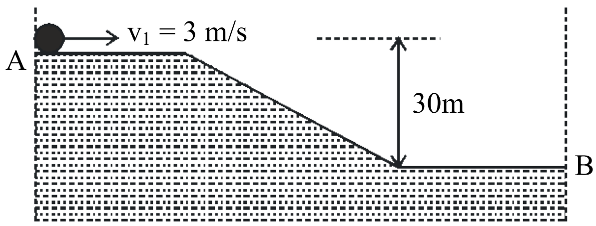
(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

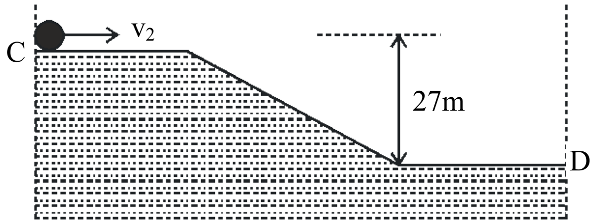
(B) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}N$

(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}N$

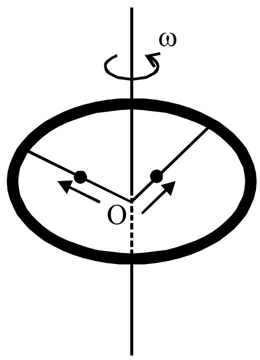
(D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

30. Two identical uniform discs roll without slipping on two different, surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2 respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$m/s, then v_2 in m/s is ($g = 10\ m/s^2$) **[JEE(Advanced) 2015]**





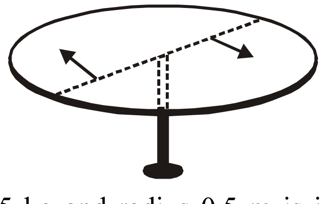
31. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is : **[JEE(Advanced) 2015]**



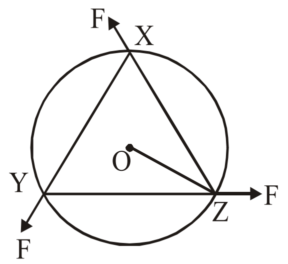
(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

32. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the indivisual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is. **[JEE(Advanced) 2015]**

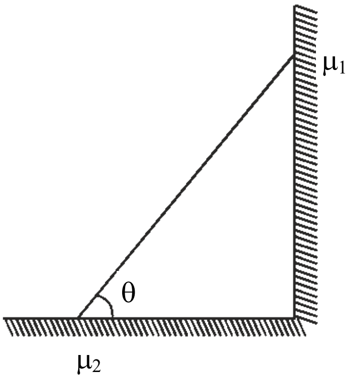
33. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **[JEE(Advanced) 2014]**



34. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is **[JEE(Advanced) 2014]**



35. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then :- **[JEE(Advanced) 2014]**



(A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

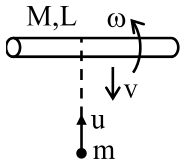
(C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

SOLUTIONS

1. Ans. (A)

Sol.



Applying angular momentum conservation about hinge

$$mv\frac{L}{2}+0=-mv\frac{L}{2}+\frac{ML^2}{3}\omega \qquad \text{....(i)}$$

Also from eq. of restitution

$$e=1=\frac{\omega\frac{L}{2}+V}{u}\Rightarrow u=\omega\frac{L}{2}+V \qquad \text{....(ii)}$$

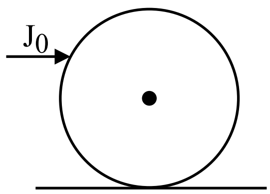
Solving (i) & (ii)

$\omega \approx 6.98$ rad/sec & v = 4.30 m/s

Hence option (A)

2. Ans. (A, B, C, D)

Sol.



$J_0 = mv$ ....(1)

$J_0 h_m = I_c\omega$ ....(2)

$v = \omega R$ ....(3)

$$\Rightarrow h_m=\frac{I_C}{mR}$$

(A) If a = 0 $I_c=\frac{1}{2}mb^2$ & R = b $\therefore$ $h_m=\frac{b}{2}$

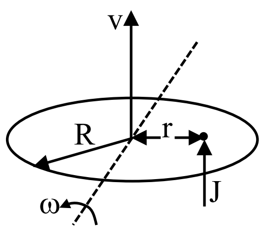
(B) If a = b $I_C = mb^2$ & R = b $\therefore$ $h_m = b$

(C) $v=\frac{J_0}{m}\Rightarrow 100=\frac{V}{R}=\frac{J_0}{mR}$

(D) Force is acting on COM $\therefore$ No rotation.

3. Ans. (30)

Sol.



$J = mv$ ....(1)

$Jr = I_c\omega$ ....(2)

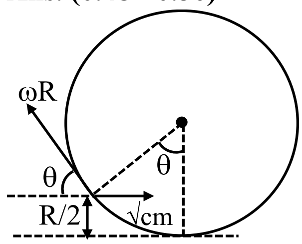
$J_C=\frac{1}{4}mR^2$ ....(3)

$t=\frac{2v}{g}$ ....(4)

$\theta = 2\pi N = \omega t$ ....(5) ($\therefore$ N = 30)

4. Ans. (0.48 - 0.56)

Sol.



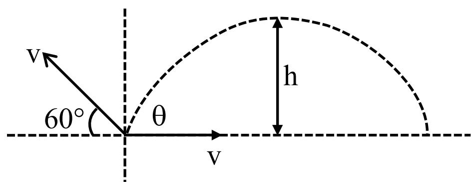
At t = 0, ω = 0

at $t=\sqrt{\pi}$, $\omega=\alpha t=\frac{2}{3}\sqrt{\pi}, v=\omega r=\frac{2}{3}\sqrt{\pi}$

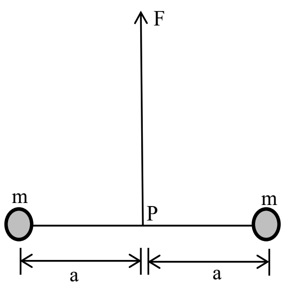
$$\theta=\frac{1}{2}\alpha t^2$$

$$\theta=\frac{1}{2}\times\frac{2}{3}\times\pi=\frac{\pi}{3}$$

$\theta = 60°$



$$v_y = v\sin 60=\frac{\sqrt{3}}{2}V$$



$$h=\frac{u_y^2}{2g}=\frac{\frac{3}{4}v^2}{2g}$$

$$h=\frac{\frac{3}{4}\times\frac{4}{9}\pi}{2g}$$

$$h=\frac{3\pi}{9\times 2g}=\frac{\pi}{6g}$$

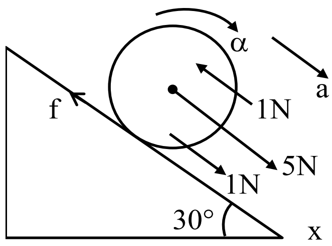
Maximum height from plane, $H=\frac{R}{2}+h$

$$H=\frac{1}{2}+\frac{\pi}{6\times 10}$$

$x=\frac{\pi}{6}$; x = 0.52

5. Ans. (2.80 - 2.92)

Sol. Solid sphere 1kg, 1m



$5 + 1 - 1 - f = 1a$

$5 - f = a$

About COM

$f\,1 - 2(1(0.5)) = \frac{2}{5}Mr^2\alpha$

$\Rightarrow f - 1 = \frac{2}{5}a \Rightarrow f = 1 + \frac{2}{5}a$

$5 - a = 1 + \frac{2}{5}a$

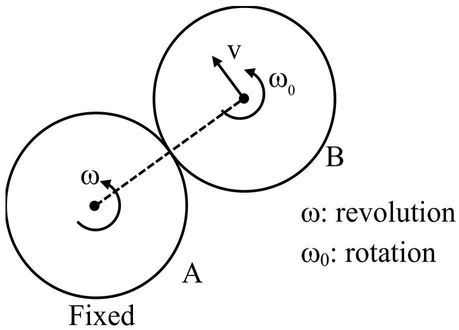
$\Rightarrow 4 = \frac{7a}{5} \quad \Rightarrow a = \frac{20}{7} = 2.86\ m/s^2$

6. Ans. (B)

Sol. $v = \omega\,(2R)$

$v = \omega_0 R$: no slipping

$\therefore\ \omega_0 = 2\omega$



$\vec{L} = m\vec{r} \times \vec{v}_c + I_c\omega_0$

$= M2Rv + \frac{1}{2}MR^2\omega_0$

$= 4MR^2\omega + \frac{1}{2}MR^2(2\omega) = 5MR^2\omega$

$\therefore\ n = 5$

7. Ans. (3)

Sol. Torque about origin is zero. So angular momentum about origin remains conserved.

$$\begin{vmatrix} i & j & k \\ \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & \frac{2}{\pi} \end{vmatrix} = \begin{vmatrix} i & j & k \\ x & y & 0.5 \\ v_x & v_y & \frac{2}{\pi} \end{vmatrix}$$

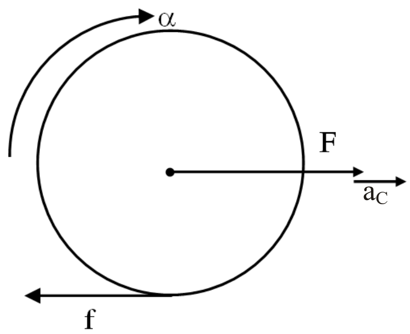
$$\hat{i}\left[\sqrt{2} \times \frac{2}{\pi}\right] - \hat{j}\left[\frac{\sqrt{2}}{\pi}\right] + \hat{k}[1+2]$$

$$i\left[\frac{y \times 2}{\pi} - 0.5v_y\right] - j\left[\frac{x \times 2}{\pi} - 0.5v_x\right] + k\left[xv_y - yv_x\right]$$

$xv_y - yv_x = 3$

8. Ans. (B, D)

Sol.



$F - f = ma_C$

$fR = I_C\alpha$

$a_C - \alpha R = 0$

$F - I_C\frac{\alpha}{R} = ma_C$

$a_C = \frac{F}{\frac{I_C}{R^2} + m}$

$f = \frac{I_C\alpha}{R} = \frac{I_C}{R^2}a_C = \frac{I_C}{R^2}\frac{F}{\left[\frac{I_C}{R^2} + m\right]}$

$f = \frac{F}{\left(m + \frac{I_C}{R^2}\right)}$

Thin walled hollow cylinder

$I_C = mR^2$

$a_C = \frac{F}{2m}$

$$fR = I_C\alpha = \frac{I_C a_C}{R}$$

$$f = \frac{I_C a_C}{R^2} \le \mu mg$$

$$a_C \le \frac{\mu mgR^2}{I_C}$$

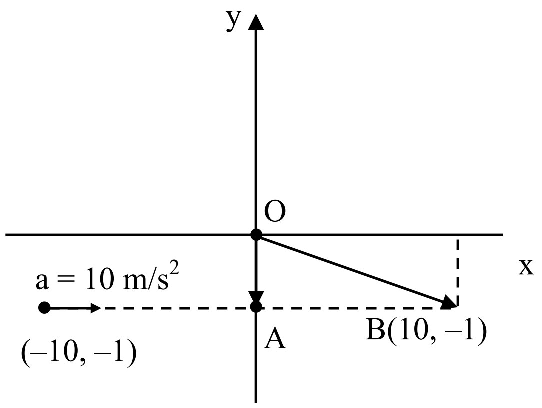
for solid cylinder $I_C = \frac{mR^2}{2}$

$$a_C \le 2\mu g$$

$$(a_C)_{max} = 2\mu g$$

9. Ans. (A, B, C)

Sol.



$$\vec{r}_A = -\hat{j}$$

$$S = \frac{1}{2}at^2$$

$$20 = \frac{1}{2}\times 10\times t^2$$

t = 2 sec

$$\vec{\tau}_0 = \vec{r}\times\vec{F}\ ;\ \vec{r}_B = 10\hat{i}-\hat{j}$$

$$\vec{F} = m\vec{a} = 0.2\times 10\hat{i} = 2\hat{i}$$

$$\vec{\tau}_0 = (10\hat{i}-\hat{j})\times(2\hat{i})$$

$$\vec{\tau}_0 = 2\hat{k}$$

$$\vec{L}_0 = \vec{r}_B\times\vec{p} = \vec{r}_B\times m\vec{v}$$

$$\vec{v} = \vec{a}t = 10\hat{i}\times 2 = 20\hat{i}$$

$$\vec{L}_0 = (0.2)\left[(10\hat{i}-\hat{j})\times 20\hat{i}\right] = 4\hat{k}$$

At point A(0, –1)

$$\vec{\tau}_0 = \vec{r}_A\times\vec{F} = (-\hat{j})\times 2\hat{i} = 2\hat{k}$$

10. Ans. (49)

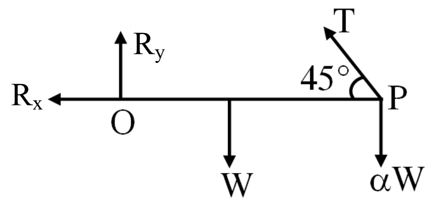
Sol. $$L = \frac{Ma^2}{3}\Omega + M\left(\frac{3a}{4}\right)^2\Omega + \frac{M\left(\frac{a}{4}\right)^2 4\Omega}{2}$$

$$L = \frac{49}{48}Ma^2\Omega$$

n = 49

11. Ans. (A, B, D)

Sol.



$$R_y + \frac{T}{\sqrt{2}} = W + \alpha W \quad ....(i)$$

$$R_x = \frac{T}{\sqrt{2}} \quad ....(ii)$$

Taking torque about 'O'

$$W\frac{\ell}{2} + \alpha W\ell = \frac{T}{\sqrt{2}}\ell$$

$$T = \sqrt{2}\left(\frac{W}{2}+\alpha W\right) \quad ....(iii)$$

$$R_x = \frac{T}{\sqrt{2}} = \left(\frac{W}{2}+\alpha W\right)$$

Taking torque about P

$$R_y\ell = W\frac{\ell}{2}$$

$$R_y = \frac{W}{2};$$

when $T = T_{max}$

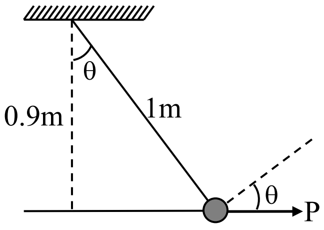
$$2\sqrt{2}W = \sqrt{2}\left(\frac{W}{2}+\alpha W\right)$$

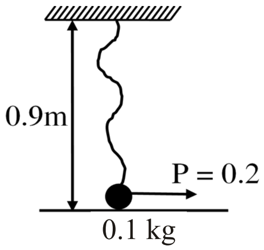
we get $\alpha = \frac{3}{2}$

12. Ans. (0.18)

13. Ans. (0.16)

Sol.





$L = P \times 0.9 = 0.18$ kgm^2/s

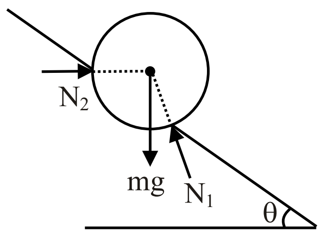
Just after string becomes taut; there will be no velocity along the string.

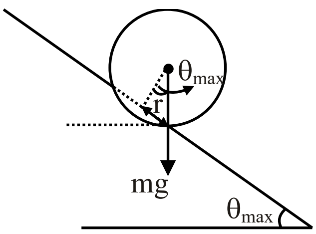
$$\therefore\ V_\perp = \frac{P\cos\theta}{m} = \frac{0.2\times0.9}{1\times0.1} = 1.8\,\text{m/s}$$

$$\therefore\ K = \frac{1}{2}mv_\perp^2 = \frac{1}{2}\times0.1\times1.8^2 = 0.162\,\text{J}$$

14. Ans. (A)

Sol.





For θ_{max}, the football is about to roll, then $N_2 = 0$ and all the forces (Mg and N_1) must pass through contact point

$$\therefore \cos(90° - \theta_{max}) = \left(\frac{r}{R}\right) \Rightarrow \sin\theta_{max} = \left(\frac{r}{R}\right)$$

15. Ans. (B)

Sol. For no slipping at the ground,

$V_{centre} = \omega R$ (R is radius of roller)

$\therefore$ Velocity of scale = $(V_{center} + \omega r)$

[r is radius of axle]

Given, $V_{center} \cdot t = 50$ cm

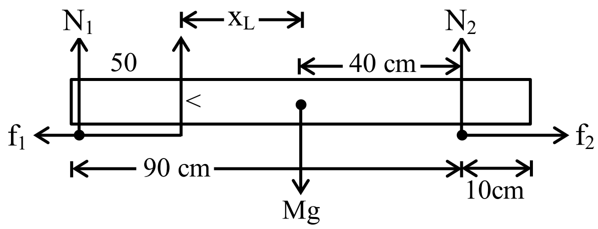
$\therefore$ Distance moved by scale = $(V_{center} + \omega r)t$

$$= \left(V_{center} + \frac{V_{center} r}{R}\right)t = \frac{3V_{center}}{2}\cdot t = 75\text{ cm}$$

Therefore relative displacement (with respect to centre of roller) is (75 – 50) cm = 25 cm

16. Ans. (25.60)

Sol.



Initially

$$N_1 + N_2 = Mg \qquad N_1 = \frac{4Mg}{9}$$

$$(\tau_N = 0)_{\text{about centre}}\ N_1(50) = N_2(40) \qquad N_2 = \frac{5Mg}{9}$$

$$5N_1 = 4N_2$$

$f_{1_K} = \mu_K N_1$ $\qquad f_{1_L} = \mu_S N_1$

$f_{1_K} = 0.32\,N_1$ $\qquad f_{1_L} = 0.4\,N_1$

$f_{2_K} = 0.32\,N_2$ $\qquad f_{2_L} = 0.4\,N_2$

Suppose x_L = distance of left finger from centre when right finger starts moving

$(\tau_n = 0)_{\text{about centre}} \Rightarrow N_1x_L = N_2(40)$

$f_{K_1} = f_{L_2} \quad \Rightarrow 0.32\,N_1 = 0.40\,N_2$

$4N_1 = 5N_2$

$$N_1x_L = \frac{4N_1}{5}(40)$$

$x_L = 32$

Now x_R = distance when right finger stops and left finger starts moving

$(\tau_n = 0)_{\text{about centre}} \Rightarrow N_1x_L = N_2(x_R)$

$f_{L_1} = f_{K_2} \quad \Rightarrow 0.4\,N_1 = 0.32\,N_2$

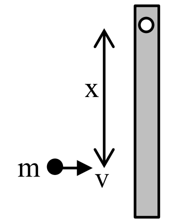
$5N_1 = 4N_2$

$$\frac{4N_2}{5}(32) = N_2x_R$$

$$x_R = \frac{128}{5} = 25.6\text{ cm}$$

17. Ans. (A, C, D)

Sol.



by the angular momentum conservation about the suspension point.

$$mvx = \left(\frac{m\ell^2}{3} + mx^2\right)\omega$$

$$\therefore\ \omega = \frac{mvx}{\frac{m\ell^2}{3} + mx^2} = \frac{2vx}{\ell^2 + 3x}$$

For maximum $\omega \Rightarrow \frac{d\omega}{dx} = 0$

$$\therefore\ x_M = \frac{\ell}{\sqrt{3}}$$

So the $\omega = \frac{V}{2\ell}\sqrt{3}$

18. Ans. (A, C, D)

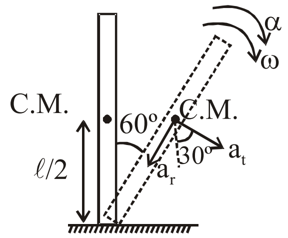
Sol. We can treat contact point as hinged.

Applying work energy theorem

$W_g = \Delta K.E.$

$$mg\frac{\ell}{4} = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{2\ell}}$$



radial acceleration of C.M. of rod

$$= \left(\frac{\ell}{2}\right)\omega^2 = \frac{3g}{4}$$

Using $\tau = I\alpha$ about contact point

$$\frac{mg\ell}{2}\sin 60^\circ = \frac{m\ell^2}{3}\alpha \quad \Rightarrow \alpha = \frac{3\sqrt{3}}{4\ell}g$$

Net vertical acceleration of C.M. of rod

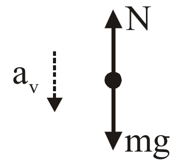
$a_v = a_r \cos 60^\circ + a_t \cos 30^\circ$

$$= \left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha\frac{\ell}{2}\right)\cos 30^\circ$$

$$= \frac{3g}{8} + \frac{3\sqrt{3}g}{4\ell}\left(\frac{\ell}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{3g}{8} + \frac{9g}{16} = \frac{15}{16}g$$

Applying $F_{net} = ma$ in vertical direction on rod as system

$$mg - N = ma_v = m\left(\frac{15}{16}g\right)$$

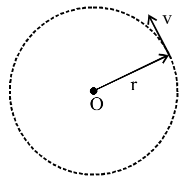


$$\Rightarrow\ N = \frac{mg}{16}$$

19. Ans. (B, C)

Sol. $V = \frac{kr^2}{2}$

$F = -kr$ (towards centre)



$$\left[F = -\frac{dV}{dr}\right]$$

At r = R,

$$kR = \frac{mv^2}{R} \quad \text{[Centripetal force]}$$

$$v = \sqrt{\frac{kR^2}{m}} = \sqrt{\frac{k}{m}}R$$

$$L = m\sqrt{\frac{k}{m}}R^2$$

20. Ans. (A, C)

Sol. $\vec{F} = (\alpha t)\hat{i} + \beta\hat{j}$ [At t = 0, v = 0, $\vec{r} = \vec{0}$]

$\alpha = 1, \beta = 1$

$\vec{F} = t\hat{i} + \hat{j}$

$$m\frac{d\vec{v}}{dt} = t\hat{i} + \hat{j}$$

On integrating

$$m\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j} \qquad [m = 1kg]$$

$$\frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j} \qquad [\vec{r} = \vec{0} \text{ at } t = 0]$$

On integrating

$$\vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$$

At t = 1 sec, $\vec{\tau} = (\vec{r} \times \vec{F}) = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j})$

$$\vec{\tau} = -\frac{1}{3}\hat{k}$$

$$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$$

At t = 1 $\vec{v}=\left(\frac{1}{2}\hat{i}+\hat{j}\right)=\frac{1}{2}\left(\hat{i}+2\hat{j}\right)\text{m/sec}$

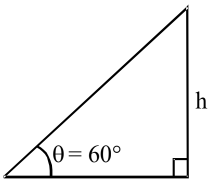
At t = 1 $\vec{s}=\vec{r}_1-\vec{r}_0=\left[\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}\right]-[\vec{0}]$

$$\vec{s}=\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}$$

$$|\vec{s}|=\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{2}\right)^2}\Rightarrow\frac{\sqrt{10}}{6}\text{m}$$

21. Ans. (0.75)

Sol. $a_c=\frac{g\sin\theta}{1+\frac{I_C}{MR^2}}$



$$a_{ring}=\frac{g\sin\theta}{2}$$

$$a_{disc}=\frac{2g\sin\theta}{3}$$

$$\frac{h}{\sin\theta}=\frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2\Rightarrow t_1=\sqrt{\frac{4h}{g\sin^2\theta}}=\sqrt{\frac{16h}{3g}}$$

$$\frac{h}{\sin\theta}=\frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2\Rightarrow t_2=\sqrt{\frac{3h}{g\sin^2\theta}}=\sqrt{\frac{4h}{g}}$$

$$\Rightarrow\sqrt{\frac{16h}{3g}}-\sqrt{\frac{4h}{g}}=\frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\sqrt{h}\left[\frac{4}{\sqrt{3}}-2\right]=2-\sqrt{3}$$

$$\sqrt{h}=\frac{(2-\sqrt{3})\sqrt{3}}{(4-2\sqrt{3})}=\frac{\sqrt{3}}{2}\Rightarrow h=\frac{3}{4}=0.75\text{m}$$

22. Ans. (A)

Sol. (P) $\vec{r}(t)=\alpha t\hat{i}+\beta t\hat{j}$

$$\vec{v}=\frac{d\vec{r}(t)}{dt}=\alpha\hat{i}+\beta\hat{j}\ \{\text{constant}\}$$

$$\vec{a}=\frac{\overrightarrow{dv}}{dt}=0$$

$\vec{P}=m\vec{v}$ (remain constant)

$k=\frac{1}{2}mv^2$ {remain constant}

$$\vec{F}=-\left[\frac{\partial U}{\partial x}\hat{i}+\frac{\partial U}{\partial y}\hat{i}\right]=0$$

$\Rightarrow U\rightarrow$ constant

E = K + U

$$\frac{d\vec{L}}{dt}=\vec{\tau}=\vec{r}\times\vec{F}=0$$

$\vec{L}$ = constant

(Q) $\vec{r}=\alpha\cos(\omega t)\hat{i}+\beta\sin(\omega t)\hat{j}$

$$\vec{v}=\frac{d\vec{r}}{dt}=-\alpha\omega\sin(\omega t)\hat{i}+\beta\omega\cos(\omega t)\hat{j}$$

$$\vec{a}=\frac{d\vec{v}}{dt}=-\alpha\omega^2\cos(\omega t)\hat{i}-\beta\omega^2\sin(\omega t)\hat{j}$$

$$=-\omega^2\left[\alpha\cos(\omega t)\hat{i}+\beta\sin(\omega t)\hat{j}\right]$$

$$\vec{a}=-\omega^2\vec{r}$$

$\vec{\tau}=\vec{r}\times\vec{F}=0$ {$\vec{r}$ and $\vec{F}$ are parallel}

$$\Delta U=-\int\vec{F}.dr=+\int_0^r m\omega^2.r.dr$$

$$\Delta U=m\omega^2\left[\frac{r^2}{2}\right]$$

$U\propto r^2$

$$r=\sqrt{\alpha^2\cos^2(\omega t)+\beta^2\sin^2(\omega t)}$$

r is a function of time (t)

U depends on r hence it will change with time

Total energy remain constant because force is central.

(R) $\vec{r}(t)=\alpha\left(\cos\omega t\hat{i}+\sin(\omega t)\hat{j}\right)$

$$\vec{v}(t)=\frac{d\vec{r}(t)}{dt}=\alpha\left[-\omega\sin(\omega t)\hat{i}+\omega\cos(\omega t)\hat{j}\right]$$

$|\vec{v}|=\alpha\omega$ (Speed remains constant)

$$\vec{a}(t)=\frac{d\vec{v}(t)}{dt}=\alpha\left[-\omega^2\cos(\omega t)\hat{i}-\omega^2\sin(\omega t)\hat{j}\right]$$

$$=-\alpha\omega^2\left[\cos(\omega t)\hat{i}+\sin(\omega t)\hat{j}\right]$$

$$\vec{a}(t)=-\omega^2(\vec{r})$$

$$\vec{\tau}=\vec{F}\times\vec{r}=0$$

$|\vec{r}|=\alpha$ (remain constant)

Force is central in nature and distance from fixed point is constant.

Potential energy remains constant

Kinetic energy is also constant (speed is constant)

(S) $\vec{r} = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \alpha t\,\hat{i} + \beta t\,\hat{j}$

(speed of particle depends on 't')

$\vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}$ {constant}

$\vec{F} = m\vec{a}$ {constant}

$\Delta U = -\int \vec{F}.d\vec{r} = -m\int_0^t \beta\hat{j}.\left(\alpha\hat{i} + \beta t\hat{j}\right)dt$

$U = \frac{-m\beta^2 t^2}{2}$

$k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\alpha^2 + \beta^2 t^2\right)$

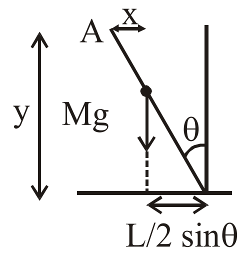
$E = k + U = \frac{1}{2}m\alpha^2$ [remain constant]

23. Ans. (A, B, C)

Sol. When the bar makes an angle θ; the height of its COM (mid point) is $\frac{L}{2}\cos\theta$

$\therefore$ displacement $= L - \frac{L}{2}\cos\theta = \frac{L}{2}(1-\cos\theta)$

Since force on COM is only along the vertical direction, hence COM is falling vertically downward.



Instantaneous torque about point of contact is

$Mg \times \frac{L}{2}\sin\theta$

Now; $x = \frac{L}{2}\sin\theta$

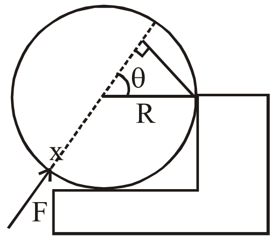
$y = L\cos\theta$

$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1$

Path of A is an ellipse.

24. Ans. (C) or (C, D)

Sol. (A) is incorrect.

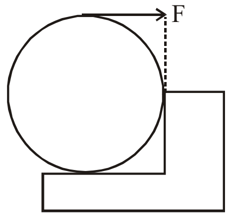


If force is applied normal to surface at point X

$\tau = F_y R \sin\theta$

Thus τ depends on θ & it is not constant

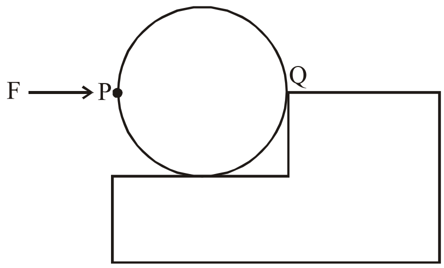
(B) is incorrect



if force applied tangentially at S

$\tau = F \times R \neq 0$

but it will climb as mentioned in question.



If force is applied normal to surface at P then line of action of force will pass from Q & thus $\tau = 0$

25. Ans. (A, B, D)

Sol. $\vec{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2\hat{i} + 2\beta t\hat{j}$

$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 6\alpha t\hat{i} + 2\beta\hat{j}$

At t = 1

(A) $\vec{v} = 3\times\frac{10}{3}\times 1\hat{i} + 2\times\beta = 5\times 1\hat{j}$

$= 10\hat{i} + 10\hat{j}$

(B) $\vec{L} = \vec{r} \times \vec{p}$

$= \left(\frac{10}{3} \times 1\hat{i} + 5 \times 1\hat{j}\right) \times 0.1\left(10\hat{i} + 10\hat{j}\right)$

$= -\frac{5}{3}\hat{k}$

(C) $\vec{F} = m \times \left(6 \times \frac{10}{3} \times 1\hat{i} + 2 \times 5\hat{j}\right) = 2\hat{i} + \hat{j}$

(D) $\vec{\tau} = r \times \vec{F}$

$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times \left(2\hat{i} + \hat{j}\right)$

$= +\frac{10}{3}\hat{k} + 10\left(-\hat{k}\right)$

$= -\frac{20}{3}\hat{k}$

26. Ans. (Bonus)

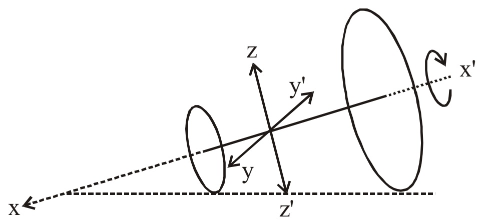
27. Ans. (B)

28. Ans. (A, D or D)

Sol. By no slip condition, here ω' is angular velocity about z axis

$\omega' x = \omega r$

$\omega' = \frac{\omega r}{x} = \omega \sin\theta = \frac{\omega}{5}$



$I_{xx'} = \frac{1}{2}ma^2 + \frac{4m(2a)^2}{2}$

$I_{zz'} = I_{yy'} = \frac{1}{4}ma^2 + \frac{1}{4} \times 4m(2a)^2$

$+m\left(\frac{4\ell}{5}\right) + 4m\left(\frac{\ell}{5}\right)^2$

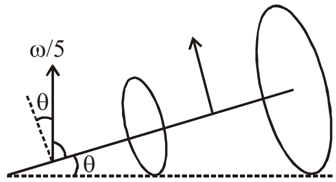
$= \frac{17}{4}ma^2 + \frac{20}{25}m \times 24a^2$

$= \frac{85 + 384}{20}ma^2$

$\omega_{xx'} = \omega - \frac{\omega}{5}\sin\theta = \frac{24\omega}{25}$

$\omega_{zz'} = \frac{\omega}{5}\cos\theta = \frac{\omega\sqrt{24}}{25}$

$\omega_{yy'} = 0$



$L_{cm} = I_{xx'}\,\omega_{xx'} + I_{yy'}\,\omega_{yy'} + I_{zz'}\,\omega_{zz'}$

$= \frac{469ma^2}{100}\sqrt{24}\,\vec{\omega}_{zz'} +$

$\frac{17ma^2}{2} \times \vec{\omega}_{xx'} = \frac{ma^2\omega}{25}\sqrt{\left(\frac{469^2 \times 24}{16} + \frac{289}{4} \times 576\right)}$

$\approx 9.36\ ma^2\omega$

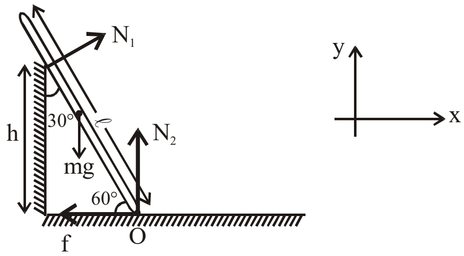
$L_0 = \vec{L}_{cm} + \vec{r} \times \vec{P}$

$= \vec{L}_{cm} + \frac{9\ell}{5} \times 5m \times \frac{9\omega a}{5} \neq 81\ ma^2\omega$

z component is $L_0 \cos\theta \neq 55\ ma^2\omega$

29. Ans. (D)

Sol.



Force equation in x-direction,

$N_1 \cos 30° - f = 0$ ...(i)

Force equation in y-direction,

$N_1 \sin 30° + N_2 - mg = 0$ ...(ii)

Torque equation about O,

$mg\frac{\ell}{2}\cos 60° - N_1\frac{h}{\cos 30°} = 0$ ...(iii)

Also, given $N_1 = N_2$ ...(iv)

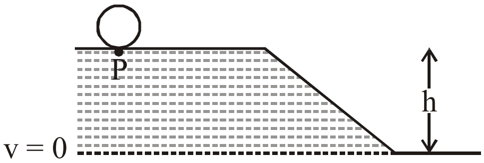
[Note taking reaction from floor as normal reaction only]

solving (i), (ii), (iii) & (iv) we have

$\frac{h}{\ell} = \frac{3\sqrt{3}}{16}$ & $f = \frac{16\sqrt{3}}{3}$

30. Ans. (7)

Sol.



Given final kinetic energy for both discs is same. (suppose K)

so applying conservation of energy

$\frac{1}{2}I_P w_1^2 + mg \times 30 = K$ (for first disc) ...(i)

$\frac{1}{2}I_P w_2^2 + mg \times 27 = K$ (for second disc) ...(ii)

from (i) & (ii)

$\frac{1}{2}\left(\frac{3}{2}mR^2\right)\left(\frac{3}{R}\right)^2 + mg \times 30$

$= \frac{1}{2}\left(\frac{3}{2}mR^2\right)\left(\frac{v_2}{R}\right)^2 + mg \times 27$

on solving

$v_2 = 7$

31. Ans. (C) or (D) or (C, D)

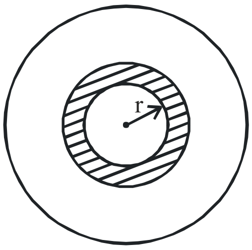
Sol. Using convervation of angular momentum about axis of ring

$$MR^2\omega = MR^2\frac{8}{9}\omega + \frac{M}{8}\left(\frac{9}{25}R^2\right)\left(\frac{8}{9}\omega\right) + \frac{Mr^2}{8}\left(\frac{8}{9}\omega\right)$$

$$r = \frac{4R}{5}$$

32. Ans. (6)

Sol.



Consider a shell of radius r and thickness dr

$dI = \frac{2}{3}(dm)r^2$

$\therefore dI = \frac{2}{3}\left[\left(\frac{kr}{R}\right)4\pi r^2 dr\right]r^2$

$\int dI = \left(\frac{8\pi}{3}\frac{k}{R}\right)\int r^5 dr$

$I_A = \left(\frac{8\pi k}{3R}\right)\left(\frac{R^6}{6}\right)$

$I_A = \left(\frac{8\pi k}{18}\right)R^5$ ....(i)

Similarly

$I_B = \frac{8k\pi}{3R^5}\int_0^R r^9 dr$

$\therefore I_B = \frac{8\pi k}{30}R^5$ ....(ii)

$\therefore \frac{I_B}{I_A} = \frac{6}{10}$ so n = 6

33. Ans. (4)

Sol. Since no external torque acts therefore angular momentum remains conserved.

Angular momentum of ball

= Angular momentum of plateform

$0.05 \times 9 \times 0.25 \times 2 = \frac{1}{2} \times 0.45 \times 0.5 \times 0.5 \times \omega$

$\omega = 4$ rad/s

34. Ans. (2)

Sol. Angular impulse = change in angular momentum

$\tau\Delta t = I\omega$

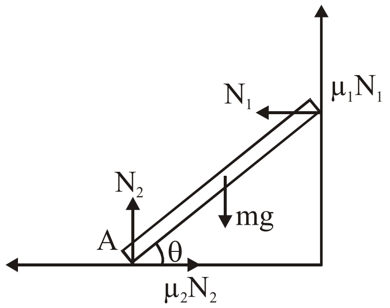
$3 \times F \times R \sin 30 \times \Delta t = I\omega$

$3 \times 0.5 \times 0.5 \times \frac{1}{2} \times 1 = \frac{1}{2} \times 1.5 \times 0.5 \times 0.5 \times \omega$

$\omega = 2$ rad/s

35. Ans. (C, D)

Sol.



(i) If $\mu_2 = 0$ equilibrium is not possible for $0 < \theta < 90$

(ii) $N_1 = \mu_2 N_2$ ...(1)

$\mu_1 N_1 + N_2 = mg$ ...(2)

Torque at A

$$N_1\ell \sin\theta + \mu_1 N_1 \ell \cos\theta = mg\frac{\ell}{2}\cos\theta$$

$$N_1\tan\theta + \mu_1 N_1 = \frac{mg}{2} \quad ...(3)$$

From (3) $\mu_1 = 0, \mu_2 \neq 0$ Ans is (D)

From (1) and (2) $(\mu_1\mu_2 + 1)N_2 = mg$ Ans is (C)