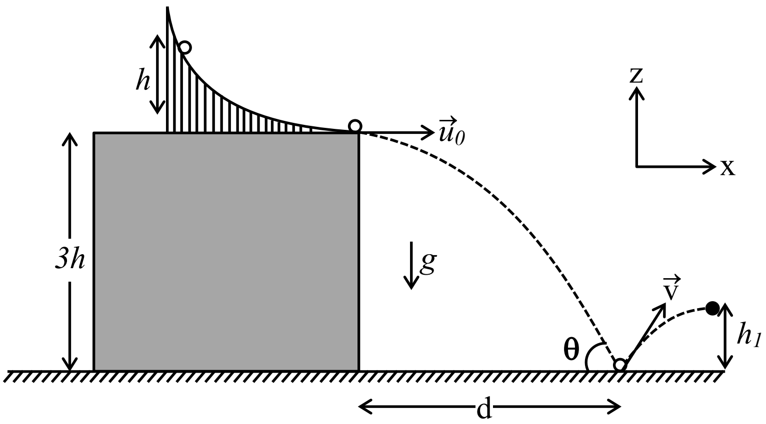
## KINEMATICS

**1.** A slide with a frictionless curved surface, which becomes horizontal at its lower end,, is fixed on the terrace of a building of height 3h from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height $h_1$. The acceleration due to gravity is g and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct? **[JEE(Advanced) 2023]**




(A) $\vec{u}_0 = \sqrt{2gh}\hat{x}$ (B) $\vec{v} = \sqrt{2gh}(\hat{x} - \hat{z})$ (C) θ = 60° (D) $d/h_1 = 2\sqrt{3}$

**2.** A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm $s^{-1}$, the speed of the tip of the person's shadow on the ground with respect to the person is _______ cm $s^{-1}$. **[JEE(Advanced) 2023]**

**3.** A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is d' = nd. The value of n is_______. **[JEE(Advanced) 2022]**

**4.** Starting at time $t = 0$ from the origin with speed 1 $ms^{-1}$, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by $a_x$ and $a_y$, respectively. Then **[JEE(Advanced) 2020]**

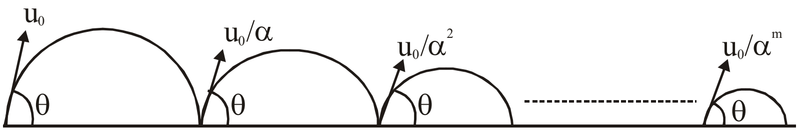
(A) $a_x = 1$ $ms^{-2}$ implies that when the particle is at the origin, $a_y = 1$ $ms^{-2}$

(B) $a_x = 0$ implies $a_y = 1$ $ms^{-2}$ at all times

(C) at $t = 0$, the particle's velocity points in the $x$-direction

(D) $a_x = 0$ implies that at $t = 1$ s, the angle between the particle's velocity and the $x$ axis is 45°

**5.** A ball is thrown from ground at an angle $\theta$ with horizontal and with an initial speed $u_0$. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is $V_1$. After hitting the ground, ball rebounds at the same angle $\theta$ but with a reduced speed of $u_0/\alpha$. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8\ V_1$, the value of $\alpha$ is______. **[JEE(Advanced) 2019]**



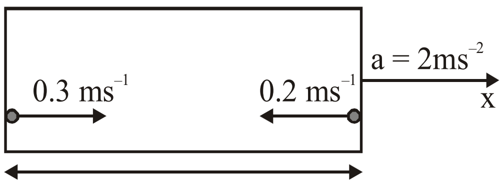

**6.** A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is______.

**[JEE(Advanced) 2018]**

**7.** Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density $\rho$ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to : **[JEE(Advanced) 2017]**

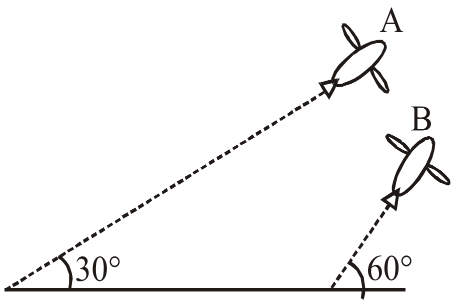
(A) $R^3$ (B) $\frac{1}{R}$ (C) R (D) $R^{2/3}$

**8.** A rocket is moving in a gravity free space with a constant acceleration of 2 $ms^{-2}$ along +x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in +x direction with a speed of 0.3 $ms^{-1}$ relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 $ms^{-1}$ from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[JEE(Advanced) 2014]**




**9.** Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}\ ms^{-1}$. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = $t_0$, A just escapes being hit by B, $t_0$ in seconds is

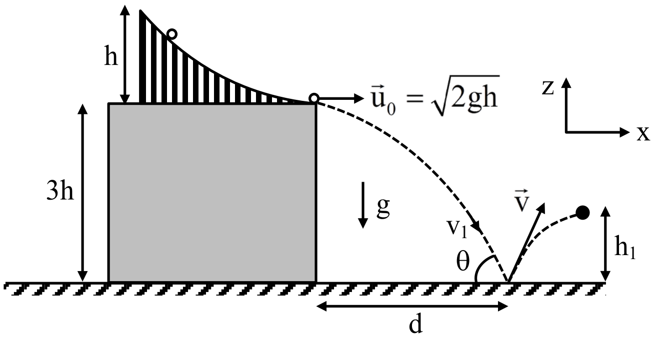
**[JEE(Advanced) 2014]**



## SOLUTIONS

**1. Ans. (A, C, D)**

**Sol.**




$$\vec{v}_1 = \sqrt{2gh}\,\hat{i} - \sqrt{2g3h}\,\hat{k}$$

$$\vec{v} = \sqrt{2gh}\,\hat{i} + \sqrt{2g3h} \times \frac{1}{\sqrt{3}}\hat{k}$$

$$= \sqrt{2gh}\,\hat{i} + \sqrt{2gh}\,\hat{k}$$

$$\tan\theta = \frac{\sqrt{2g3h}}{\sqrt{2gh}} = \sqrt{3} \qquad (\theta = 60°)$$

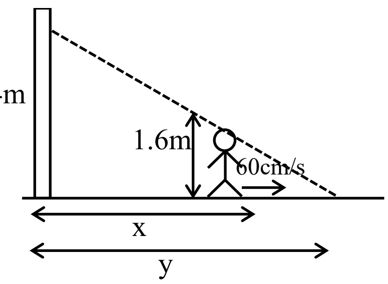
$$h_1 = \frac{v_{1y}^2}{2g} = \frac{2gh}{2g} = h$$

$$d = v_x t = \sqrt{2gh} \times \sqrt{\frac{2\times 3h}{g}}$$

$$= \sqrt{2gh}\sqrt{\frac{6h}{g}} = 2\sqrt{3}h \;=\; \frac{d}{h_1} = 2\sqrt{3}$$

**2. Ans. (40)**

**Sol.**




$$\frac{4}{y} = \frac{1.6}{y-x}$$

$4y - 4x = 1.6y$

$2.4\,y = 4x$

$X = 0.6y$

$$\frac{dx}{dt} = 0.6 \times \frac{dy}{dt}$$

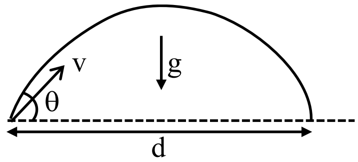
$$60 = 0.6 \times \frac{dy}{dt}$$

$$\therefore \frac{dy}{dt} = 100 \text{ cm/s}$$

Speed of tip of person's

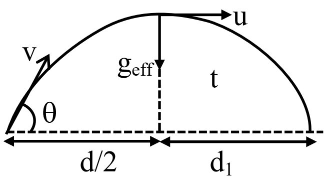
Shadow w.r.t person = 100 – 60 = 40 cm/s

**3. Ans. (0.93 - 0.97)**




**Sol.**

$$d = \frac{v^2 \sin 2\theta}{g}$$




$$H_{max} = \frac{v^2\sin^2\theta}{2g};\ \frac{1}{2}g_{eff}t^2 = H_{max}$$

$$\Rightarrow t^2 = \frac{2H_{max}}{g_{eff}};$$

$$t = \sqrt{\frac{v^2\sin^2\theta \times 0.81}{g^2}};$$

$$t = \frac{0.9v\sin\theta}{g}$$

$$t^2 = \frac{2\times v^2\sin^2\theta}{2g\left(\frac{g}{0.81}\right)}$$

$$d' = \text{New range} = \frac{d}{2} + d_1$$

$d_1 = v\cos\theta t$

$$= \frac{v^2\sin^2\theta\cos\theta \times 0.9}{g};$$

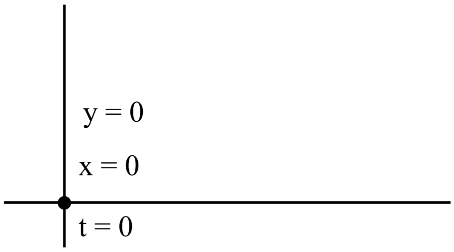
$$d' = \frac{v^2\sin 2\theta}{2g} + \frac{v^2\sin 2\theta \times 0.9}{2g}$$

$$= \frac{v^2\sin 2\theta}{g}\left(\frac{1.0}{2}\right) = 0.95d$$

n = 0.95

**4. Ans. (A, B, C, D)**

**Sol.**




$$y = \frac{x^2}{2}$$

at t = 0, $\left.\begin{matrix} x = 0, y = 0 \\ u = 1 \end{matrix}\right\}$ given

$$y = \frac{x^2}{2}$$

$$\frac{dy}{dt} = \frac{1}{2}.2x\frac{dx}{dt}$$

$\Rightarrow v_y = xv_x$

differentiate wrt time

$$a_y = \frac{dx}{dt}.V_x + xa_x$$

$a_y = v_x^2 + xa_x$

Option

(A) If $a_x = 1$ and particle is at origin

(x = 0, y =0)

$a_y = v_x^2$

$a_y = 1^2 = 1$

At origin, at t = 0 sec

speed = 1 given

(B) Option

$a_y = v_x^2 + xa_x$

given in option B, $a_x = 0$

$\Rightarrow a_y = v_x^2$

If $a_x = 0$, $v_x$ = constant = 1, (all the time)

$\Rightarrow a_y = I^2 = 1$ (all the time)

(C) at t = 0, x = 0 $\quad v_y = xv_x$

speed = 1

$v_y = 0$

$v_x = 1$

(D) $a_y = v_x^2 + xa_x$

$v_y = xv_x$

$a_x = 0$ (given in D option)

$\Rightarrow a_y = v_x^2$

If $a_x = 0 \Rightarrow V_x$ = constant initially ($v_x = 1$)

$\Rightarrow a_y = 1^2 = 1$

at t = 1 sec

$v_y = 0 + a_y \times t = 1 \times 1 = 1$

$$\tan\theta = \frac{v_y}{v_x} = x$$

($\theta \rightarrow$ angle with x axis)

$$\tan\theta = \frac{v_y}{v_x} = \frac{1}{1} = 1$$

$\theta = 45°$

**5. Ans. (4.00)**

**Sol.** $\underset{<v>}{\text{Average velocity}} = \frac{\text{Total displacement}}{\text{Total time}}$

Total time taken = $t_1 + t_2 + t_3 + .............$

$$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + ..........$$

$$\text{Total time} = \frac{t_1}{1 - \frac{1}{\alpha}}$$

Total displacement = $v_1t_1 + v_2t_2 + ...........$

$$= v_1t_1 + \frac{v_1}{\alpha}.\frac{t_1}{\alpha} + ..........$$

$$= \frac{v_1t_1}{1 - \frac{1}{\alpha^2}}$$

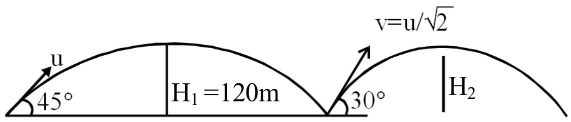
On solving

$$<v> = \frac{v_1\alpha}{\alpha + 1} = 0.8v_1$$

$$\boxed{\alpha = 4.00}$$

**6. Ans. (30.00)**

**Sol.**




$$H_1 = \frac{u^2 \sin^2 45}{2g} = 120$$

$$\Rightarrow \frac{u^2}{4g} = 120 \qquad \text{....(i)}$$

when half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \frac{u^2}{16g} \qquad \text{....(ii)}$$

from (i) & (ii)

$$H_2 = \frac{H_1}{4} = 30 \text{ m on } 30.00$$

**7.** **Ans. (C)**

**Sol.** Density of sphere is $\rho = \frac{m}{v} = \frac{3m}{4\pi R^3}$

$$\Rightarrow \frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{R}\frac{dR}{dt}$$

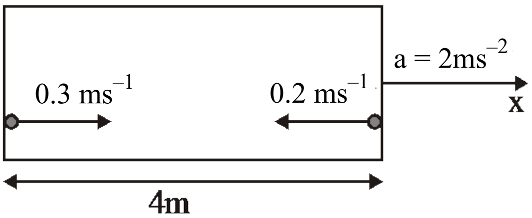
Since $\Rightarrow \frac{1}{\rho}\frac{d\rho}{dt}$ is constant

$$\therefore \frac{dR}{dt} \propto R$$

Velocity of any point on the circumfrence V is equal to $\frac{dR}{dt}$ (rate of change of radius of outer layer).

**8.** **Ans. (2 or 8)**

**Sol.** Assuming open chamber




$V_{relative} = 0.5$ m/s

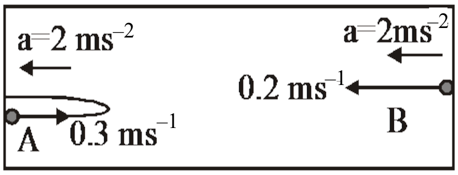
$S_{relative} = 4$m

$$\text{time} = \frac{4}{0.5} = 8 \text{ m/s}$$

**Alternate**

Assuming closed chamber

In the frame of chamber :




Maximum displacement of ball A from its left end is $\frac{u_A^2}{2a} = \frac{(0.3)^2}{2(2)} = 0.0225$ m

This is negligible with respect to the length of chamber i.e. 4m. So, the collision will be very close to the left end.

Hence, time taken by ball B to reach left end will be given by

$$S = u_B t + \frac{1}{2}at^2$$

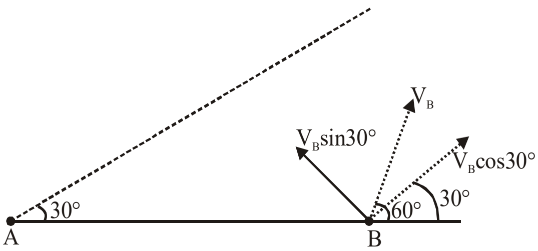
$$4 = (0.2)(t) + \frac{1}{2}(2)(t)^2$$

Solving this, we get

$t \approx 2$ s

**9.** **Ans. (5)**

**Sol.**




As observed from A, B moves perpendicular to line of motion of A. It means velocity of B along A is equal to velocity of A

$V_B \cos 30 = 100\sqrt{3}$

$V_B = 200$

If A is observer A remains stationary therefore

$$t = \frac{500}{V_B \sin 30} = \frac{500}{100} = 5$$