GEOMETRICAL OPTICS

1. A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta = 60°$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{min} = 30°$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct? **[JEE(Advanced) 2023]**

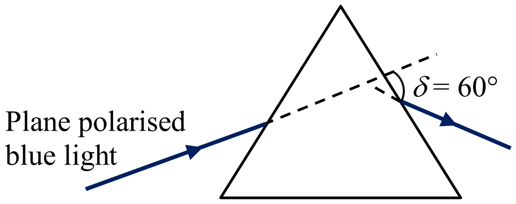


Figure-1

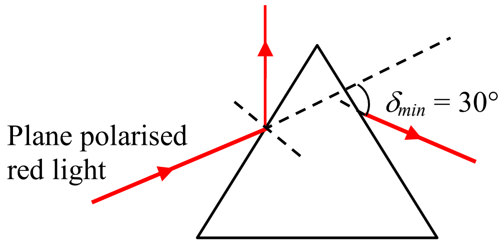


Figure-2

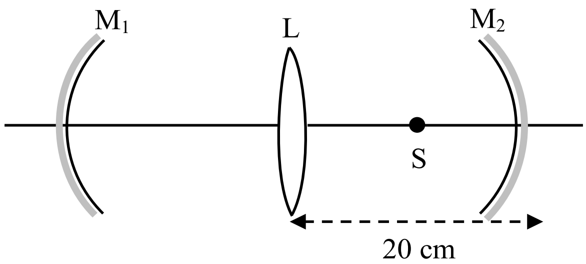
(A) The blue light is polarized in the plane of incidence.

(B) The angle of the prism is 45°.

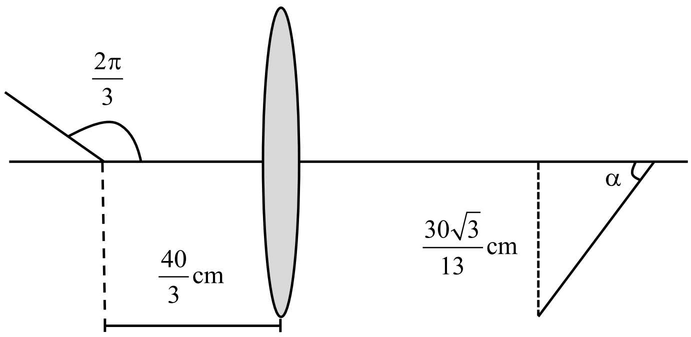
(C) The refractive index of the material of the prism for red light is $\sqrt{2}$.

(D) The angle of refraction for blue light in air at the exit plane of the prism is 60°.

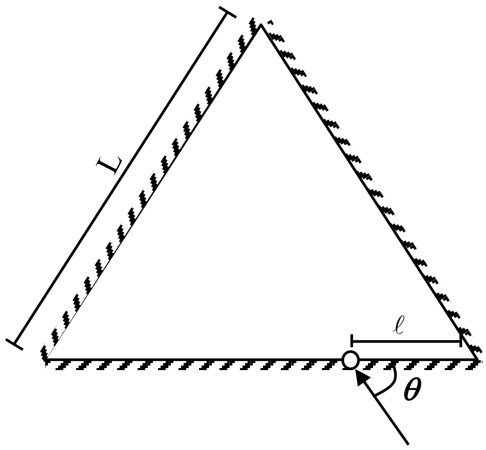
2. An optical arrangement consists of two concave mirrors M_1 and M_2, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of M_1 and M_2 are 20 cm and 24 cm, respectively. The distance between L and M_2 is 20 cm. A point object S is placed at the mid-point between L and M_2 on the axis. When the distance between L and M_1 is n/7 cm, one of the images coincides with S. The value of n is _________. **[JEE(Advanced) 2023]**



3. A rod of length 2 cm makes an angle $\frac{2\pi}{3}$ rad with the principal axis of a thin convex lens. The lens has a focal length of 10 cm and is placed at a distance of $\frac{40}{3}$ cm from the object as shown in the figure. The height of the image is $\frac{30\sqrt{3}}{13}$ cm and the angle made by it with respect to the principal axis is α rad. The value of α is $\frac{\pi}{n}$ rad, where n is _________. **[JEE(Advanced) 2022]**



4. Three plane mirrors form an equilateral triangle with each side of length L. There is a small hole at a distance $l > 0$ from one of the corners as shown in the figure. A ray of light is passed through the hole at an angle θ and can only come out through the same hole. The cross section of the mirror configuration and the ray of light lie on the same plane. **[JEE(Advanced) 2022]**



Which of the following statement(s) is(are) correct?

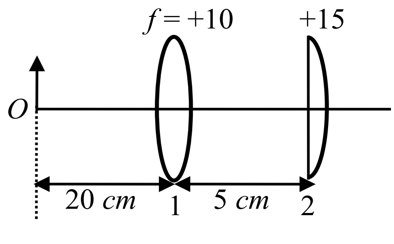
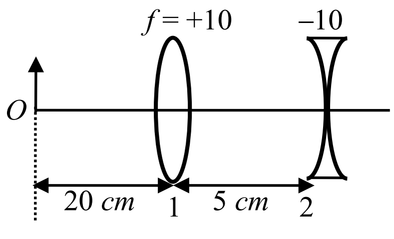
(A) The ray of light will come out for θ = 30°, for $0 < l < L$.

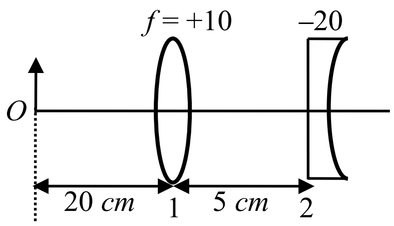
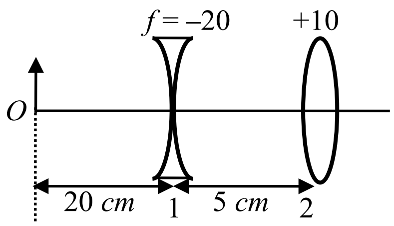
(B) There is an angle for $l = \frac{L}{2}$ at which the ray of light will come out after two reflections.

(C) The ray of light will **NEVER** come out for θ = 60° and $l = \frac{L}{3}$.

(D) The ray of light will come out for θ = 60°, and $0 < l < \frac{L}{2}$ after six reflections.

5. List-I contains four combinations of two lenses (1 and 2) whose focal lengths (in cm) are indicated in the figures. In all cases, the object is placed 20 *cm* from the first lens on the left, and the distance between the two lenses is 5 cm. List-II contains the positions of the final images. **[JEE(Advanced) 2022]**

List-I		**List-II**	
(I)		(P)	Final image is farmed at 7.5 *cm* on the right side of lens 2.
(II)		(Q)	Final image is formed at 60.0 cm on the right side of lens 2.

(III)		(R)	Final image is formed at 30.0 *cm* on the left side of lens 2.
(IV)		(S)	Final image is formed at 6.0 *cm* on the right side of lens 2.
		(T)	Final image is formed at 30.0 *cm* on the right side of lens 2.

Which one of the following options is correct ?

(A) I → P, II → R, III → Q, IV → T

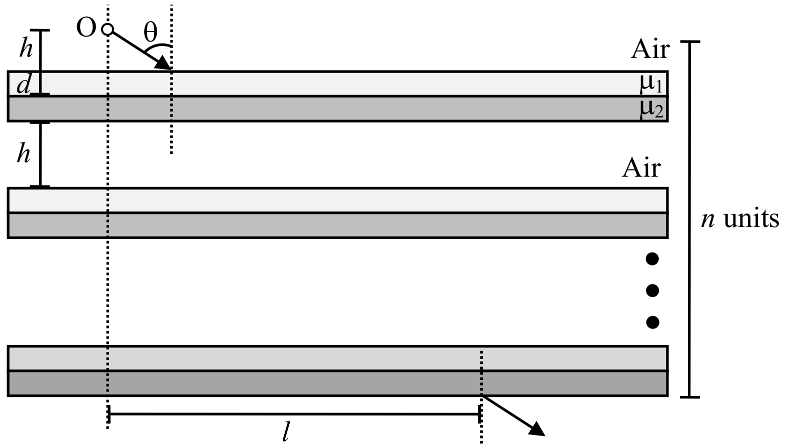
(B) I → Q, II → P, III → T, IV → S

(C) I → P, II → T, III → R, IV → Q

(D) I → T, II → S, III → Q, IV → R

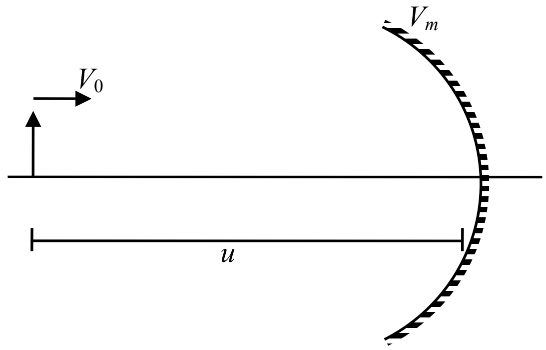
6. Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height h = $\frac{1}{3}$ cm, and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2}$ cm , and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of θ = 60° to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}}$ cm, as shown in the figure. The value of n is ________.

[JEE(Advanced) 2022]

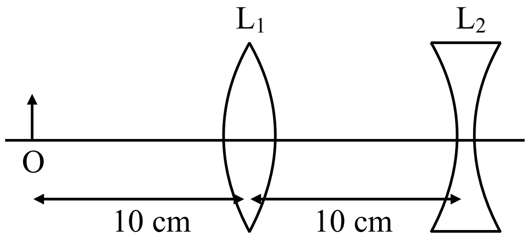


7. An object and a concave mirror of focal length f = 10 cm both move along the principal axis of the mirror with constant speeds. The object moves with speed V_0 = 15 cm s^{-1} towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u. When u = 30 cm, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is ________ cm s^{-1}.

[JEE(Advanced) 2022]

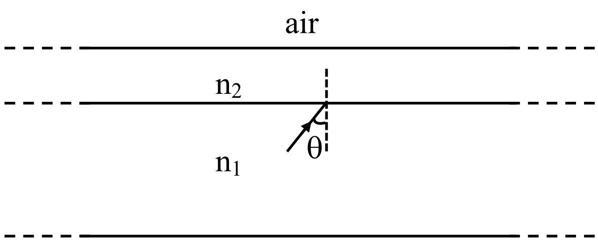


8. An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is **[JEE(Advanced) 2021]**



(A) 0.4 (B) 0.8 (C) 1.3 (D) 1.6

9. A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where sin θ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is(are) correct? **[JEE(Advanced) 2021]**



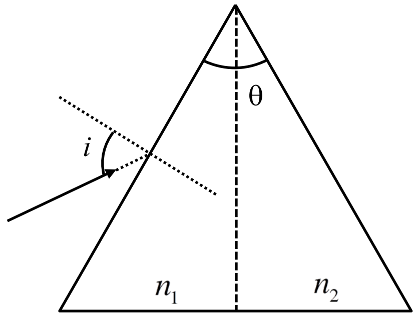
(A) The light ray enters air if $n_2 = n_1$

(B) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$

(C) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$

(D) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

10. For a prism of prism angle θ = 60º, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence i is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2$ = n = 1.5. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where Δn << n), the angle of emergence e = i + Δe. Which of the following statement(s) is (are) correct ? **[JEE(Advanced) 2021]**



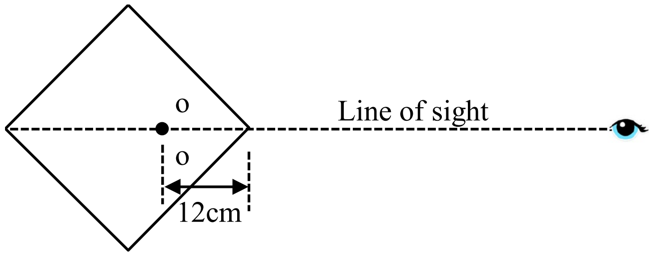
(A) The value of Δe (in radians) is greater than that of Δn

(B) Δe is proportional to Δn

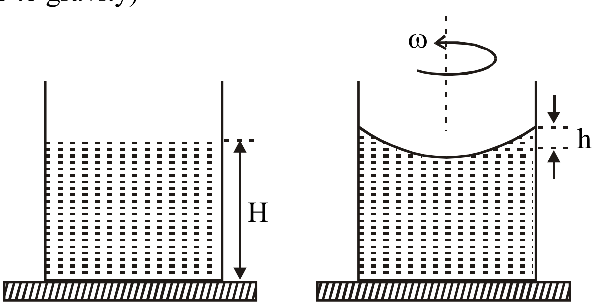
(C) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

(D) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

11. A large square container with thin transparent vertical walls and filled with water (refractive index $\frac{4}{3}$) is kept on a horizontal table. A student holds a thin straight wire vertically inside the water 12 cm from one of its corners, as shown schematically in the figure. Looking at the wire from this corner, another student sees two images of the wire, located symmetrically on each side of the line of sight as shown. The separation (in cm) between these images is_________. **[JEE(Advanced) 2020]**



12. A beaker of radius r is filled with water (refractive inde x $\frac{4}{3}$) up to a height H as shown in the figure on the left. The beaker is kept on a horizontal table rotating with angular speed ω. This makes the water surface curved so that the difference in the height of water level at the center and at the circumference of the beaker is h (h ≪ H, h ≪ r), as shown in the figure on the right. Take this surface to be approximately spherical with a radius of curvature R. Which of the following is/are correct ?
(g is the acceleration due to gravity) **[JEE(Advanced) 2020]**



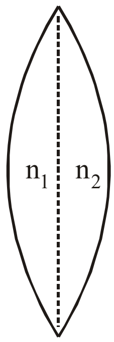
(A) $R = \frac{h^2 + r^2}{2h}$

(B) $R = \frac{3r^2}{2h}$

(C) Apparent depth of the bottom of the beaker is close to $\frac{3H}{2}\left(1 + \frac{\omega^2 H}{2g}\right)^{-1}$

(D) Apparent depth of the bottom of the beaker is close to $\frac{3H}{4}\left(1 + \frac{\omega^2 H}{4g}\right)^{-1}$

13. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right *spherical* surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are : **[JEE(Advanced) 2019]**



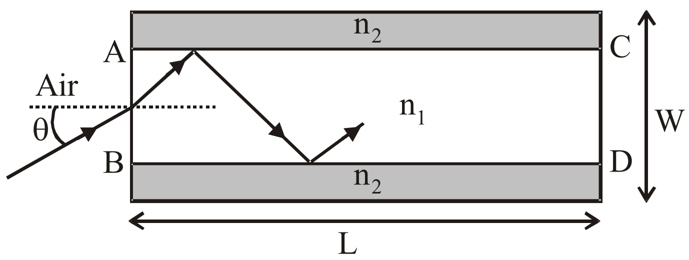
(A) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

(B) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

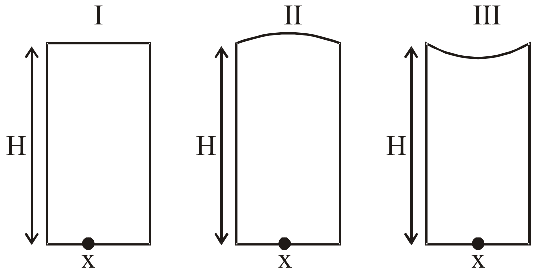
(C) For n = 1.5, $\Delta n = 10^{-3}$ and f = 20 cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place)

(D) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

14. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is _______. [Speed of light $c = 3 \times 10^8$ m/s] **[JEE(Advanced) 2019]**



15. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surfaces shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are **[JEE(Advanced) 2019]**



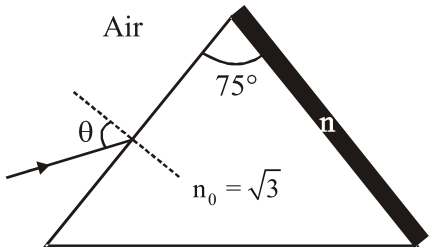
(A) $H_3 > H_1$

(B) 0.8 cm < $(H_2 - H_1)$ < 0.9 cm

(C) $H_2 > H_3$

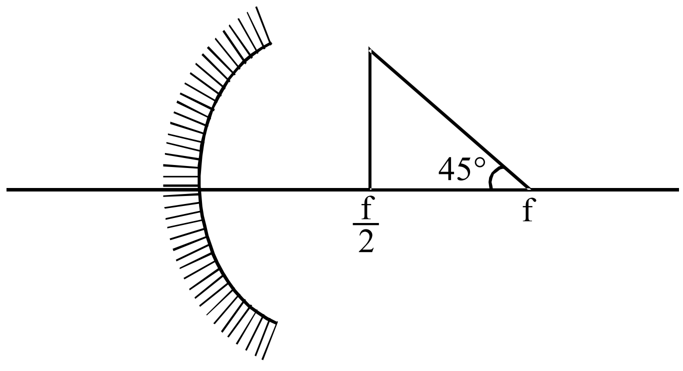
(D) $H_2 > H_1$

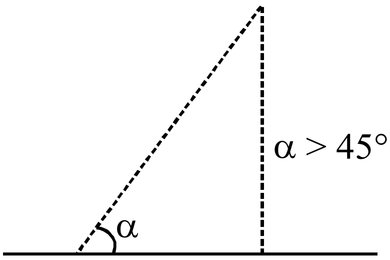
16. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of a prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is_________. **[JEE(Advanced) 2019]**

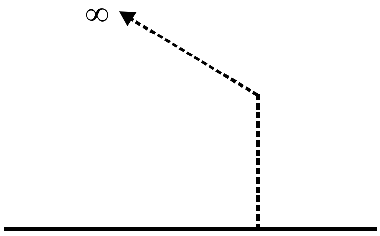


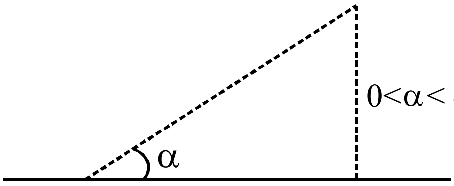
17. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is _________. **[JEE(Advanced) 2018]**

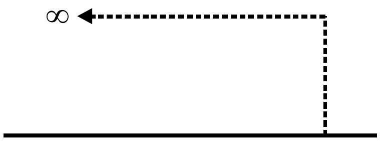
18. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire ? (These figures are not to scale.) ? **[JEE(Advanced) 2018]**



(A) 

(B) 

(C) 

(D) 

19. For an isosceles prism of angle A and *refractive* index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct ? **[JEE(Advanced) 2017]**

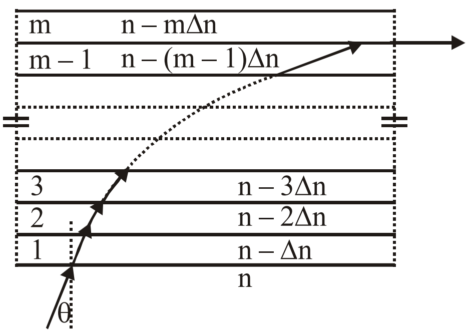
(A) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(B) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

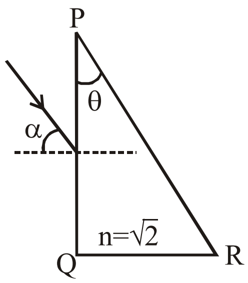
(C) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1}-\cos A\right]$

(D) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

20. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle θ = 30°. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m ? **[JEE(Advanced) 2017]**



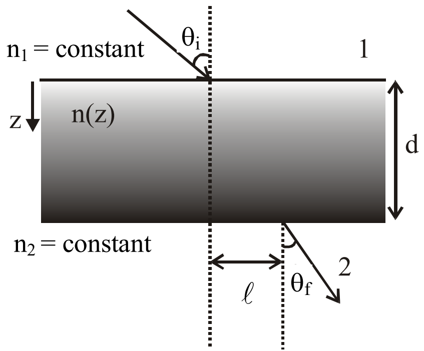
21. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is : **[JEE(Advanced) 2016]**



(A) 15° (B) 22.5° (C) 30° (D) 45°

22. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 (> n_1), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement ℓ.

Which of the following statement(s) is(are) true ? **[JEE(Advanced) 2016]**



(A) ℓ is independent of n_2 (B) ℓ is dependent on n(z)

(C) $n_1\sin\theta_i = (n_2 - n_1)\sin\theta_f$ (D) $n_1\sin\theta_i = n_2\sin\theta_f$

23. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true? **[JEE(Advanced) 2016]**

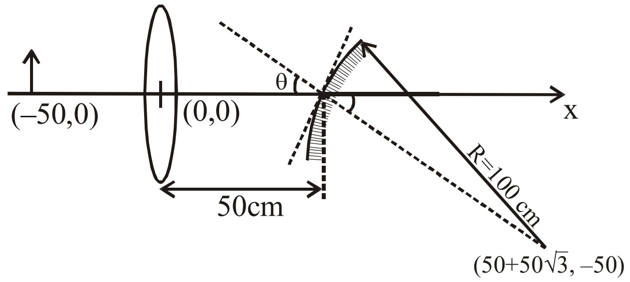
(A) The refractive index of the lens is 2.5

(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

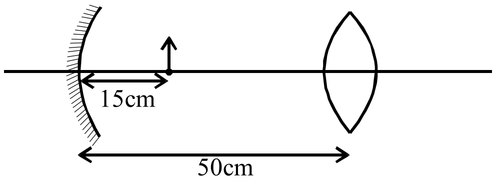
(D) The focal length of the lens is 20 cm.

24. A small object is placed 50 cm to the left of thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure. If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are : **[JEE(Advanced) 2016]**

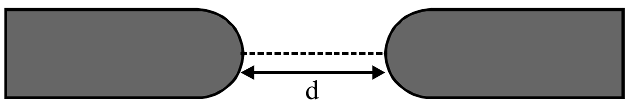


(A) $(25, 25\sqrt{3})$ (B) $\left(\frac{125}{3}, \frac{25}{\sqrt{3}}\right)$ (C) $(50-25\sqrt{3}, 25)$ (D) (0, 0)

25. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6 the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is. **[JEE(Advanced) 2015]**

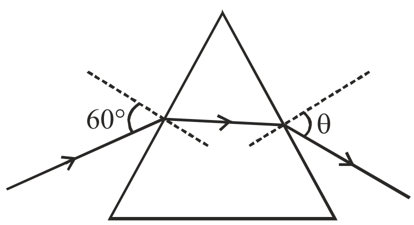


26. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashded line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is : **[JEE(Advanced) 2015]**



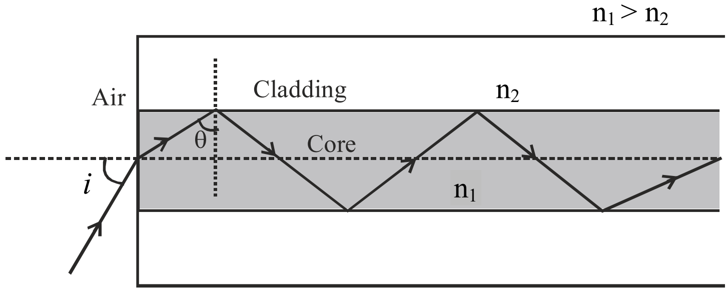
(A) 60 cm (B) 70 cm (C) 80 cm (D) 90 cm

27. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle θ(n) with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is **[JEE(Advanced) 2015]**



Paragraph for Question No. 28 and 29

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value of i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m.



28. For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is (are)

[JEE(Advanced) 2015]

(A) NA of S_1 immersed in water is the same as that of S_2 immersed in liquid of refractive index $\frac{16}{3\sqrt{15}}$.

(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water.

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$.

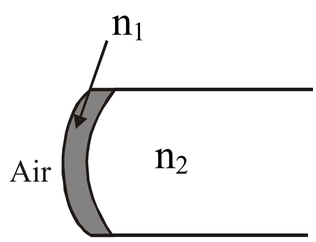
(D) NA of S_1 placed in air is the same as that of S_2 placed in water.

29. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

[JEE(Advanced) 2015]

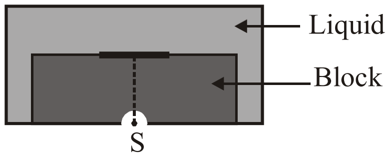
(A) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (B) $NA_1 + NA_2$ (C) NA_1 (D) NA_2

30. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then **[JEE(Advanced) 2014]**



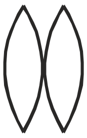
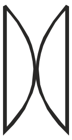
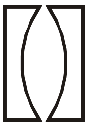
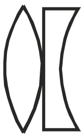
(A) $|f_1| = 3R$ (B) $|f_1| = 2.8\,R$ (C) $|f_2| = 2R$ (D) $|f_2| = 1.4\,R$

31. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is :- **[JEE(Advanced) 2014]**



(A) 1.21 (B) 1.30 (C) 1.36 (D) 1.42

32. Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists. **[JEE(Advanced) 2014]**

List–I	List–II
(P) 	(1) 2r
(Q) 	(2) $\frac{r}{2}$
(R) 	(3) –r
(S) 	(4) r

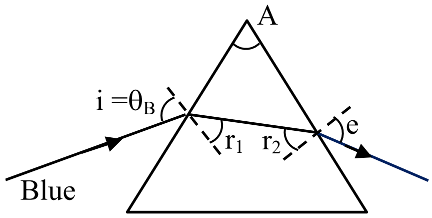
Code :

(A) P-1, Q-2, R-3, S-4
(B) P-2, Q-4, R-3, S-1
(C) P-4, Q-1, R-2, S-3
(D) P-2, Q-1, R-3, S-4

SOLUTIONS

1. Ans. (A, C, D)

Sol.



$\tan\theta_B = \mu_B = \sqrt{3}$

$i = \theta_B = 60°$

$1\sin 60° = \sqrt{3}\sin r_1$

$r_1 = 30°$

$r_1 + r_2 = A$

$\delta = (i + e) - A$

$60° = 60° + e - A$

$e = A$

$\sqrt{3}\sin r_2 = 1\sin e$

$\sqrt{3}\sin(A - 30) = \sin A$

Solving

$A = 60°$

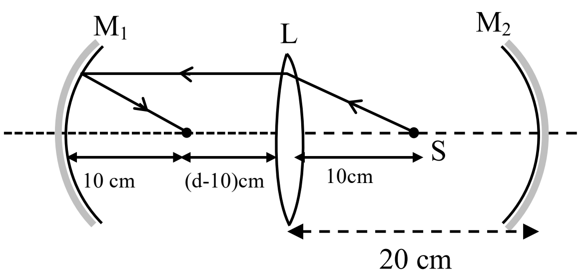
$\therefore\ e = 60°$

For red light

$$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}} = \sqrt{2}$$

2. Ans. (80 or 150 or 220)

Sol.



Two cases are possible if Ist refraction on lens:

Since object is at focus ⇒ light will become parallel.

Ist reflection at M_1 :-

Light is parallel ⇒ Image will be at focus.

IInd refraction from L :-

$u = -(d - 10)$

$f = 10$ cm

$$\frac{1}{v} - \frac{1}{\mu} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{d-10} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{(d-10)} \quad \dots (i)$$

This v will be object for M_2, and image should be at 10 cm

$$\frac{1}{\mu} + \frac{1}{v_1} = \frac{1}{f}$$

$$-\frac{1}{(20-v)} - \frac{1}{10} = -\frac{1}{12}$$

$$\frac{1}{12} - \frac{1}{10} = \frac{1}{20-v}$$

$$-\frac{2}{120} = \frac{1}{20-v}$$

$20 - v = -60$

$v = 80$ cm

From equation (i)

$$\frac{1}{80} = \frac{1}{10} - \frac{1}{d-10}$$

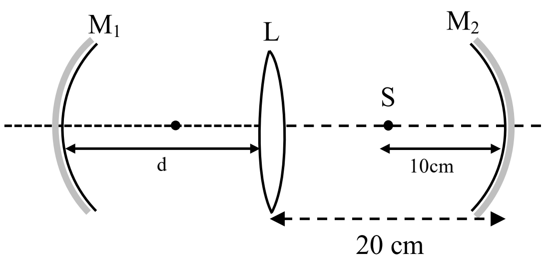
$$\frac{1}{d-10} = \frac{1}{10} - \frac{1}{80}$$

$$\frac{1}{d-10} = \frac{80-10}{800} = \frac{70}{800}$$

$$d - 10 = \frac{80}{7} \Rightarrow d = 10 + \frac{80}{7} = \frac{150}{7}$$

$\boxed{n = 150}$

Case-2: If 1st reflection on mirror m_2



For m_2

$$\frac{1}{V_1} + \frac{1}{-10} = \frac{1}{-12}$$

$V_1 = 60$ cm

Then refraction on lens L

$u_2 = -80$ cm

$$\frac{1}{V_2}-\frac{1}{-60}=\frac{1}{10}$$

$$V_2=\frac{80}{7}$$

Then reflection on m_2

Either V_2 is at centre (normal incidence)

$$d-\frac{80}{7}=20$$

$$d=\frac{220}{7}$$

$$\frac{n}{7}=\frac{220}{7},$$

$$\boxed{n=220}$$

V_2 is at pole of m_2

$$d-\frac{80}{7}=0$$

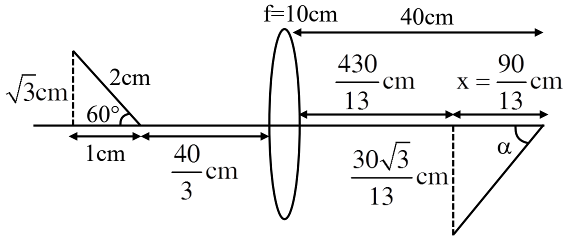
$$d=\frac{80}{7}$$

$$\frac{n}{7}=\frac{80}{7}$$

$$\boxed{n=80}$$

3. Ans. (5.95 - 6.05)

Sol.



$$\frac{h_i}{h_0}=\frac{v}{u}\Rightarrow\frac{-\frac{30\sqrt{3}}{13}}{\sqrt{3}}=\frac{v}{-\frac{43}{3}}\Rightarrow v_1=\frac{430}{13}\text{cm}$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{10}-\frac{3}{40}\Rightarrow v=40\text{cm}$$

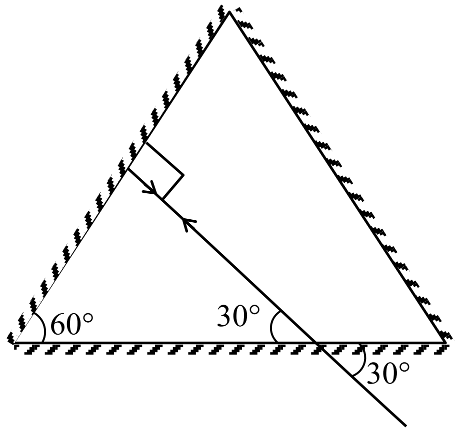
$$x=40-\frac{430}{13}=\frac{90}{13}\text{cm}$$

$$\tan\alpha=\frac{\frac{30\sqrt{3}}{13}}{\frac{90}{13}}=\frac{1}{\sqrt{3}}\Rightarrow\alpha=30°=\frac{\pi}{6}$$

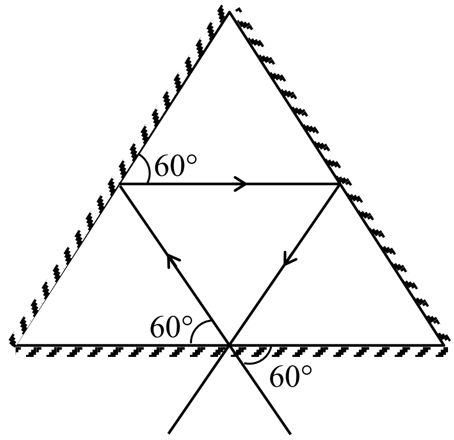
N = 6 Ans.

4. Ans. (A, B)

Sol. (A) Ray will come out after one reflection for $\theta=30°$ & $0<\ell<L$

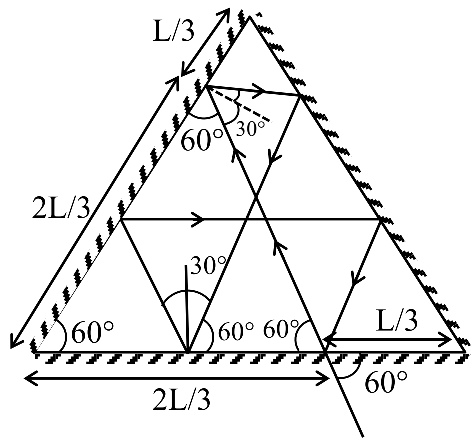


(B)

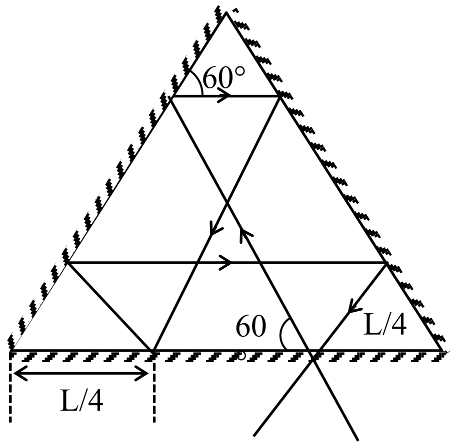


for $\theta=60°$ & $\ell=\frac{L}{2}$, ray will come out after two reflections.

(C) For $\ell=\frac{L}{3}$ & $\theta=60°$ ray will come out after five reflections.



(D) For $\theta=60°$ & $0<\ell<\frac{L}{2}$, ray will come out after five reflections



5. Ans. (A)

Sol. (I) $v_1 = \frac{uf}{u+f}$

$= \frac{(-20)(10)}{(-20)+(10)} = +20$

$u_2 = +15$

$v_2 = \frac{(15)(15)}{(15)+(15)} = +7.5$

(II) $v_1 = +20$

$u_2 = +15$

$v_2 = \frac{(15)(-10)}{(15)+(-10)} = -30$

(III) $v_1 = +20$

$u_2 = +15$

$v_2 = \frac{(15)(-20)}{(15)+(-20)} = 60$

(IV) $v_1 = \frac{(-20)(-20)}{(-20)+(-20)} = -10$

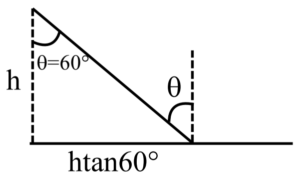
$u_2 = -15$

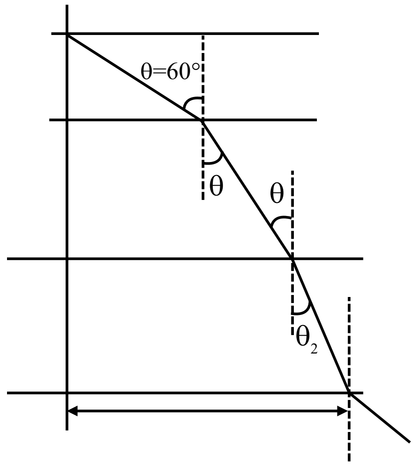
$v_2 = \frac{(-15)(10)}{(-15)+(10)} = 30$

Ans. (A), I-P, II-R, III-Q, IV-T

6. Ans. (4)

Sol.





$1\sin 60° = \sqrt{\frac{3}{2}}\sin\theta$

$\Rightarrow \theta_1 = 45°$

$\sqrt{\frac{3}{2}}\sin 45° = \sqrt{3}\sin\theta_2$

$= \sqrt{\frac{3}{2}}\frac{1}{\sqrt{2}} = \sqrt{3}\sin\theta_2$

$= \theta_2 = 30°$

h tan60° + d tan45° + d tan 30°

$\frac{1}{3}\sqrt{3} + \left(\frac{\sqrt{3}-1}{2}\right) + \left(\frac{\sqrt{3}-1}{2}\right)\frac{1}{\sqrt{3}}$

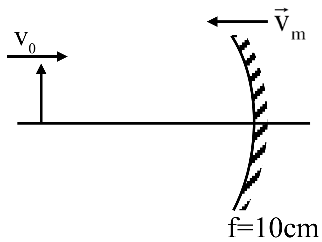
$\frac{2\sqrt{3}+3\sqrt{3}-3+3-\sqrt{3}}{6} = \frac{4\sqrt{3}}{6}$

$\therefore n\frac{4\sqrt{3}}{6} = \frac{8}{\sqrt{3}}$

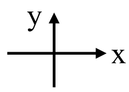
n = 4

7. Ans. (3)

Sol.



Let



u = –30 cm

f = –10 cm

$v = \frac{f_0}{u-f} = -15\text{cm}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{du}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$$

$$\vec{v}_{Im} = -\left(\frac{v}{u}\right)^2 \vec{v}_{om}$$

Given $\vec{v}_I = \vec{0}$

$$\vec{v}_I - \vec{v}_m = -\left(\frac{-15}{-30}\right)^2 (\vec{v}_{O/m})$$

$$\vec{v}_I - \vec{v}_m = -\frac{1}{4}\vec{v}_0 + \frac{1}{4}\vec{v}_m$$

$$\vec{v}_0 = 15\text{cm / s } \hat{i}$$

$$\vec{v}_I = \vec{0}\text{cm / s}$$

$$\frac{5}{4}\vec{v}_m = \frac{\vec{v}_0}{4}$$

$$\vec{v}_m = \frac{\vec{v}_0}{4} = \frac{15\text{cm / s}\,\hat{i}}{5} = 3\text{m / s } \hat{i}$$

$$|\vec{v}_m|_m \text{ cm / s} = 3$$

8. Ans. (B)

Sol. Focal length of convex lens (f_1)

$$\frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= (1.5 - 1)\left[\frac{1}{20} - \left(\frac{1}{-20}\right)\right]$$

$$\frac{1}{f_1} = \frac{1}{20}$$

$\Rightarrow$ $\boxed{f_1 = +20 \text{ cm}}$

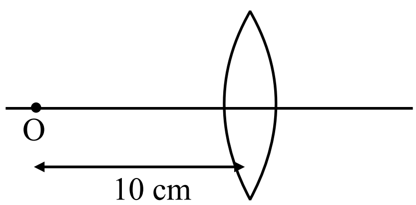
Focal length of concave lens (f_2)

$$\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_2} = (1.5 - 1)\left[-\frac{1}{20} - \frac{1}{20}\right] = \frac{1}{-20}$$

$\Rightarrow$ $\boxed{f_2 = -20 \text{ cm}}$

For lens 1

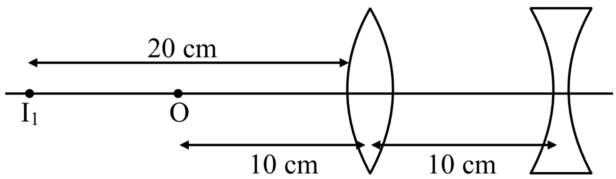


$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow$ $\boxed{v = -20 \text{ cm}}$

$$m_1 = \frac{v}{u} = \frac{-20}{-10} = 2$$

For lens 2



$$u = -30,\ f = -20,\ \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\boxed{v = -12 \text{ cm}}$

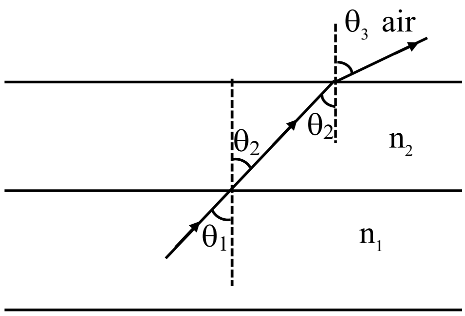
$$m_2 = \frac{v}{u} = \frac{-12}{-30} = \frac{2}{5}$$

Net magnification

$$m = m_1 m_2 = 2 \times \frac{2}{5} = \frac{4}{5} = 0.8$$

9. Ans. (B, C, D)

Sol.



$$\sin\theta > \frac{1}{n_1} (\text{Given})$$

i.e. $\sin\theta_1 > \dfrac{1}{n_1}$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

If $n_1 = n_2$ then $\theta_2 = \theta_1$

$n_2 \sin\theta_2 = (1) \sin\theta_3$

$$\sin\theta_3 = n_2 \sin\theta_2$$

$$\sin\theta_3 = n_1 \sin\theta_1$$

$$\sin\theta_1 = \frac{\sin\theta_3}{n_1} > \frac{1}{n_1}$$

$$\sin\theta_3 > 1$$

$\theta_3 > 90°$

This means ray cannot enter air

For $n_1 > n_2$; $\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2 > \frac{1}{n_1}$

$\sin\theta_2 > \frac{1}{n_2}$

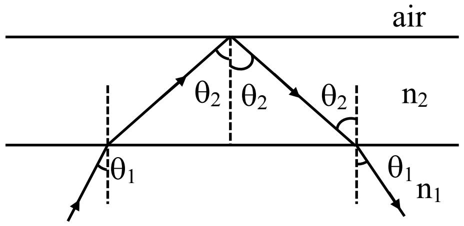
for surface 2 – air interface

$n_2 \sin\theta_2 = \sin\theta_3$

$\sin\theta_2 = \frac{\sin\theta_3}{n_2} > \frac{1}{n_2}$

$\theta_2 > 90°$

It means ray is reflected back in medium-2



for surface 1 – surface 2 interface

$n_2 \sin\theta_2 = n_1 \sin\theta_1$

$\sin\theta_{2C} = \frac{n_1}{n_2}$

θ_{2C} : critical angle

for ray to enter medium-1

$\theta_2 < \theta_{2C}$

$\sin\theta_2 < \sin 2\theta_C$

$\frac{n_1}{n_2}\sin\theta_1 < \frac{n_1}{n_2}$

$\sin\theta_1 < 1$

$\theta_1 < 90°$, which is true

Hence ray enters medium-1

For $n_2 > n_1$

$\frac{n_2}{n_1}\sin\theta_2 > \frac{n_2}{n_1}$

$\sin\theta_2 > \frac{1}{n_2}$

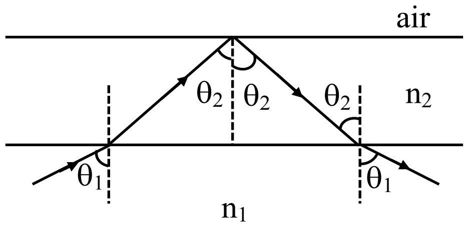
For surface 2 – air interface

$n_2 \sin\theta_2 = \sin\theta_3$

$\sin\theta_2 = \frac{\sin\theta_3}{n_2} > \frac{1}{n_2}$

$\theta_2 > 90$

It means ray is reflected back in medium-2



$n_2 \sin\theta_2 = n_1 \sin\theta_1$

$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2$

$\sin\theta_{2c} = \frac{n_1}{n_2}$; $\theta_{2c} \rightarrow$ critical angle

For ray to enter medium - 1

$\theta_2 < \theta_{2c}$

$\sin\theta_2 < \sin\theta_{2c}$

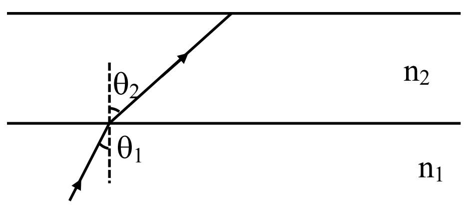
$\frac{n_1}{n_2}\sin\theta_1 < \frac{n_1}{n_2}$

$\sin\theta_1 < 1$

$\theta_1 < 90°$, which is true

Hence ray enters medium - 1

Let $n_2 = 1$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_2 = 1$

$n_1 \sin\theta_1 = \sin\theta_2$

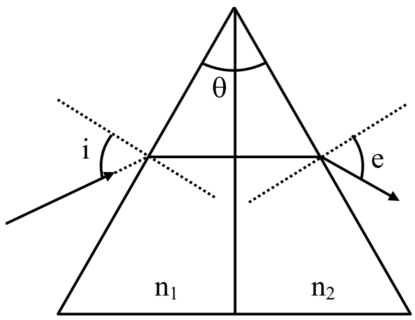
$\sin\theta_1 = \frac{\sin\theta_2}{n_1} > \frac{1}{n_1}$

$\sin\theta_2 > 1 \Rightarrow \theta_2 > 90°$

ray is reflected back in medium –1

10. Ans. (B, C)

Sol.



$$1\times\sin i=\mu\sin\left(\frac{A}{2}\right)$$

$$\sin i=\frac{3}{4}$$

$n_1\sin 30° = 1\sin(e)$

on differentiating both sides

$dn\sin 30° = de\cos(e)$

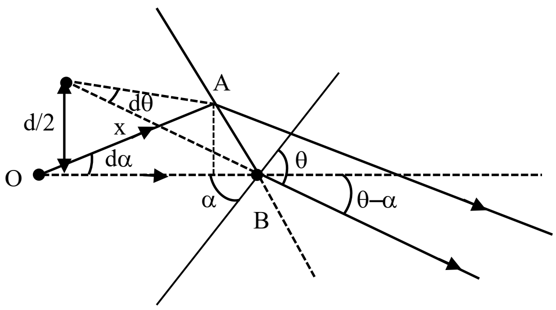
$$de=\frac{dn}{2\cos(e)}=\frac{dn}{2\times\frac{\sqrt{7}}{4}}$$

$$de=\frac{2}{\sqrt{7}}dn\Rightarrow de<dn$$

$$de=\frac{2.8\times10^{-3}\times2}{\sqrt{7}}=2.11\text{ mrad}$$

11. Ans. (BONUS)

Sol.



We will assume that observer sees the image of object through edge $\Rightarrow \alpha = 45°$

$$AB=\frac{12d\alpha}{\cos\alpha}=\frac{xd\theta}{\cos\theta}$$

By applying Snell's Law

$$\frac{4}{3}\sin\alpha=1\sin\theta$$

$$\frac{4}{3}\cos\alpha d\alpha=\cos\theta d\theta$$

$$\Rightarrow\frac{9}{\cos^2\alpha}=\frac{x}{\cos^2\theta}$$

$$1\sin\theta=\frac{4}{3}\sin\alpha$$

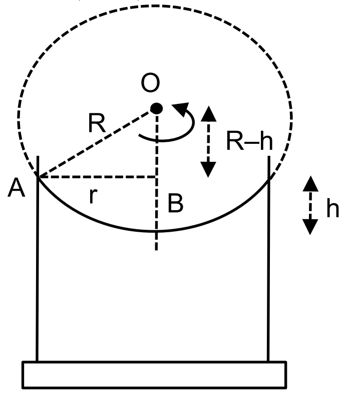
$$\Rightarrow\sin\theta=\frac{2\sqrt{2}}{3}\Rightarrow x=18\times\frac{1}{9}=2$$

$d = 2x\sin(\theta-\alpha)$

$$=4\times\frac{1}{\sqrt{2}}\left(\frac{2\sqrt{2}}{3}-\frac{1}{3}\right)=\frac{8-2\sqrt{2}}{3}\approx1.73\approx2$$

12. Ans. (A, D)

Sol.



In ΔOAB

$R^2 = (R-h)^2 + r^2$

$R^2 = R^2 - 2hR + h^2 + r^2$

$\Rightarrow 2hR = h^2 + r^2$

$$\Rightarrow R=\frac{h^2+r^2}{2h}$$

Now considering equation of surface

$$y=y_0+\frac{\omega^2r^2}{2g}$$

$$h=\frac{\omega^2r^2}{2g}$$

Now using : $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$$\Rightarrow\frac{1}{v}+\frac{4}{3(H-h)}=\frac{1-4/3}{-R}$$

$$\Rightarrow\frac{1}{v}=\frac{1}{3R}-\frac{4}{3H}$$

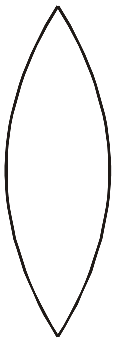
$$\Rightarrow\frac{1}{v}=\frac{2h}{3r^2}-\frac{4}{3H}$$

$$\Rightarrow\frac{1}{v}=-\frac{4}{3H}\left[1-\frac{\omega^2H}{4g}\right]$$

$$\Rightarrow v=\frac{3H}{4}\left[1+\frac{\omega^2H}{4g}\right]^{-1}$$

13. Ans. (A, C, D)

Sol. When $n_1 = n_2 = n$

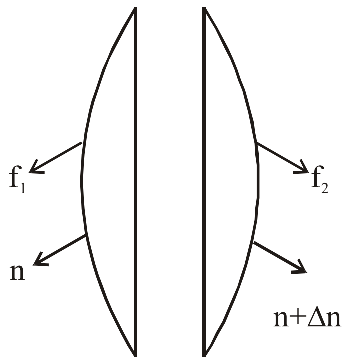


$$\frac{1}{f} = (n-1)\times\frac{2}{R}$$

So $f = \dfrac{R}{2(n-1)}$ ....(1)

2^{nd} case :

$$\frac{1}{f_1} = \frac{n-1}{R}$$



$$\frac{1}{f_2} = \frac{(n+\Delta n)-1}{R}$$

$$\frac{1}{f_2} = \frac{(n+\Delta n)-1}{R}$$

$$\frac{1}{f_{eq}} = \frac{1}{f+\Delta f}$$

$$= \left(\frac{n-1}{R}\right) + \frac{(n+\Delta n)-1}{R}$$

$$= \frac{2(n-1)+\Delta n}{R}$$

$$\Delta f = \left(\frac{R}{2(n-1)+\Delta n}\right) - \left(\frac{R}{2(n-1)}\right)$$

$$= \frac{R}{2}\left[\frac{(n-1)-(n-1+\Delta n)}{(n-1+\Delta n)(n-1)}\right] = \frac{-\Delta n}{(n-1)^2}\times\frac{R}{2}$$

$$\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \quad ....(2)$$

(A) Relation between $\dfrac{\Delta f}{f}$ and $\dfrac{\Delta n}{n}$ is independent of R.
So (A) is correct.

(B) $2n - 2 < n$ because $n < 2$

$$\Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\left|\frac{\Delta n}{n-1}\right| > \frac{\Delta n}{n}$$

So $\dfrac{\Delta f}{f} > \left|\dfrac{\Delta n}{n}\right|$

So (B) is wrong

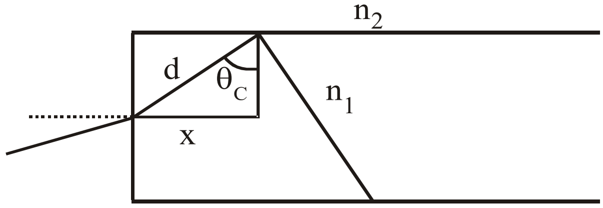
(C) $|\Delta f| = \dfrac{f\Delta n}{(n-1)} = \dfrac{(20\times10^{-3})}{1.5-1} = 40\times10^{-3} = 0.04$

So (3) is wrong

(D) If $\dfrac{\Delta n}{n} < 0$ then $\dfrac{\Delta f}{f} > 0$ from equation (2)

14. Ans. (49 to 51)

Sol. For maximum time the ray of light must undergo TIR at all surfaces at minimum angle i.e. θ_C



For TIR $n_1 \sin\theta_C = n_2$

$$\sin\theta_C = \frac{1.44}{1.5}$$

In above Δ $\sin\theta_C = \dfrac{x}{d}$

$$d = \frac{x}{\sin\theta_C}$$

Similarly $D = \dfrac{L}{\sin\theta_C}$

where,
L = length of tube
D = length of path of light
Time taken by light

$$t = \frac{D}{C} = \frac{L/\sin\theta_C}{2\times10^8}$$

$t = 50 \times 10^{-9}$s

15. Ans. (C, D)

Sol. $H_1 = \dfrac{2H}{3} = \dfrac{2}{3}\times\dfrac{3}{10} = \dfrac{1}{5}$m

For 2^{nd}

$$\frac{1}{v} + \frac{3}{2H} = \frac{-1}{2(-3)}$$

$$\frac{1}{v} = \frac{1}{6} - \frac{10}{2} = \frac{1}{6} - \frac{30}{6} = \frac{-29}{6}$$

$$H_2 = \frac{6}{26} > H_1$$

For 3^{rd}

$$\frac{1}{v}+\frac{3}{2H}=\frac{-1}{2(3)}$$

$$\frac{1}{v}=\frac{-1}{6}-5=\frac{-31}{6}$$

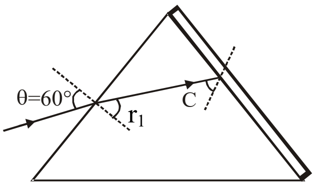
$$H_3=\frac{6}{31}$$

so $\boxed{H_3 < H_1 < H_2}$ & $(H_2 - H_1)$

$$=\frac{6}{29}-\frac{6}{31}=0.68\,\text{cm}$$

16. Ans. (1.50)

Sol. At θ = 60° ray incidents at critical angle at second surface



So, $\sin\theta = \sqrt{3}\sin r_1$

$$\frac{\sqrt{3}}{2}=\sqrt{3}\sin r_1$$

$r_1 = 30°$

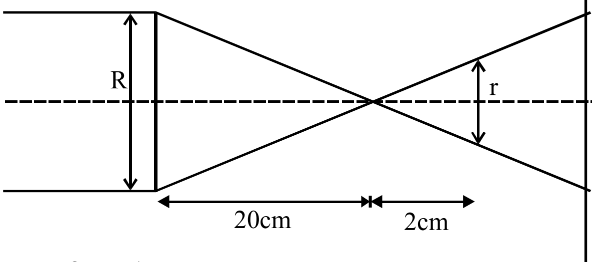
$r_2 = 45° = C$

$$\sqrt{3}\sin 45° = n\sin 90°$$

$$n=\sqrt{\frac{3}{2}} \Rightarrow n^2=\frac{3}{2}$$

17. Ans. (130)

Sol.



$$\frac{r}{R}=\frac{2}{20}=\frac{1}{10}$$

$\therefore$ Ratio of area $=\frac{1}{100}$

Let energy incident on lens be E.

$\therefore$ Given $\frac{E}{A}=1.3$

So final, $\frac{E}{a}=??$

$E = A \times 1.30$

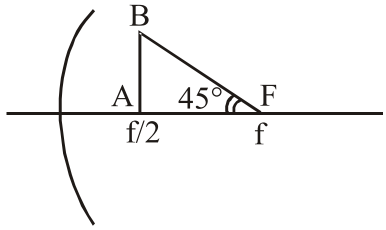
Also $\frac{a}{A}=\frac{1}{100}$

$\therefore$ Average intensity of light at 22 cm =

$$\frac{E}{a}=\frac{A\times 1.3}{a}=100\times 1.3=130\,\text{kW/m}^2$$

18. Ans. (D)

Sol.



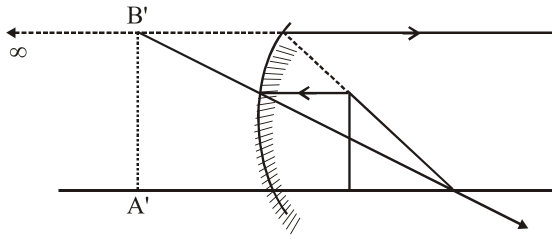
Distance of point A is f/2

Let A' is the image of A from mirror, for this image

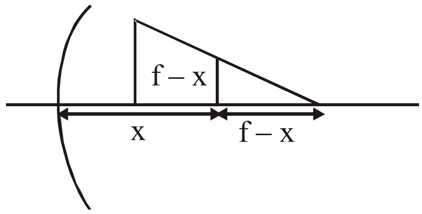
$$\frac{1}{v}+\frac{1}{-f/2}=\frac{1}{-f}$$

$$\frac{1}{v}=\frac{2}{f}-\frac{1}{f}=\frac{1}{f}$$

image of line AB should be perpendicular to the principal axis & image of F will form at infinity, therefore correct image diagram is



OR



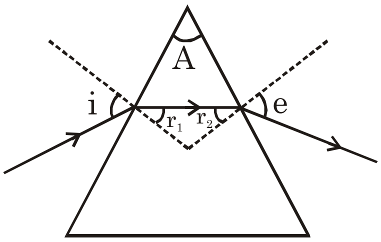
$$\frac{f}{f-u}=\frac{h_2}{h_1}$$

$$h_2=\frac{-f(f-x)}{-f+x}$$

$h_2 = f$

19. Ans. (A, C, D)

Sol.



i = e (for minimum deviation)

$r_1 + r_2 = A$, $r_1 = r_2$

(A) $\delta_m = 2i - A = A$ (given)

$\Rightarrow i = A$

$\Rightarrow r_1 = \frac{A}{2} = \frac{i}{2}$

(B)

$$\mu = \frac{\sin(A)}{\sin(A/2)} = 2\cos\frac{A}{2} \Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

(C) $\mu \sin (r_2) = 1$

$\sin (r_2) =$

$r_1 + r_2 = A$

$r_1 = A - r_2$

$= A - \sin^{-1}\left[\frac{1}{\mu}\right]$

$\sin (i) = \mu \sin (r_1)$

$i = \sin^{-1}[\mu \sin [A - \sin^{-1}\left[\frac{1}{\mu}\right]]]$

$i_g = \sin^{-1}\left[\sqrt{\mu^2 - 1}\sin A - \cos A\right]$

$= \sin^{-1}\left[\mu \sin\left(A - \theta_C\right)\right]$

(Here $\mu = 2\cos\frac{A}{2}$)

(D) Condition of min. deviation i = e &

$r_1 = r_2 = \frac{A}{2}$

Rays will be parallel to base.

20. Ans. (8)

Sol. Applying snell's law between entry & exit surfaces,

$n \sin \theta = \mu \sin \left(\frac{\pi}{2}\right)$

$\Rightarrow 1.6 \sin 30° = \mu \sin \left(\frac{\pi}{2}\right)$

$\therefore \mu = 0.8$

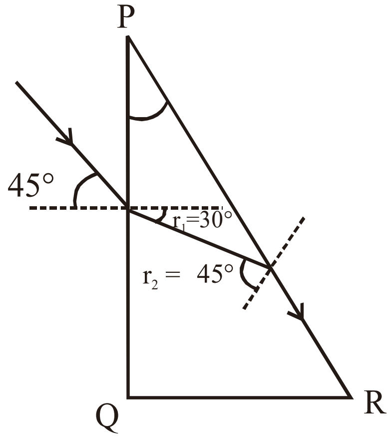
$\therefore 0.8 = n - m\Delta n$

$= 1.6 - m \times 0.1$

$\therefore m = 8$

21. Ans. (A)

Sol.



$1 \sin 45° = \sqrt{2} \sin r_1$

$r_2 - r_1 = \theta$

$\theta = 45° - 30°$

$\Rightarrow \theta = 15°$

22. Ans. (A, B, D)

Sol. For parallel slab

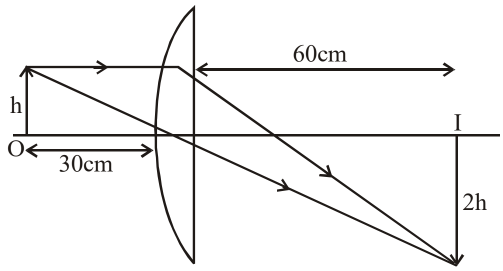
$n_1 \sin\theta_i = n_2 \sin\theta_f$

and ℓ depends on refractive angle in slab

$\therefore$ ℓ depends on refractive index of slab and independent of n_2

23. Ans. (A, D)

Sol. For lens

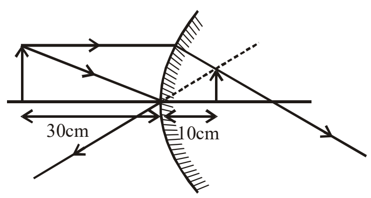


$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f} \Rightarrow f = 20\,\text{cm} \qquad \text{...(i)}$$

Also $\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{(n-1)}{R}$ ...(ii)

For reflection from convex mirror (curved surface)



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{+10}+\frac{1}{-30}=\frac{1}{f}=\frac{2}{R} \qquad ...(iii)$$

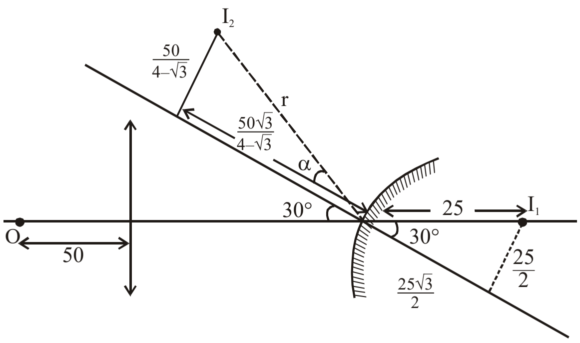
R = 30 cm

from (i), (ii) & (iii)

n = 2.5, faint image erect & virtual

24. Ans. (A)

Sol.



For lens $V=\frac{(-50)(30)}{-50+30}=75$

For mirror $V=\frac{\left(\frac{25\sqrt{3}}{2}\right)(50)}{\frac{25\sqrt{3}}{2}-50}=\frac{-50\sqrt{3}}{4-\sqrt{3}}$

$$m=\frac{v}{u}=\frac{h_2}{h_1}\Rightarrow h_2=-\left(\frac{\frac{-50\sqrt{3}}{4-\sqrt{3}}}{\frac{25\sqrt{3}}{2}}\right)\cdot\frac{25}{2}$$

$$h_2=\frac{+50}{4-\sqrt{3}}$$

The x coordinate of the images =

50 – v cos 30 + h_2 cos 60 25

The y coordinate of the images =

v sin 30 + h_2 sin 60 $25\sqrt{3}$

25. Ans. (7)

Sol. For reflection from concave mirror,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{15}=\frac{-1}{10}$$

$$\frac{1}{v}=\frac{1}{15}-\frac{1}{10}=\frac{-1}{30}$$

$\therefore$ v = –30

magnification (m_1) = $-\frac{v}{u}=-2$

Now for refraction from lens,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$\therefore$ magnification (m_2) = $\frac{v}{u}=-1$

$\therefore$ $M_1 = m_1m_2 = 2$

Now when the set-up is immersed in liquid, no effect for the image formed by mirror.

we have $(\mu_L-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{10}$

$$\Rightarrow\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{5}$$

when lens is immersed in liquid,

$$\frac{1}{f_{lens}}=\left(\frac{\mu_L}{\mu_S}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{2}{7}\times\frac{1}{5}=\frac{2}{35}$$

$$\therefore\ \frac{1}{v}-\frac{1}{u}=\frac{1}{f_{Liquid}}$$

$$\Rightarrow\frac{1}{v}=\frac{2}{35}-\frac{1}{20}=\frac{8-7}{140}=\frac{1}{140}$$

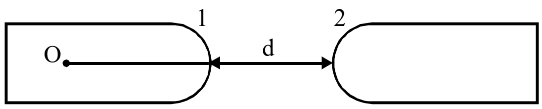
$\therefore$ magnification = $-\frac{140}{20}=-7$

$\therefore$ $M_2 = 2 \times 7 = 14$

$$\therefore\ \left|\frac{M_2}{M_1}\right|=7$$

26. Ans. (B)

Sol.



For first surface

$$\frac{1}{V}-\frac{1.5}{-50}=\frac{1-1.5}{-10}$$

V = 50 cm

for second surface

$$\frac{1.5}{\infty}-\frac{1}{-(d-50)}=\frac{1.5-1}{10}$$

d = 70 cm

$\therefore$ (B)

27. Ans. (2)

Sol. By snell's law

1 sin 60 = n sin r_1

$\Rightarrow \sin r_1 = \frac{1}{2} \Rightarrow r_1 = 30°$ ....(i)

By differentiating 'w.r.t' n

$O = \sin r_1 + n \cos r_1 \left(\frac{dr_1}{dn}\right)$

$O = \frac{1}{2} + \sqrt{3}\left(\sqrt{\frac{3}{2}}\right)\frac{dr_1}{dn}$

$\frac{dr_1}{dn} = \frac{1}{3}$ ....(ii)

By applying snell's law

n sin r_2 = 1 sin θ

n sin (60 – r_1) = 1 sin θ [∴ A = r_1 + r_2]

By diffrentiating 'w.r.t' n

$\sin(60 - r_1) - n\cos(60 - r_1)\frac{dr_1}{dn} = \cos\theta\frac{d\theta}{dn}$

By substituting value of 'r_1' and $\frac{dr_1}{dn}$

from (1) and (2)

$\frac{d\theta}{dn} = 2$

28. Ans. (A, C)

Sol. Let the whole structure is placed in a medium of refractive index n', then

n' sin i = n_1 sin (90 – θ)

n' sin i = n_1 cos θ ...(i)

Here for i_m ; θ = C and $\sin C = \frac{n_2}{n_1}$

from eq. (i), n' sin i_m

$= n_1\sqrt{\frac{1-n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$

$\Rightarrow \sin i_m = \frac{\sqrt{n_1^2 - n_2^2}}{n'}$

Now, for (A)

$(NA)_{s\,1} = \frac{3}{4}\sqrt{\frac{45}{16} - \frac{9}{4}} = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

$(NA)_{s\,2} = \frac{3\sqrt{15}}{16}\sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{3\sqrt{15}}{16}\frac{1}{5}\sqrt{15} = \frac{9}{16}$

For (B)

$(NA)_{s\,1} = \frac{\sqrt{15}}{6} \times \frac{3}{4} = \frac{\sqrt{15}}{8}$

$(NA)_{s\,2} = \frac{3}{4} = \frac{\sqrt{15}}{5}$ Not equal

For (C)

$(NA)_{s\,1} = 1 \times \frac{3}{4} = \frac{3}{4}$

$(NA)_{s\,2} = \frac{\sqrt{15}}{4} \times \frac{\sqrt{15}}{5} = \frac{15}{4 \times 5} = \frac{3}{4}$

For (D)

$(NA)_{s\,1} = \frac{3}{4}$

$(NA)_{s\,2} = \frac{3}{4}\;\frac{\sqrt{15}}{5}$ Not equal

29. Ans. (D)

Sol. It is given that $NA_2 < NA_1 \Rightarrow i_{m\,2} < i_{m\,1}$

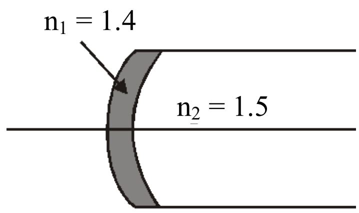
Hence if the combination can be placed both ways i.e. 1st structure & then 2nd structure and then reversed also, then the condition of TIR is satisfied for lower i_m then it can be satisfied for all other less angles as well.

Hence NA_2 will be the numerical aperture of the combined structure.

30. Ans. (A, C)

Sol. When rays are moving from air to glass,

$\frac{1.5}{f_1} = \frac{(1.4-1)}{+R} + \frac{(1.5-1.4)}{+R}$



$\frac{1.5}{f_1} = \frac{0.4}{R} + \frac{0.1}{R} = \frac{0.5}{R}$

$|f_1| = 3R$

When rays are moving from glass to air,

$\frac{1}{F_2} = \frac{(1-1.4)}{-R} + \frac{(1.4-1.5)}{-R} = \frac{0.5}{R}$

$|f_2| = 2R$

31. Ans. (C)

Sol. From the given situation, at critical angle,

$$\tan\theta = \frac{(d/2)}{h} = \frac{5.77}{10}$$

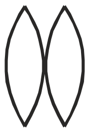
$$\therefore \ \sin\theta_C \approx \frac{1}{2}$$

$$\mu_{denser}\sin\theta_C = \mu_{rarer}\sin(\pi/2)$$

$$\Rightarrow 2.72\times\frac{1}{2} = \mu_r \Rightarrow \mu_r = 1.36$$

32. Ans. (B)

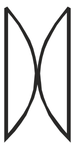
Sol. $P \to 2$



$$\frac{1}{F_{eq}} = \frac{1}{f_1}+\frac{1}{f_2} = (1.5-1)\left(\frac{2}{r}\right)\times 2 = \frac{2}{r}$$

$F_{eq} = r/2$

$Q \to 4$



$$\frac{1}{F_{eq}} = (1.5-1)\left(\frac{2}{r}\right) = \frac{1}{r}$$

$F_{eq} = r$

$R \to 3$

$$\frac{1}{F_{eq}} = (1.5-1)\left(-\frac{2}{r}\right) = -\frac{1}{r}$$

$F_{eq} = -r$