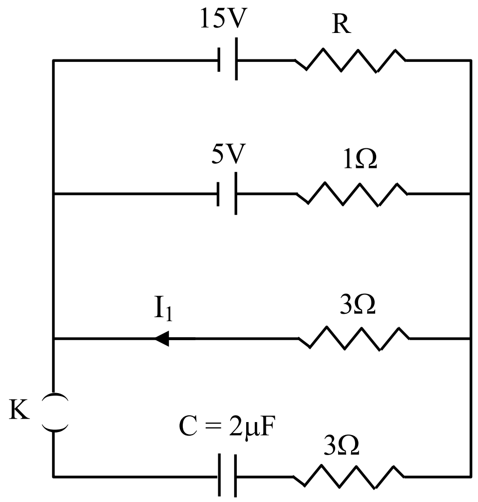
## CAPACITANCE

**1.** In a circuit shown in the figure, the capacitor C is initially uncharged and the key K is open. In this condition, a current of 1 A flows through the 1Ω resistor. The key is closed at time $t = t_0$. Which of the following statement(s) is(are) correct? [Given: $e^{-1} = 0.36$] **[JEE(Advanced) 2023]**




(A) The value of the resistance R is 3Ω.

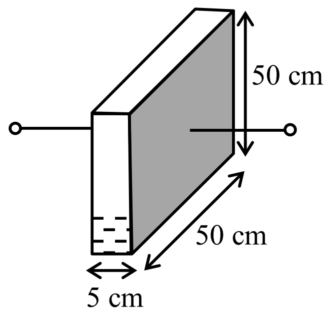
(B) For $t < t_0$, the value of current $I_1$ is 2A.

(C) At $t = t_0 + 7.2$ μs, the current in the capacitor is 0.6 A.

(D) For $t \to \infty$, the charge on the capacitor is 12 μC.

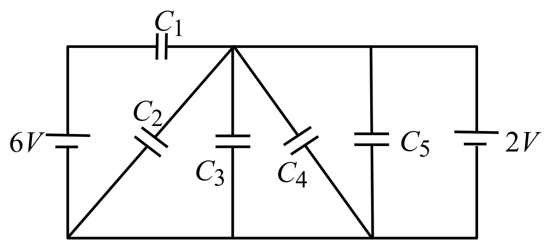
**2.** A container has a base of 50 cm × 5 cm and height 50 cm, as shown in the figure. It has two parallel electrically conducting walls each of area 50 cm × 50 cm. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of 250 $cm^3 s^{-1}$. What is the value of the capacitance of the container after 10 seconds?

[Given: Permittivity of free space $\epsilon_0 = 9 \times 10^{-12} C^2 N^{-1} m^{-2}$, the effects of the non-conducting walls on the capacitance are negligible] **[JEE(Advanced) 2023]**




(A) 27 pF (B) 63 pF (C) 81 pF (D) 135 pF

**3.** In the following circuit $C_1 = 12$ μF, $C_2 = C_3 = 4$ μF and $C_4 = C_5 = 2$ μF. The Charge stored in $C_3$ is _______ μC. **[JEE(Advanced) 2022]**



**4.** A medium having dielectric constant $K > 1$ fills the space between the plates of a parallel plate capacitor. The plates have large area, and the distance between them is $d$. The capacitor is connected to a battery of voltage $V$ as shown in Figure (a). Now, both the plates are moved by a distance of $\frac{d}{2}$ from their original positions, as shown in Figure (b). **[JEE(Advanced) 2022]**

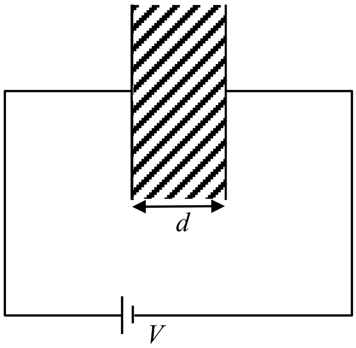


Figure (a)

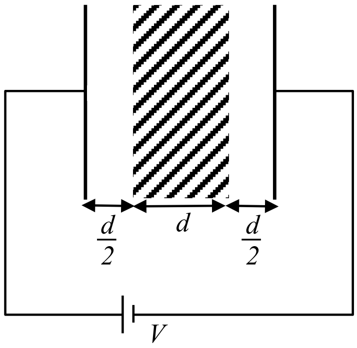


Figure (b)

In the process of going from the configuration depicted in Figure (a) to that in Figure (b), which of the following statement(s) is(are) correct?

(A) The electric field inside the dielectric material is reduced by a factor of $2K$.

(B) The capacitance is decreased by a factor of $\frac{1}{K+1}$.

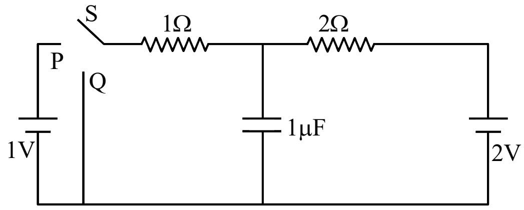
(C) The voltage between the capacitor plates is increased by a factor of $(K + 1)$.

(D) The work done in the process **DOES NOT** depend on the presence of the dielectric material.

**Question Stem for Question Nos. 5 and 6**

**Question Stem**

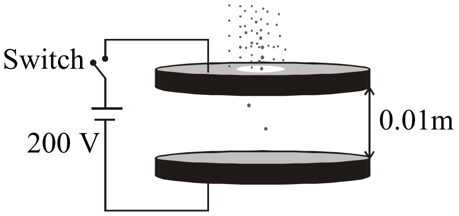
In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1$ μC. Then S is *switched* to position Q. After a long time, the charge on the capacitor is $q_2$ μC.



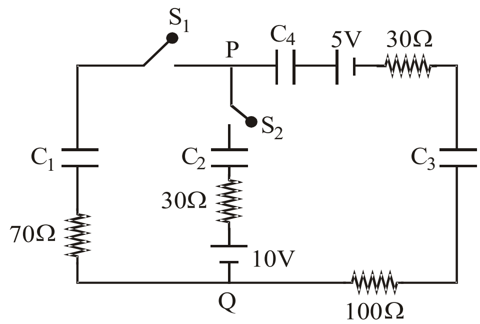

**5.** The *magnitude* of $q_1$ is ______ . **[JEE(Advanced) 2021]**

**6.** The *magnitude* of $q_2$ is ______ . **[JEE(Advanced) 2021]**

**7.** Two large circular discs separated by a distance of 0.01 m are connected to a battery via a switch as shown in the figure. Charged oil drops of density 900 kg $m^{-3}$ are released through a tiny hole at the center of the top disc. Once some oil drops achieve terminal velocity, the switch is closed to apply a voltage of 200 V across the discs. As a result, an oil drop of radius $8\times10^{-7}$ m stops moving vertically and floats between the discs. The number of electrons present in this oil drop is _____. (neglect the buoyancy force, take acceleration due to gravity =10 $ms^{-2}$ and charge on an electron (e) = $1.6 \times 10^{-19}$C) **[JEE(Advanced) 2020]**



**8.** In the circuit shown, initially there is no charge on capacitors and keys $S_1$ and $S_2$ are open. The values of the capacitors are $C_1 = 10\ \mu F$, $C_2 = 30\ \mu F$ and $C_3 = C_4 = 80\ \mu F$. **[JEE(Advanced) 2019]**




Which of the statement(s) is/are correct ?

(A) The keys $S_1$ is kept closed for long time such that capacitors are fully charged. Now key $S_2$ is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).

(B) If key $S_1$ is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

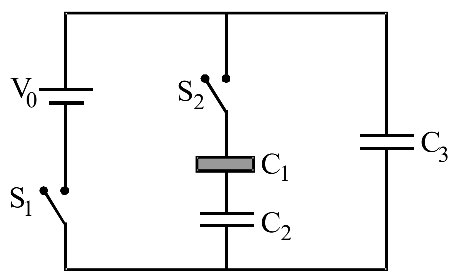
(C) At time t = 0, the key $S_1$ is closed, the instantaneous current in the closed circuit will be 25 mA.

(D) If key $S_1$ is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors $C_1$ will be 4V.

**9.** A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the $m^{th}$ layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N (> $10^3$), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d\ell n2}\right)$. The value of $\alpha$ will be _____. **[JEE(Advanced) 2019]**

[$\in_0$ is the permittivity of free space]

**10.** Three identical capacitors $C_1$, $C_2$ and $C_3$ have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and $C_1$ is then filled completely with a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) $V_0$ = 8V. First the switch $S_1$ is closed while the switch $S_2$ is kept open. When the capacitor $C_3$ is fully charged, $S_1$ is opened and $S_2$ is closed simultaneously. When all the capacitors reach equilibrium, the charge on $C_3$ is found to be 5μC. The value of $\in_r$. **[JEE(Advanced) 2018]**



**Paragraph for Question Nos. 11 and 12**

Consider a simple RC circuit as shown in figure 1.

**Process 1 :** In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage $V_0$ (i.e., charging continues for time T >> RC). In the process some dissipation ($E_D$) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is $E_C$.

**Process 2 :** In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to $V_0$ and the capacitor is charged to the same final voltage $V_0$ as in Process 1.

These two processes are depicted in Figure 2.

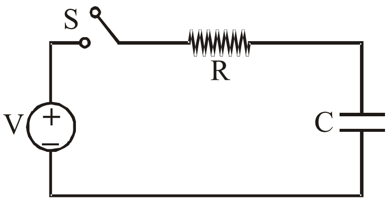



Figure 1

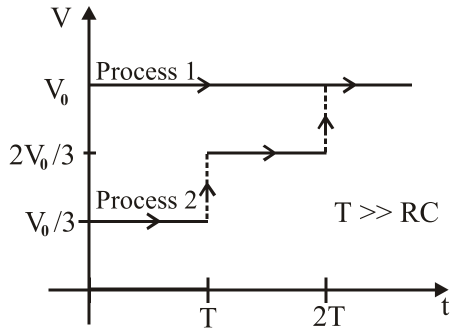



Figure 2

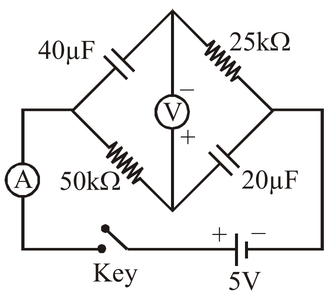
**11.** In Process 1, the energy stored in the capacitor $E_C$ and heat dissipated across resistance $E_D$ are related by :- **[JEE(Advanced) 2017]**

(A) $E_C = E_D$ (B) $E_C = 2E_D$ (C) $E_C = \frac{1}{2}E_D$ (D) $E_C = E_D \ln 2$

**12.** In Process 2, total energy dissipated across the resistance $E_D$ is :- **[JEE(Advanced) 2017]**

(A) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (C) $E_D = \frac{1}{2}CV_0^2$ (D) $E_D = 3CV_0^2$

**13.** In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true ? **[JEE(Advanced) 2016]**



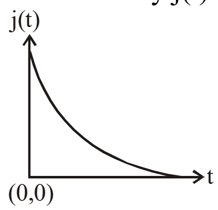

(A) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time

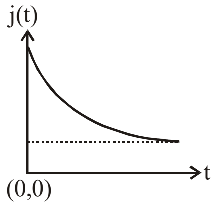
(B) The voltmeter will display 0 V at time t = ln 2 seconds

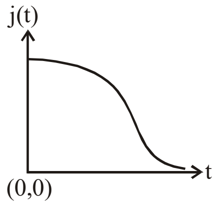
(C) The current in the ammeter becomes 1/e of the initial value after 1 second

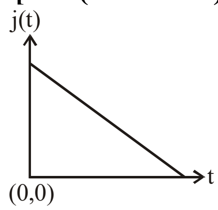
(D) The current in the ammeter becomes zero after a long time

**14.** An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material? **[JEE(Advanced) 2016]**

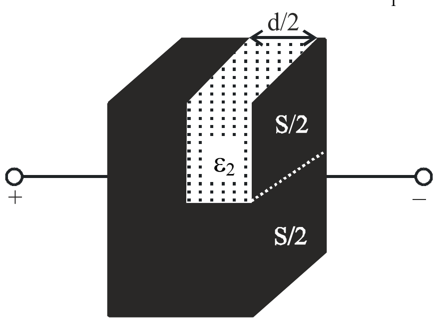
(A) 


(B) 


(C) 

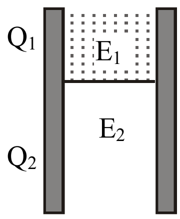

(D) 


**15.** A parallel plate capacitor having plates of area S and plate separation d, has capacitance $C_1$ in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes $C_2$. The ratio $\frac{C_2}{C_1}$ is :- **[JEE(Advanced) 2015]**




(A) $\frac{6}{5}$ (B) $\frac{5}{3}$ (C) $\frac{7}{5}$ (D) $\frac{7}{3}$

**16.** A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is $C_1$. When the capacitor is charged, the plate area covered by the dielectric gets charge $Q_1$ and the rest of the area gets charge $Q_2$. The electric field in the dielectric is $E_1$ and that in the other portion is $E_2$. Choose the correct option/options, ignoring edge effects.
**[JEE(Advanced) 2014]**

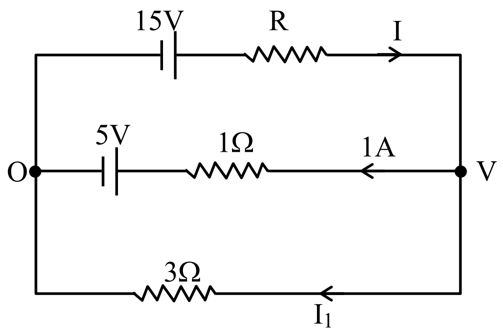



(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

## SOLUTIONS

**1. Ans. (A, B, C, D)**

**Sol.**




By writing voltage drop across 1Ω

⇒ $0 + 5 + 1 \times 1 = V$

$V = 6$

⇒ Similarly across R

$0 + 15 - I \times R = 6$

$IR = 9$

⇒ across 3Ω

$6 - 3 I_1 = 0$

$I_1 = 2A$

Hence option (B) is correct

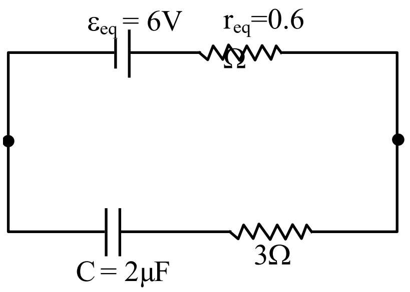
⇒ $I = 1 + 2$ (by KCL)

$I = 3$

$IR = 9$

$R = 3\Omega$

Option (A) is correct




$$\varepsilon = \frac{\frac{15}{3}+\frac{5}{1}+\frac{0}{3}}{\frac{1}{3}+\frac{1}{1}+\frac{1}{3}} = 10\times\frac{3}{5} = 6V$$

$q_{max} = 2 \times 6 = 12\mu C$

$$i = \frac{6}{3.6}e^{-\frac{t}{\tau}}$$

$$=\frac{5}{3}e^{-\frac{7.2}{7.2}} = \frac{5}{3}e^{-1} \approx 0.6A$$

**2. Ans. (B)**

**Sol.** In t = 10 sec volume of liquid is

$V = 2500$ cc

$$h = \frac{2500}{50\times 5} = 10cm$$

$$C_d = \frac{A_d\varepsilon_0 k}{d}$$

$$= \frac{50\times10^{-2}\times10\times10^{-2}\varepsilon_0\times3}{5\times10^{-2}} = 3\varepsilon_0$$

$$C_a = \frac{A_a\varepsilon_0}{d} = \frac{50\times10^{-2}\times40\times10^{-2}\varepsilon_0}{5\times10^{-2}} = 4\varepsilon_0$$

$C = C_a + C_d = 7\varepsilon_0$

$= 7 \times 9 \times 10^{-12} = 63$ Pf

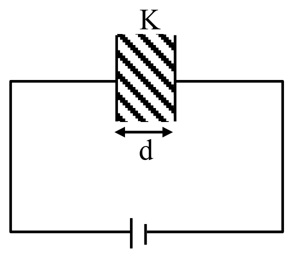
**3. Ans. (7.9 - 8.1)**

**Sol.** Potential difference across the terminals of $C_3$ is 2V.

$\therefore Q_3 = CV = (4\mu)(2) = 8\mu C$

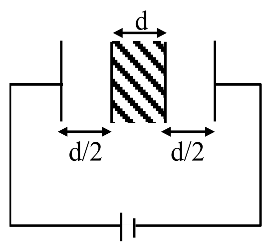
**4. Ans. (B)**

**Sol.** For figure(a)




$$E_0 = \frac{V}{d};\ C = \frac{K\varepsilon_0 A}{d}$$

For figure(b)



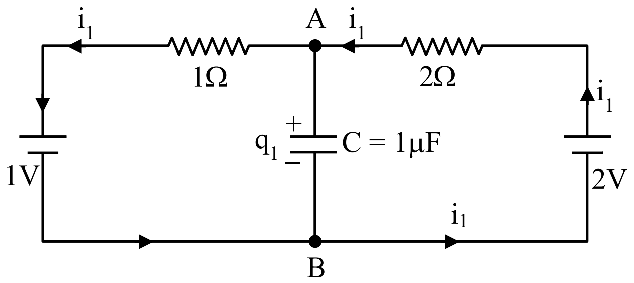

$$C' = \frac{\varepsilon_0 A}{2d - d + d/k};$$

$$C' = \frac{K\varepsilon_0 A}{(K+1)d};\ C' = \frac{C}{K+1}$$

**5. Ans. (1.33)**

**6. Ans. (0.67)**

**Sol.**




Switch connected to position 'P'

$V_A - 1 \cdot i_1 - 1 + 2 - 2i_1 = V_A$

$3i_1 = 1$

$i_1 = \frac{1}{3}A$

$V_A - 1 \cdot i_1 - 1 = V_B$

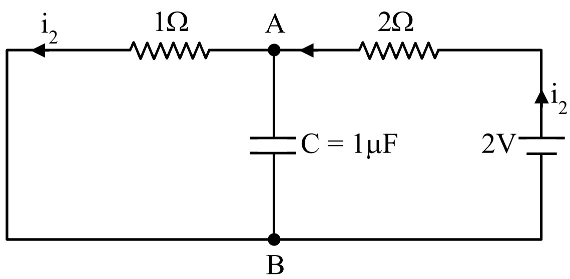
$V_A - V_B = 1 + i_1 = \frac{4}{3}\text{volt}$

Potential drop across capacitor $\Delta V = \frac{4}{3}\text{volt}$

$\therefore$ Charge on capacitor $q_1 = C\Delta V$

$= 1 \times \frac{4}{3}\mu C$

$q_1 = 1.33\ \mu C$




Switch at Position 'Q'

$V_A - 1 \cdot i_2 + 2 - 2i_2 = V_A$

$3i_2 = 2$

$i_2 = \frac{2}{3}A$

$V_A - i_2 \times 1 = V_B$

$V_A - V_B = i_2 \times 1 = \frac{2}{3}\text{volt}$

Potential difference across capacitor

$\Delta V = \frac{2}{3}\text{volt}$

$\therefore$ Charge on capacitor $q_2 = C\Delta V$

$= 1 \times \frac{2}{3} = 0.67\mu C$

**7. Ans. (6)**

**Sol.** $E = \frac{V}{d} = \frac{200}{0.01} = 2 \times 10^4\ V/m$

When terminal velocity is achieved

$qE = mg$

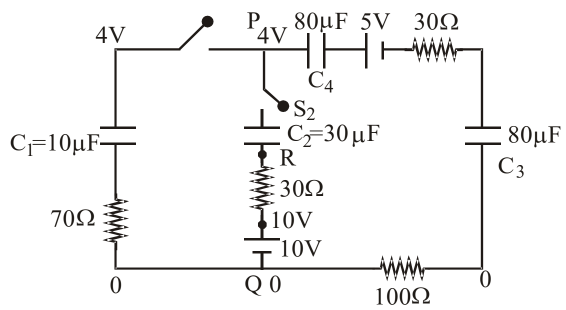
$\Rightarrow n \times 1.6 \times 10^{-19} \times 2 \times 10^4$

$= \frac{4\pi}{3}\left(8 \times 10^{-7}\right)^3 \times 900 \times 10$

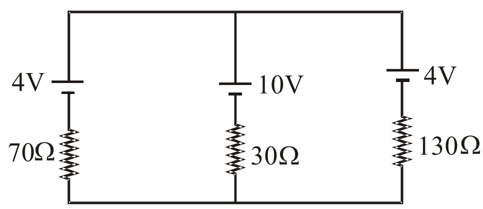
$\Rightarrow n \approx 6$

**8. Ans. (C, D)**

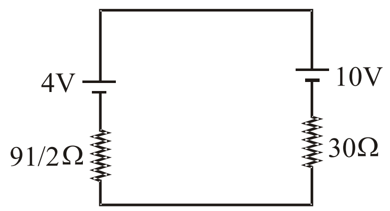
**Sol.**




(A) at t = 0, capacitor $C_1$ acts as a battery of 4V, $C_4$ & $C_3$ of $\frac{1}{2}$V each, $C_2$ is shorted Circuit is




$\Rightarrow$




$\Rightarrow i = \frac{6}{30 + \frac{91}{2}} = 0.079A$

(B) and (D)

At steady state,

When capacitor is fully charged it behave as open circuit and current through it zero.

Hence, Charge on each capacitor is same.

$Q = C_{eq} V$

$= (8\ \mu F) \times 5$

$Q = 40\ \mu C$

Now,

$$V_P - \frac{40}{10} = V_Q$$

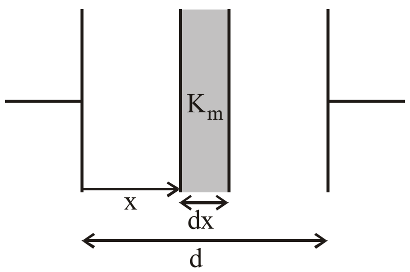
$$\boxed{V_P - V_Q = 4V}$$

(C) At t = 0, $S_1$ is closed, capacitor act as short circuit.

$$i = \frac{V}{R_{eq}} = \frac{5}{200} = 25\,mA$$

**9. Ans. (0.99 to 1.01)**

**Sol.** $\delta = dx = \frac{d}{N}$ & $\frac{m}{N} = \frac{x}{d}$




$$K_m = K\left(1 + \frac{m}{N}\right)$$

$$\Rightarrow K_m = K\left(1 + \frac{x}{d}\right)$$

$$C' = \frac{K_m A \in_0}{dx}$$

$$\frac{1}{C_{eq}} = \int_0^d \frac{dx}{K_m A \in_0} = \frac{1}{KA \in_0}\int_0^d \frac{dx}{\left(1 + \frac{x}{d}\right)}$$

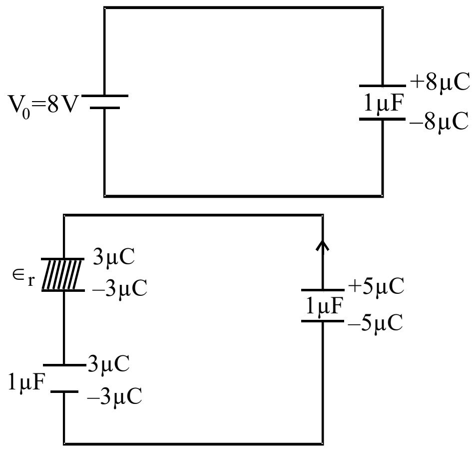
$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA \in_0}\left[\ell n\left(1 + \frac{x}{d}\right)\right]_0^d$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA \in_0}\left[\ell n 2 - \ell n(1)\right]$$

$$\Rightarrow C_{eq} = \frac{KA \in_0}{d \ell n 2} \quad \Rightarrow \alpha = 1$$

**10. Ans. (1.50)**

**Sol.**



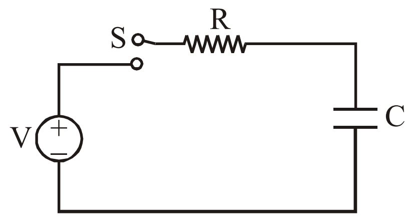

Applying loop rule

$$\frac{5}{1} - \frac{3}{\in_r} - \frac{3}{1} = 0 \qquad \frac{3}{\in_r} = 2$$

$\in_r = 1.50$

**11. Ans. (A)**

**Sol.**




When switch is closed for a very long time capacitor will get fully charged & charge on capacitor will be q = CV

Energy stored in capacitor

$$\in_C = \frac{1}{2}CV^2 \qquad ...(i)$$

Work done by battery (ω) = Vq = VCV = $CV^2$

dissipated across resistance $\in_D$ = (work done by battery) – (energy stored)

$$\in_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2 \qquad ...(ii)$$

from (i) & (ii)

$\in_D = \in_C$

**12. Ans. (A)**

**Sol.** For process (1)

Charge on capacitor = $\frac{CV_0}{3}$

energy stored in capacitor = $\frac{1}{2}C\frac{V_0^2}{9} = \frac{CV_0^2}{18}$

work done by battery = $\frac{CV_0}{3} \times \frac{V}{3} = \frac{CV_0^2}{9}$

Heat loss = $\frac{CV_0^2}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$

For process (2)

Charge on capacitor = $\frac{2CV_0}{3}$

Extra charge flow through battery = $\frac{CV_0}{3}$

Work done by battery : $\frac{CV_0}{3}.\frac{2V_0}{3} = \frac{2CV_0^2}{9}$

Final energy stored in capacitor :

$$\frac{1}{2}C\left(\frac{2V_0}{3}\right)^2 = \frac{4CV_0^2}{18}$$

energy stored in process 2 :

$$\frac{4CV_0^2}{18} - \frac{CV_0^2}{18} = \frac{3CV_0^2}{18}$$

Heat loss in process (2) = work done by battery in process (2) – energy stored in capacitor process (2) = $\frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$

For process (3)

Charge on capacitor = $CV_0$

extra charge flow through battery :

$$CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$$

work done by battery in this process :

$$\left(\frac{CV_0}{3}\right)(V_0) = \frac{CV_0^2}{3}$$

find energy stored in capacitor : $\frac{1}{2}CV_0^2$

energy stored in this process :

$$\frac{1}{2}CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$$

heat loss in process (3) :

$$\frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$$

Now total heat loss ($E_D$) :

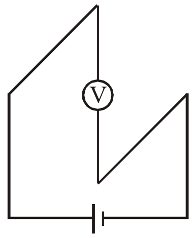
$$\frac{CV_0^2}{18} + \frac{CV_0^2}{18} + \frac{CV_0^2}{18} = \frac{CV_0^2}{6}$$

final energy stored in capacitor : $\frac{1}{2}CV_0^2$

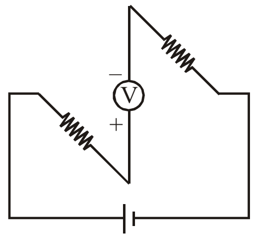
so we can say that $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

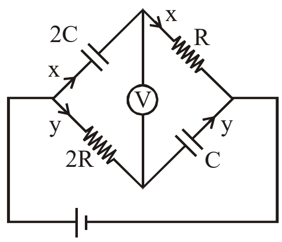
**13. Ans. (A, B, C, D)**

**Sol.** (A) At t = 0, capacitor acts as short-circuit

$\therefore$ 


At $t \to \infty$, capacitor acts as open circuit & no current flows through voltmeter.

$\therefore$ 


(B) 


$q_x = 2CV\left(1 - e^{-t/2CR}\right) \qquad x = \frac{V}{R}e^{-t/2CR}$

$q_y = CV\left(1 - e^{-t/2CR}\right) \qquad y = \frac{V}{2R}e^{-t/2CR}$

$$\Delta V = -y2R + \frac{q_x}{2C}$$

$$= V\left[1 - 2e^{-t/2CR}\right] = 0$$

(C) $\tau$ = 1 sec

So by $i = i_0e^{-t/\tau}$ current at t = 1 sec is = $i_0/e$

(D) After long time no current flows since both capacitor & voltmeter does not allow.

**14. Ans. (A)**

**Sol.** This is the problem of RC circuit where the product RC is a constant.

So due to leakage current, charge & current density will exponentially decay & will become zero at infinite time. So correct answer is (A).

for any small element

Resistance $R = \dfrac{dr}{\sigma(2\pi r\ell)}$

Capacitance $C = \dfrac{\in 2\pi r\ell}{dr}$

Product $R \times C = \dfrac{\in}{\sigma}$ = constant

$$q = q_0 e^{-\left(\frac{t\sigma}{\in}\right)}$$

$$I = \frac{dq}{dt} = \frac{q_0\sigma}{\in} e^{-\left(\frac{t\sigma}{\in}\right)}$$

$$\text{Current density} = \frac{I}{A} = \frac{q_0 \frac{\sigma}{\in} e^{-\frac{t\sigma}{\in}}}{2\pi r\ell}$$

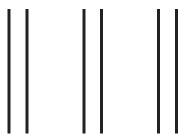
$$j \propto e^{-\frac{t\sigma}{\in}}$$

**15. Ans. (D)**

**Sol.** $\dfrac{5\in_0}{d} = C_1$

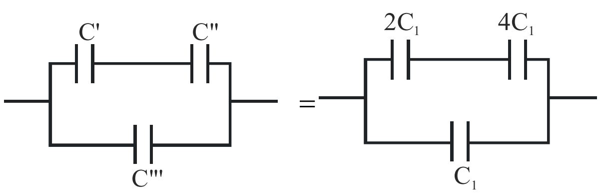
in new setup we have 3 different setups

$5/2, \dfrac{d}{2}, \in_1 \quad 5/2, \dfrac{d}{2}, \in_2 \quad 5/2, d, \in_1$



$$C' = \frac{\frac{5}{2}\in_1\in_0}{d/2} = 2C_1 \Rightarrow C'' = \frac{\frac{5}{2}\in_2\in_0}{d/2} = 4C_1$$

$$C''' = \frac{\frac{5}{2}\in_1\in_0}{d} = C_1$$




$$C_2 = \left(\frac{1}{2C_1} + \frac{1}{4C_1}\right)^{-1} + C_1 = \frac{14}{6}C_1 = \frac{7}{3}C_1$$

$$\therefore \frac{C_2}{C_1} = \frac{7}{3}$$

**16. Ans. (A, D)**

**Sol.** $C_1 = \dfrac{K\varepsilon_0(A/3)}{d}$ (With dielectric)

& let $C_2 = \dfrac{\in_0(2A/3)}{d}$ (without dielectric)

$$C = \frac{K\varepsilon_0 A/3}{d} + \frac{\varepsilon_0 2A/3}{d} = \frac{\varepsilon_0 A/3}{d}[K+2]$$

$$\therefore \frac{C}{C_1} = \frac{K+2}{K}$$

As potential difference is same and gap is same.

$\therefore E_1 = E_2$ , $\therefore \dfrac{E_1}{E_2} = 1$

$Q_1 = C_1V$, $Q_2 = C_2V$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{K}{2}$$