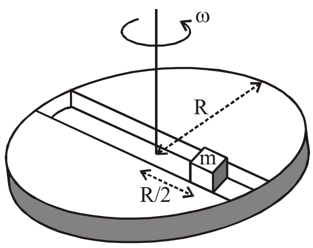
CIRCULAR MOTION

Paragraph for Question Nos. 1 and 2

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is $\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{\upsilon}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega}$,

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega\hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.



1. The distance r of the block at time t is : **[JEE(Advanced) 2016]**

(A) $\frac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$ (B) $\frac{R}{2}\cos 2\omega t$ (C) $\frac{R}{2}\cos\omega t$ (D) $\frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

2. The net reaction of the disc on the block is : **[JEE(Advanced) 2016]**

(A) $-m\omega^2 R\cos\omega t\hat{j} - mg\hat{k}$ (B) $m\omega^2 R\sin\omega t\hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$ (D) $\frac{1}{2}m\omega^2 R\left(e^{2\omega t} - e^{-2\omega t}\right)\hat{j} + mg\hat{k}$

SOLUTIONS

1. Ans. (D)

Sol. Force on block along slot = $m\omega^2 r = ma$

$$= m\left(\frac{vdv}{dr}\right) \Rightarrow \int_0^v vdv = \int_{R/2}^{r} \omega^2 r dr$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right) \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\Rightarrow \int_{R/4}^{r} \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega dt$$

$$\ell n\left(\frac{r + \sqrt{r^2 - \frac{R^2}{4}}}{\frac{R}{2}}\right) - \ell n\left(\frac{R/2 + \sqrt{\frac{R^2}{4} - \frac{R^2}{4}}}{\frac{R}{2}}\right) = \omega t$$

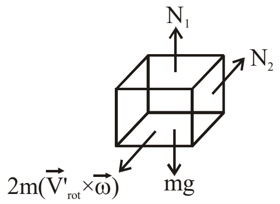
$$\Rightarrow r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

$$\Rightarrow r = \frac{\frac{R^2}{4}e^{2\omega t} + \frac{R^2}{4}}{Re^{\omega t}} = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

2. Ans. (C)

Sol.



$$\vec{N}_1 = mg\hat{k}$$

$$\vec{N}_2 = 2m\left(V'_{rot} \times \vec{\omega}\right)\hat{j}$$

$$= 2m\left[\frac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)\right]\omega\hat{j}$$

$$= \frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

Total reaction on block = $\vec{N}_1 + \vec{N}_2$

$$= \frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$