CENTRE OF MASS

Question Stem for Question Nos. 1 and 2

Question Stem

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}$ m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity g = 10 m/s^2.

1. The value of t is ___ . **[JEE(Advanced) 2021]**

2. The value of x is ___ . **[JEE(Advanced) 2021]**

3. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct? **[JEE(Advanced) 2019]**

(A) The rate at which the particle strikes the piston is v/L

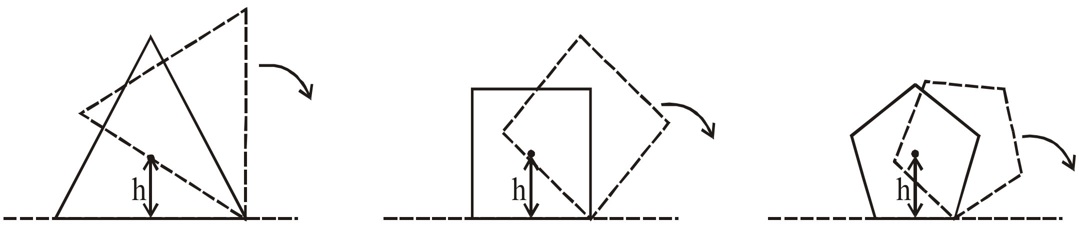
(B) After each collision with the piston, the particle speed increases by 2V

(C) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(D) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

4. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4 kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is________. (Take $e^{-1} = 0.37$) ? **[JEE(Advanced) 2018]**

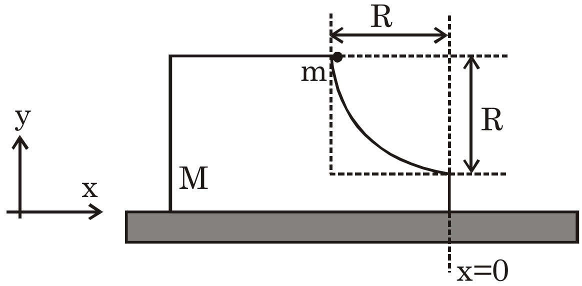
5. Consider regular polygons with number of sides n = 3, 4, 5 ..... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as : **[JEE(Advanced) 2017]**



(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (B) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

(C) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$ (D) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

6. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are **correct** ? **[JEE(Advanced) 2017]**



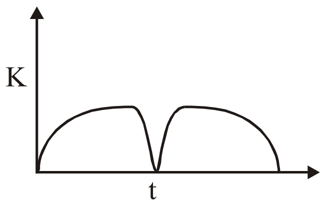
(A) The x component of displacement of the centre of mass of the block M is : $-\frac{mR}{M+m}$

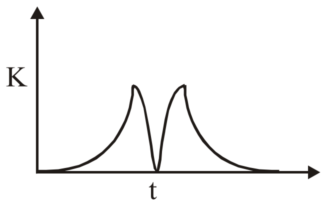
(B) The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$

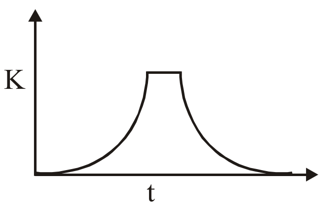
(C) The velocity of the point mass m is : $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

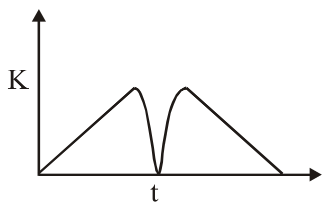
(D) The velocity of the block M is : $V = -\frac{m}{M}\sqrt{2gR}$

7. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately ? The figures are only illustrative and not to the scale. **[JEE(Advanced) 2014]**

(A)


(B)


(C)


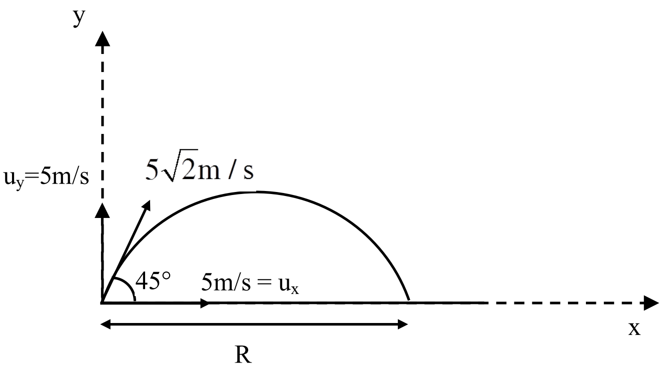
(D)


SOLUTIONS

1. **Ans. (0.50)**

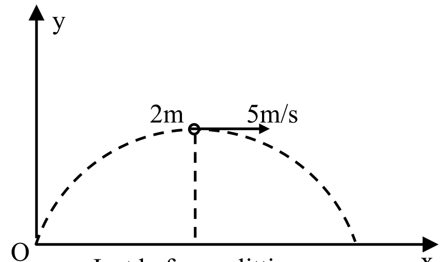
2. **Ans. (7.50)**

Sol.

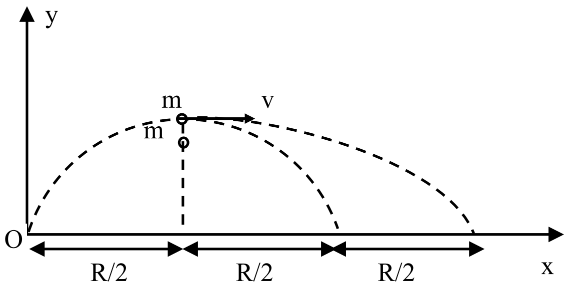


Range $R = \frac{2u_xu_y}{g} = \frac{2\times5\times5}{10} = 5m$

Time of flight $T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1\,sec$



Just before splitting



Just after splitting

∵ Time of motion of one part falling vertically downwards is = 0.5 sec = $\frac{T}{2}$

⇒ Time of motion of another part, $t = \frac{T}{2}$ = 0.5 sec

From momentum conservation ⇒ $P_i = P_f$

2m × 5 = m × v ⇒ v = 10 m/s

Displacement of other part in 0.5 sec in horizontal direction= $v\frac{T}{2}$= 10 × 0.5 = 5 m = R

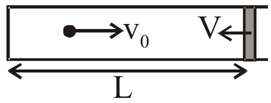
∴ Total distance of second part from point 'O' is,

$x = \frac{3R}{2} = 3\times\frac{5}{2}$

x = 7.5 m ⇒ t = 0.5 sec.

3. **Ans. (B, C)**

Sol.



(A) average rate of collision = $\frac{2L}{v}$

(B) speed of particle after collision = 2V + v_0

change in speed = (2V + v_0) – v_0

after each collision = 2V

no. of collision per unit time (frequency) = $\frac{v}{2L}$

change in speed in dt time = 2V × number of collision in dt time

$\Rightarrow dv = 2V\left(\frac{v}{2L}\right).\frac{dL}{V}$

$\boxed{dv = \frac{vdL}{L}}$

Now, $dv = -\frac{vdL}{L}$ {as L decrease}

$\int_{v_0}^{v}\frac{dv}{v} = -\int_{L_0}^{L_0/2}\frac{dL}{L}$

$\Rightarrow [\ln v]_{v_0}^{v} = -[\ln L]_{L}^{L_0/2} \Rightarrow v = 2v_0$

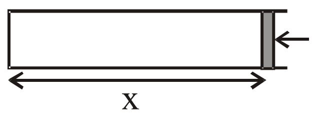
$\Rightarrow KE_{L0} = \frac{1}{2}mv_0^2$ $\boxed{\frac{KE_{L_0/2}}{KE_0} = 4}$

$KE_{L0/2} = \frac{1}{2}m(2v_0)^2$

or

$(dt)\left(\frac{v}{2x}\right)\frac{2mv}{dt} = F$

$F = \frac{mv^2}{x}$



$-m\,v\frac{dv}{dx} = \frac{mv^2}{x}$

$-\frac{dv}{v} = \frac{dx}{x}$

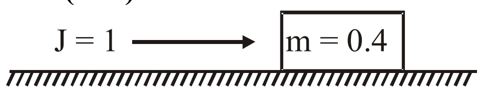
$\ln\frac{v_2}{v_1} = \ln\frac{x_1}{x_2}$

vx = constant ⇒ on decreasing length to half K.E. becomes 4 times

vdx + xdv = 0

4. **Ans. (6.30)**

Sol.



$v = v_0 e^{-t/\tau}$

$v_0 = \frac{J}{m} = 2.5 \text{ m/s}$

$v = v_0 e^{-t/\tau}$

$\frac{dx}{dt} = v_0 e^{-t/\tau}$

$\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt \quad \int e^{-x} dx = \frac{e^{-x}}{-1}$

$x = v_0 \left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}} \right]_0^\tau$

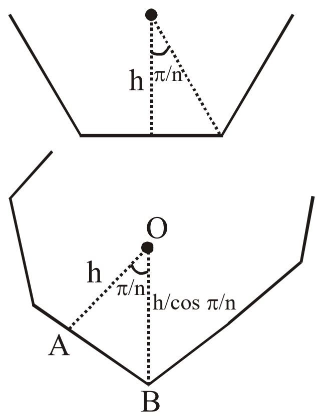
$x = 2.5\,(-4)\,(e^{-1} - e^0)$

$x = 25\,(-4)\,(0.37 - 1)$

$x = 6.30$

5. **Ans. (C)**

Sol.



$OA = h$

$OB = \frac{h}{\cos\frac{\pi}{n}}$

Initial height of COM = h

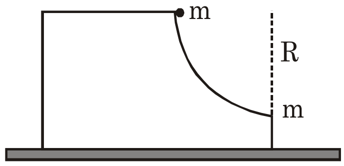
Final height of COM = $\frac{h}{\cos\left(\frac{\pi}{n}\right)}$

$\therefore \Delta = \frac{h}{\cos\frac{\pi}{n}} - h$

$\Delta = h\left[\frac{1}{\cos\frac{\pi}{n}} - 1\right]$

6. **Ans. (A, C)**

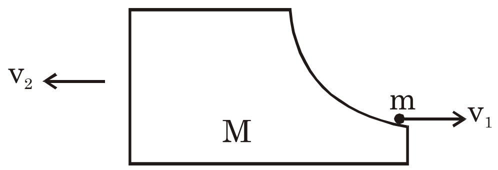
Sol.



$M_s \Delta\bar{x}_{cm} = m_1 \Delta\bar{x} + m_2 \Delta\bar{x}_2$

$0 = m\,(+R + \bar{x}) + m\,\bar{x}$

$\bar{x} = \frac{-mR}{M+m}$



$0 = m\bar{v}_1 + M\bar{v}_2$

$\bar{v}_2 = -\frac{m\bar{v}_1}{M}$

$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$

$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{mv_1}{M}\right)^2$

$mgR = \frac{1}{2}mv_1^2\left(1 + \frac{m}{M}\right)$

$\sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}} = v_1$

7. **Ans. (B)**

Sol. During fall, $v = 0 + gt$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\,(gt)^2$

$KE \propto t^2$, so graph is upward parabola.

During collision, KE will decrease in compression and increase in reformation.

Finally, during going up KE will decrease.