TRIGONOMETRIC EQUATION

1. Consider the following lists:

	List-I		List-II
(I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P)	has two elements
(II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R)	has four elements
(IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

The correct option is: **[JEE(Advanced) 2022]**

(A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)
(B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)
(C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)
(D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

2. Let $f : [0, 2] \to \mathbb{R}$ be the function defined by **[JEE(Advanced) 2020]**

$$f(x) = (3 - \sin(2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$$

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \ge 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is _____.

3. Answer the following by appropriately matching the lists based on the information given in the paragraph.

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\}$$
$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List-I contains the sets X,Y,Z and W. List -II contains some information regarding these sets.

[JEE(Advanced) 2019]

List-I		List-II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

Options :

(A) (II), (R), (S) (B) (I), (P), (R) (C) (II), (Q), (T) (D) (I), (Q), (U)

4. Answer the following by appropriately matching the lists based on the information given in the paragraph.

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order : **[JEE(Advanced) 2019]**

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$.

$Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$.

List-I contains the sets X,Y,Z and W. List -II contains some information regarding these sets.

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		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

Options :

(A) (IV), (Q), (T) (B) (IV), (P), (R), (S) (C) (III), (R), (U) (D) (III), (P), (Q), (U)

5. Let a, b, c be three non-zero real numbers such that the equation **[JEE(Advanced) 2018]**

$$\sqrt{3}a\cos x + 2b\sin x = c, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is ______.

6. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solution of the equation

$\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to - **[JEE(Advanced) 2016]**

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

7. The number of distinct solutions of equation **[JEE(Advanced) 2015]**

$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

in the interval $[0,2\pi]$ is

8. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has **[JEE(Advanced) 2014]**

(A) infinitely many solutions (B) three solutions

(C) one solution (D) no solution

SOLUTIONS

1. Ans. (B)

Sol.

(I) $\left\{x \in \left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$

cosx + sinx = 1

$\Rightarrow \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$; $n \in Z$

$\Rightarrow x = 2n\pi$; $x = 2n\pi + \frac{\pi}{2}$; $n \in Z$

$\Rightarrow x \in \left\{0, \frac{\pi}{2}\right\}$ in given range has two solutions

(II) $\left\{x \in \left[\frac{-5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$

$\sqrt{3}\tan 3x = 1 \Rightarrow \tan 3x = \frac{1}{\sqrt{3}}$

$\Rightarrow 3x = n\pi + \frac{\pi}{6}$

$\Rightarrow x = (6n+1)\frac{\pi}{18}$; $n \in Z$

$\Rightarrow x \in \left\{\frac{\pi}{18}, \frac{-5\pi}{18}\right\}$ in given range has two solutions

(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$

$2\cos 2x = \sqrt{3}$

$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{6}$; $n \in Z$

$\Rightarrow x = n\pi \pm \frac{\pi}{12}$; $n \in Z$

$x \in \left\{\pm\frac{\pi}{12}, \pi \pm \frac{\pi}{12}, -\pi \pm \frac{\pi}{12}\right\}$

Six solutions in given range

(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$

cosx – sinx = –1

$\Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

$\Rightarrow x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$; $n \in Z$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi - \pi$; $n \in Z$

$\Rightarrow x \in \left\{\frac{\pi}{2}, \frac{-3\pi}{2}, \pi, -\pi\right\}$

four solutions in given range

2. Ans. (1.00)

Sol. Let $\pi x - \frac{\pi}{4} = \theta \in \left[\frac{-\pi}{4}, \frac{7\pi}{4}\right]$

So, $\left(3 - \sin\left(\frac{\pi}{2} + 2\theta\right)\right)\sin\theta \geq \sin(\pi + 3\theta)$

$\Rightarrow (3 - \cos 2\theta)\sin\theta \geq -\sin 3\theta$

$\Rightarrow \sin\theta[3 - 4\sin^2\theta + 3 - \cos 2\theta] \geq 0$

$\Rightarrow \sin\theta(6 - 2(1 - \cos 2\theta) - \cos 2\theta) \geq 0$

$\Rightarrow \sin\theta(4 + \cos 2\theta) \geq 0$

$\Rightarrow \sin\theta \geq 0$

$\Rightarrow \theta \in [0, \pi] \Rightarrow 0 \leq \pi x - \frac{\pi}{4} \leq \pi$

$\Rightarrow x \in \left[\frac{1}{4}, \frac{5}{4}\right]$

$\Rightarrow \beta - \alpha = 1$

3. Ans. (C)

4. Ans. (B)

Sol. f(x) = sin (π cos x)

X : {x : f(x) = 0}

f(x) = 0 ⇒ sin (π cos x) = 0 ⇒ cos x = n ⇒ cos x = 1, –1, 0 ⇒ $x = \frac{n\pi}{2}$

$X = \left\{\frac{n\pi}{2} : n \in N\right\} = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, ...\right\}$

g(x) = cos (2π sin x)

Z = {x : g(x) = 0}

cos (2π sin x) = 0 ⇒ 2π sin x = (2n + 1) $\frac{\pi}{2}$

$\Rightarrow \sin x = \dfrac{(2n+1)}{4}$

$\sin x = -\dfrac{1}{4}, \dfrac{1}{4}, \dfrac{-3}{4}, \dfrac{3}{4}$

$Z = \left\{n\pi \pm \sin^{-1}\left(\dfrac{1}{4}\right), n\pi \pm \sin^{-1}\left(\dfrac{3}{4}\right), n \in I\right\}$

Y = {x : f'(x) = 0}

f(x) = sin (π cos x) ⇒ f '(x)

= cos (π cos x) . (–π sin x) = 0

sin x = 0 ⇒ x = nπ.

$\cos(\pi \cos x) = 0 \Rightarrow \pi \cos x = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \cos x = \dfrac{(2n+1)}{2} \Rightarrow \cos x = -\dfrac{1}{2}, \dfrac{1}{2}$

$Y = \left\{n\pi, n\pi \pm \dfrac{\pi}{3}\right\} = \left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \pi, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}, 2\pi, ...\right\}$

W = {x : g'(x) = 0}

g(x) = cos (2π sin x) ⇒ g'(x)

= –sin (2π sin x).(2π cos x) = 0

$\cos x = 0 \Rightarrow x = (2n+1)\dfrac{\pi}{2}$

$\sin(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = n\pi \Rightarrow \sin x = \dfrac{n}{2}$

$= -1, -\dfrac{1}{2}, 0, \dfrac{1}{2}, 1$

$W = \left\{\dfrac{n\pi}{2}, n\pi \pm \dfrac{\pi}{6}, n \in I\right\}$

$= \left\{\dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \pi, \dfrac{7\pi}{6}, \dfrac{3\pi}{2}, ...\right\}$

Now check the options

5. **Ans. (0.5)**

Sol. $\sqrt{3}\cos x + \dfrac{2b}{a}\sin x = \dfrac{c}{a}$

Now, $\sqrt{3}\cos\alpha + \dfrac{2b}{a}\sin\alpha = \dfrac{c}{a}$ .....(1)

$\sqrt{3}\cos\beta + \dfrac{2b}{a}\sin\beta = \dfrac{c}{a}$ .....(2)

$\sqrt{3}[\cos\alpha - \cos\beta] + \dfrac{2b}{a}(\sin\alpha - \sin\beta) = 0$

$\sqrt{3}\left[-2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)\right] +$

$\dfrac{2b}{a}\left[2\cos\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)\right] = 0$

$-\sqrt{3} + 2\sqrt{3}.\dfrac{b}{a} = 0$

$\dfrac{b}{a} = \dfrac{1}{2} = 0.5$

6. **Ans. (C)**

Sol. $\sqrt{3}\sin x + \cos x = 2\cos 2x$

$\Rightarrow \cos 2x = \cos\left(x - \dfrac{\pi}{3}\right)$

$\Rightarrow 2x = 2n\pi \pm \left(x - \dfrac{\pi}{3}\right)$

$x = (6n-1)\dfrac{\pi}{3}$ or $(6n+1)\dfrac{\pi}{9}$

$\Rightarrow x = -\dfrac{\pi}{3}, \dfrac{\pi}{9}, \dfrac{7\pi}{9}$ and $-\dfrac{5\pi}{9}$ in $(-\pi, \pi)$

⇒ sum = 0

7. **Ans. (8)**

Sol. Given equation

$\dfrac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\dfrac{5}{4}\cos^2 2x + 1 - 2\sin^2 x\cos^2 x + 1 - 3\sin^2 x\cos^2 x = 2$

$\dfrac{5}{4}\cos^2 2x - \dfrac{1}{2}(1-\cos^2 2x) - \dfrac{3}{4}(1-\cos^2 2x) = 0$

$\dfrac{5}{2}\cos^2 2x = \dfrac{5}{4} \Rightarrow \cos^2 2x = \dfrac{1}{2} \Rightarrow \cos 4x = 0$

Clearly having 8 solutions in [0, 2π]

8. **Ans. (D)**

Sol. sinx – sin3x + 2sin2x = 3

–2sinx cos2x + 4sinx cosx = 3

2sinx{–cos2x + 2cosx} = 3

$2\sin x\{-(2\cos^2 x - 1) + 2\cos x\} = 3$

$2\sin x\{-2\cos^2 x + 2\cos x + 1\} = 3$

$\underbrace{2\sin x}_{\le 2}\underbrace{\left\{\dfrac{3}{2} - 2\left(\cos x - \dfrac{1}{2}\right)^2\right\}}_{\le \frac{3}{2}} = 3$

for equality to hold

$\sin x = 1$ & $\cos x = \dfrac{1}{2}$

which is not possible

hence no solution