STRAIGHT LINE

Question Stem for Question No. 1 and 2

Question Stem

Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line y = 2x + 1 meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the **square** of the distance between R' and S'.

1. The value of λ^2 is __________. **[JEE(Advanced) 2021]**

2. The value of D is __________. **[JEE(Advanced) 2021]**

3. Let a, λ, m ∈ ℝ. Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is(are) correct ? **[JEE(Advanced) 2016]**

(A) If a = –3, then the system has infinitely many solutions for all values of λ and μ

(B) If a ≠ –3, then the system has a unique solution for all values of λ and μ

(C) If λ + μ = 0, then the system has infinitely many solutions for a = –3

(D) If λ + μ ≠ 0, then the system has no solution for a = –3

4. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines x – y = 0 and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is **[JEE(Advanced) 2014]**

5. For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then **[JEE(Advanced) 2014]**

(A) a + b – c > 0 (B) a – b + c < 0 (C) a – b + c > 0 (D) a + b – c < 0

SOLUTIONS

1. Ans. (9.00)

Sol. $\left|\dfrac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\dfrac{\sqrt{2}x-y+1}{\sqrt{3}}\right|=\lambda^2$

$\left|\dfrac{2x^2-(y-1)^2}{3}\right|=\lambda^2,\ C:\left|2x^2-(y-1)^2\right|=3\lambda^2$

line y = 2x + 1, $RS=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$,

$R(x_1,y_1)$ and $S(x_2,y_2)$

$y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1$

$\Rightarrow (y_1 - y_2) = 2(x_1 - x_2)$

$RS=\sqrt{5(x_1-x_2)^2}=\sqrt{5}|x_1-x_2|$

solve curve C and line y = 2x + 1 we get

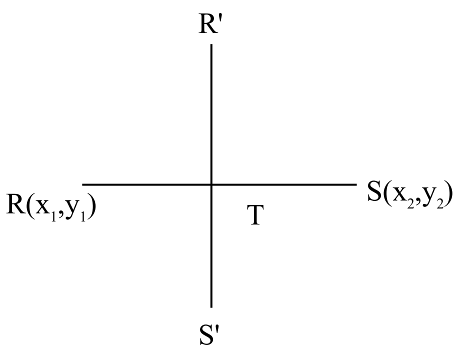
$\left|2x^2-(2x)^2\right|=3\lambda^2 \Rightarrow x^2=\dfrac{3\lambda^2}{2}$

$RS=\sqrt{5}\left|\dfrac{2\sqrt{3}\lambda}{\sqrt{2}}\right|=\sqrt{30}\lambda=\sqrt{270}$

$\Rightarrow 30\lambda^2=270\Rightarrow\lambda^2=9$

2. Ans. (77.14)

Sol.



$\perp$ bisector of RS

$T\equiv\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

Here $x_1 + x_2 = 0$

T = (0,1)

Equation of

R'S' : $(y-1)=-\dfrac{1}{2}(x-0)\Rightarrow x+2y=2$

$R'(a_1,b_1)$ $S'(a_2,b_2)$

$D = (a_1 - a_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

solve $x+2y=2$ and $\left|2x^2-(y-1)^2\right|=3\lambda^2$

$\left|8(y-1)^2-(y-1)^2\right|=3\lambda^2\Rightarrow(y-1)^2=\left(\dfrac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$

$y-1=\pm\dfrac{\sqrt{3}\lambda}{\sqrt{7}}\Rightarrow b_1=1+\dfrac{\sqrt{3}\lambda}{\sqrt{7}},b_2=1-\dfrac{\sqrt{3}\lambda}{\sqrt{17}}$

$D=5\left(\dfrac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2=\dfrac{5\times4\times3\lambda^2}{7}$

$=\dfrac{5\times4\times27}{7}=77.14$

3. Ans. (B, C, D)

Sol. ax + 2y = λ

3x – 2y = μ

for a = –3 above lies will be parallel or coincident

parallel for λ + μ ≠ 0 and coincident if λ + μ = 0

and if a ≠ –3 lies are intersecting

⇒ unique solution.

4. Ans. (6)

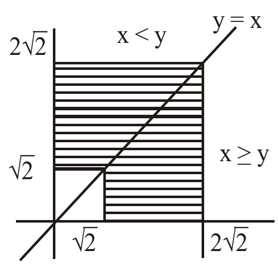
Sol. Let P(x,y) is the point in I quad.

Now $2\le\left|\dfrac{x-y}{\sqrt{2}}\right|+\left|\dfrac{x+y}{\sqrt{2}}\right|\le4$

$2\sqrt{2}\le|x-y|+|x+y|\le4\sqrt{2}$

Case-I : $x\ge y$

$2\sqrt{2}\le(x-y)+(x+y)\le4\sqrt{2}$



⇒ $x\in[\sqrt{2},2\sqrt{2}]$

Case-II : x < y

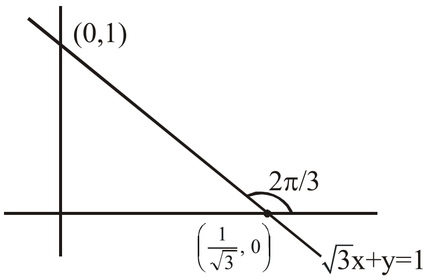
$2\sqrt{2}\le y-x+(x+y)\le4\sqrt{2}$

$y\in[\sqrt{2},2\sqrt{2}]$

$A=\left(2\sqrt{2}\right)^2-\left(\sqrt{2}\right)^2=6$

5. **Ans. (A or C or A, C)**

Sol.



Point of intersection of both lines is

$$\left(-\frac{c}{(a+b)}, -\frac{c}{(a+b)}\right)$$

Distance between $\left(-\frac{c}{(a+b)}, -\frac{c}{(a+b)}\right)$

& (1,1) is

Distance $= \sqrt{\frac{(a+b+c)^2}{(a+b)^2} \times 2} < 2\sqrt{2}$

$a + b + c < 2(a + b)$

$a + b - c > 0$

According to given condition option (C) also correct.