PERMUTATION & COMBINATION

1. The number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits 0, 2, 3, 4, 6, 7 is _______. **[JEE(Advanced) 2022]**

2. Consider 4 boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen? **[JEE(Advanced) 2022]**

(A) 21816 (B) 85536 (C) 12096 (D) 156816

3. Let

$S_1 = \{(i, j, k) : i, j, k \in \{1, 2,\ldots,10\}\}$

$S_2 = \{(i, j) : 1 \le i < j + 2 \le 10,\ i, j \in \{1, 2,\ldots,10\}\}$,

$S_3 = \{(i, j, k, l) : 1 \le i < j < k < l,\ i, j, k, l \in \{1, 2,\ldots,10\}\}$.

and

$S_4 = \{(i, j, k, l) : i, j, k$ and l are distinct elements in $\{1, 2,\ldots,10\}\}$.

If the total number of elements in the set S_r is n_r, r = 1, 2, 3, 4, then which of the following statements is (are) **TRUE**? **[JEE(Advanced) 2021]**

(A) $n_1 = 1000$ (B) $n_2 = 44$ (C) $n_3 = 220$ (D) $\frac{n_4}{12} = 420$

4. An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that no two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is _______. **[JEE(Advanced) 2020]**

5. In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is _______. **[JEE(Advanced) 2020]**

6. Five person A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is **[JEE(Advanced) 2019]**

7. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is _______. **[JEE(Advanced) 2018]**

8. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is _______. **[JEE(Advanced) 2018]**

9. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boy and 2 girls.

(ii) Let α_2 be the total number of ways in which the committe can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committe can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the commitee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

[JEE(Advanced) 2018]

LIST-I	LIST-II
P. The value of α_1 is	**1.** 136
Q. The value of α_2 is	**2.** 189
R. The value of α_3 is	**3.** 192
S. The value of α_4 is	**4.** 200
	5. 381
	6. 461

The correct option is :-

(A) $P \to 4; Q \to 6, R \to 2; S \to 1$

(B) $P \to 1; Q \to 4; R \to 2; S \to 3$

(C) $P \to 4; Q \to 6, R \to 5; S \to 2$

(D) $P \to 4; Q \to 2; R \to 3; S \to 1$

10. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$ **[JEE(Advanced) 2017]**

11. Let S = {1, 2, 3,.....,9}. For k = 1,2, ..., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$ **[JEE(Advanced) 2017]**

(A) 125 (B) 252 (C) 210 (D) 126

12. A debate club consists of 6 girls and 4 boy. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 member) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is **[JEE(Advanced) 2016]**

(A) 380 (B) 320 (C) 260 (D) 95

13. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

[JEE(Advanced) 2015]

14. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. The the number of such distinct arrangements (n_1,n_2,n_3,n_4,n_5) is **[JEE(Advanced) 2014]**

15. Let $n \geq 2$ b an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is **[JEE(Advanced) 2014]**

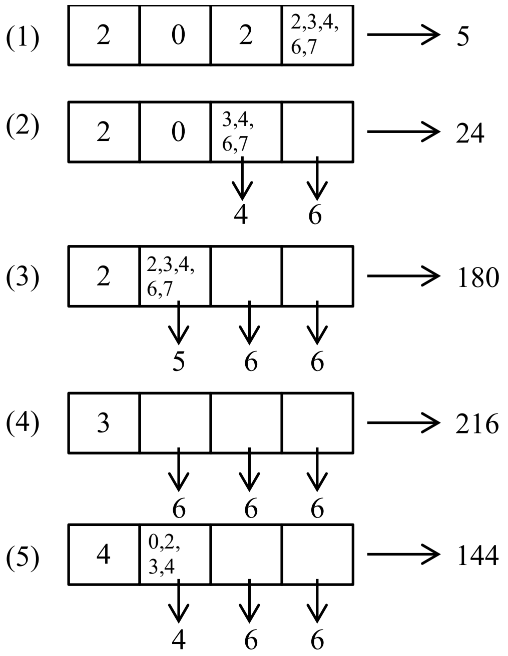
16. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 in always placed in envelope numbered 2. Then the number of ways it can be done is - **[JEE(Advanced) 2014]**

(A) 264 (B) 265 (C) 53 (D) 67

SOLUTIONS

1. Ans. (569.00)

Sol.



Number of 4 digit integers in [2022,4482]

= 5 + 24 + 180 + 216 + 144 = 569

2. Ans. (A)

Sol.

3R 2B	3R 2B	3R 2B	3R 2B
B-1	B-2	B-3	B-4

Case-I : when exactly one box provides four balls (3R 1B or 2R 2B)

Number of ways in this case

$^5C_4\,(^3C_1 \times {}^2C_1)^3 \times 4$

Case-II : when exactly two boxes provide three balls (2R 1B or 1R 2B) each

Number of ways in this case

$(^5C_3 - 1)^2\,(^3C_1 \times {}^2C_1)^2 \times 6$

Required number of ways = 21816

Language ambiguity : If we consider at least one red ball and exactly one blue ball, then required number of ways is 9504. None of the option is correct.

3. Ans. (A, B, D)

Sol. (A) $n_1 = 10 \times 10 \times 10 = 1000$

(B) As per given condition

$1 \le i < j + 2 \le 10 \Rightarrow j \le 8 \;\&\; i \ge 1$

for i = 1, 2,

j = 1, 2, 3, ..., 8 $\rightarrow$ (8 + 8) possibilities

for i = 3, j = 2, 3, ..., 8 $\rightarrow$ 7 possibilities

i = 4, j = 3, ..., 8 $\rightarrow$ 6 possibilities

i = 9, j = 1 $\rightarrow$ 1 possibility

So $n_2 = (1 + 2 + 3 + ...+ 8) + 8 = 44$

(C) $n_3 = {}^{10}C_4$ (Choose any four)

$= 210$

(D) $n_4 = {}^{10}C_4.\,4! = (210)\,(24)$

$\Rightarrow \dfrac{n_4}{12} = 420$

So correct Ans. (A), (B), (D)

4. Ans. (495.00)

Sol. Selection of 4 days out of 15 days such that no two of them are consecutive

$= {}^{15-4+1}C_4 = {}^{12}C_4$

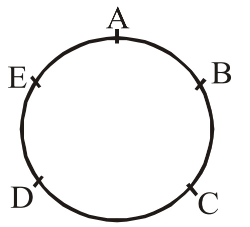
$= \dfrac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2} = 11 \times 5 \times 9 = 495$

5. Ans. (1080.00)

Sol. required ways $= \dfrac{6!}{2!\,2!\,1!\,1!\,2!\,2!} \times 4! = 1080$

6. Ans. (30.00)

Sol.



When 1R, 2B, 2G

$5C_1 \times 2 = 10$

Other possibilities

1B, 2R, 2G

or 1G, 2R, 2B

So total no. of ways = 3 × 10 = 30

7. Ans. (625)

Sol. Option for last two digits are (12), (24), (32), (44) are (52).

$\therefore$ Total No. of digits

$= 5 \times 5 \times 5 \times 5 = 625$

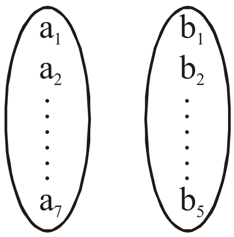
8. Ans. (119)

Sol. n(X) = 5

n(Y) = 7

$\alpha \to$ Number of one-one function = $^7C_5 \times 5!$

$\beta \to$ Number of onto function Y to X



1, 1, 1, 1, 3   1, 1, 1, 2, 2

$$\frac{7!}{3!4!}\times 5! + \frac{7!}{(2!)^3 3!}\times 5!$$

$$=\left({}^7C_3 + 3.{}^7C_3\right)5! = 4\times {}^7C_3 \times 5!$$

$$\frac{\beta-\alpha}{5!} = 4\times {}^7C_3 - {}^7C_5 = 4\times 35 - 21 = 119$$

9. Ans. (C)

Sol. (1) $\alpha_1 = \binom{6}{3}\binom{5}{2} = 200$

So P $\to$ 4

(2) $\alpha_2 = \binom{6}{1}\binom{5}{1} + \binom{6}{2}\binom{5}{2} +$

$\binom{6}{3}\binom{5}{3} + \binom{6}{4}\binom{5}{4} + \binom{6}{5}\binom{5}{5}$

$= \binom{11}{5} - 1$

= 461

So Q $\to$ 6

(3) $\alpha_3 = \binom{5}{2}\binom{6}{3} + \binom{5}{3}\binom{6}{2} + \binom{5}{4}\binom{6}{1} + \binom{5}{5}\binom{6}{0}$

$= \binom{11}{5} - \binom{5}{0}\binom{6}{5} - \binom{5}{1}\binom{6}{4}$

= 381

So R $\to$ 5

(4) $\alpha_4 = \binom{5}{2}\binom{6}{2} - \binom{4}{1}\binom{5}{1} +$

$\binom{5}{3}\binom{6}{1} - \binom{4}{2}\binom{1}{1} + \binom{5}{4} = 189$

So S $\to$ 2

10. Ans. (5)

Sol. x = 10!

$$y = {}^{10}C_1\, {}^9C_8\, \frac{10!}{2!}$$

$$\frac{y}{9x} = \frac{5.9.10!}{9.10!} = 5$$

11. Ans. (D)

Sol. $N_1 + N_2 + N_3 + N_4 + N_5$

= Total ways – {when no odd}

Total ways = $^9C_5$

Number of ways when no odd, is zero

($\because$ only available even are 2, 4, 6, 8)

$\therefore$ **Ans :** $^9C_5$ – zero = 126

12. Ans. (A)

Sol. $({}^6C_4 + {}^6C_3.{}^4C_1).{}^4C_1 = 380$

13. Ans. (5)

Sol. n = 5!6!

m = 5! $^6C_2.{}^5C_4.{}^2C_1$.4!

$$\therefore \frac{m}{n} = 5$$

14. Ans. (7)

Sol. as $n_1 \ge 1, n_2 \ge 2, n_3 \ge 3, n_4 \ge 4, n_5 \ge 5$

Let $n_1 - 1 = x_1 \ge 0$,

$n_2 - 2 = x_2 \ge 0$ ....

$n_5 - 5 = x_5 \ge 0$

$\Rightarrow$ New equation will be

$x_1 + 1 + x_2 + 2 + ... x_5 + 5 = 20$

$\Rightarrow$ $x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 15 = 5$

Now $x_1 \le x_2 \le x_3 \le x_4 \le x_5$

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So, 7 possible cases will be there.

15. Ans. (5)

Sol. Number of red line segments = $^nC_2 - n$

Number of blue line segments = n

$\therefore\ ^nC_2 - n = n$

$$\frac{n(n-1)}{2} = 2n \Rightarrow n = 5 \text{ Ans.}$$

16. Ans. (C)

Sol Total number of dearrangement

$$6!\left[\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right]$$

= 360 – 120 + 30 – 6 + 1

= 240 + 25 = 265

There are equal chances that card 1 goes into any envelope from 2 to 6

$$\therefore\ \frac{1}{5}(265) = 53$$