ALLEN®

LOGARITHM

1. The product of all positive real values of x satisfying the equation

$$x^{(16(\log_5 x)^3 - 68\log_5 x)} = 5^{-16}$$

is_____.   **[JEE(Advanced) 2022]**

2. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is _____.   **[JEE(Advanced) 2018]**

SOLUTIONS

1. **Ans. (1)**

Sol. $x^{16(\log_5 x)^3 - 68\log_5 x} = 5^{-16}$

Take log to the base 5 on both sides and put

$\log_5 x = t$

$16t^4 - 68t^2 + 16 = 0$

$\Rightarrow 4t^4 - 17t^2 + 4 = 0 \begin{cases} t_1 \\ t_2 \\ t_3 \\ t_4 \end{cases}$

$t_1 + t_2 + t_3 + t_4 = 0$

$\log_5 x_1 + \log_5 x_2 + \log_5 x_3 + \log_5 x_4 = 0$

$x_1 x_2 x_3 x_4 = 1$

2. **Ans. (8)**

Sol. $\log_2 9^{\frac{2}{\log_2(\log_2 9)}} \times 7^{\frac{1/2}{\log_4 7}}$

$= (\log_2 9)^{2\log_{\log_2 9}^2} \times 7^{\frac{1}{2}\log_7 4}$

$= 4 \times 2 = 8$