DETERMINANT

1. Let α, β and γ be real numbers. consider the following system of linear equations

$x + 2y + z = 7$ **[JEE(Advanced) 2023]**

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	List-II
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1) a unique solution
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solution
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3) infinitely many solutions
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4) $x = 11, y = -2$ and $z = 0$ as a solution
	(5) $x = -15, y = 4$ and $z = 0$ as a solution

The correct option is :

(A) (P) → (3) (Q) → (2) (R) → (1) (S) → (4)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
(D) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)

2. Let p, q, r be nonzero real numbers that are, respectively, the 10^{th}, 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equations **[JEE(Advanced) 2022]**

$$x + y + z = 1$$
$$10x + 100y + 1000z = 0$$
$$qr\,x + pr\,y + pq\,z = 0.$$

List-I		List-II	
(I)	If $\frac{q}{r} = 10$, then the system of linear equations has	(P)	$x = 0, y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II)	If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q)	$x = \frac{10}{9}, y = -\frac{1}{9}, z = 0$ as a solution
(III)	If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R)	infinitely many solutions
(IV)	If $\frac{p}{q} = 10$, then the system of linear equations has	(S)	no solution
		(T)	at least one solution

The correct option is:

(A) (I) → (T); (II) → (R); (III) → (S); (IV) → (T)

(B) (I) → (Q); (II) → (S); (III) → (S); (IV) → (R)

(C) (I) → (Q); (II) → (R); (III) → (P); (IV) → (R)

(D) (I) → (T); (II) → (S); (III) → (P); (IV) → (T)

3. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is **[JEE(Advanced) 2016]**

4. Which of the following values of α satisfy the equation $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$?

[JEE(Advanced) 2015]

(A) –4 (B) 9 (C) –9 (D) 4

SOLUTIONS

1. Ans. (A)

Sol. Given $x + 2y + z = 7$ .... (1)

$x + \alpha z = 11$ .... (2)

$2x - 3y + \beta z = \gamma$ .... (3)

Now, $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 7\alpha - 2\beta - 3$

$\therefore$ if $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \boxed{\Delta = 0}$

Now, $\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix}$

$= 21\alpha - 22\beta + 2\alpha\gamma - 33$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_x = 0$

$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix}$

$\Delta_y = 4\beta + 14\alpha - \alpha\gamma + \gamma - 22$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_y = 0$

Now, $\Delta_z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = 56 - 2\gamma$

If $\gamma = 28$

$\Rightarrow \boxed{\Delta_z = 0}$

$\therefore$ if $\gamma = 28$ and $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow$ system has infinite solution

and if $\gamma \neq 28$

$\Rightarrow$ system has no solution

$\Rightarrow P \to (3)$; $Q \to (2)$

Now if $\beta \neq \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \Delta \neq 0$

and for $\alpha = 1$ clearly

$y = -2$ is always be the solution

$\therefore$ if $\gamma \neq 28$

System has a unique solution

if $\gamma = 28$

$\Rightarrow$ $x = 11$, $y = -2$ and

$z = 0$ will be one of the solution

$\therefore R \to 1$; $S \to 4$

$\therefore$ option 'A' is correct

2. Ans. (B)

Sol. If $\frac{q}{r} = 10 \Rightarrow A = D \Rightarrow D_x = D_y = D_z = 0$

So, there are infinitely many solutions

Look of infinitely many solutions can be given as

$x + y + z = 1$

& $10x + 100y + 1000z = 0$

$\Rightarrow$ $x + 10y + 100z = 0$

Let $z = \lambda$

then $x + y = 1 - \lambda$

and $x + 10y = -100\lambda$

$\Rightarrow x = \frac{10}{9} + 10\lambda$; $y = \frac{-1}{9} - 11\lambda$

i.e., $(x, y, z) \equiv \left(\frac{10}{9} + 10\lambda, \frac{-1}{9} - 11\lambda, \lambda\right)$

$Q\left(\frac{10}{9}, \frac{-1}{9}, 0\right)$ valid for $\lambda = 0$

$P\left(0, \frac{10}{9}, \frac{-1}{9}\right)$ not valid for any λ.

(I) $\to$ Q,R,T

(II) If $\frac{p}{r} \neq 100$, then $D_y \neq 0$

So no solution

(II) $\to$ (S)

(III) If $\frac{p}{q} \neq 10$, then $D_z \neq 0$ so, no solution

(III) $\to$ (S)

(IV) If $\frac{p}{q} = 10$ $\Rightarrow D_z = 0 \Rightarrow D_x = D_y = 0$

so infinitely many solution

(IV) $\to$ Q,R,T

3. **Ans. (2)**

Sol. $$x^3\begin{vmatrix}1 & 1 & 1+x^3\\ 2 & 4 & 1+8x^3\\ 3 & 9 & 1+27x^3\end{vmatrix}=10$$

$$\Rightarrow \quad x^3\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 3 & 9\end{vmatrix}+x^3.x^3\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 8\\ 3 & 9 & 27\end{vmatrix}=0$$

$\Rightarrow \quad x^3(25-23)+6x^6.2=10$

$\Rightarrow \quad 6x^6+x^3-5=0$

$\Rightarrow \quad x^3=\frac{5}{6},-1$

two real solutions

4. **Ans. (B, C)**

Sol. $$\begin{vmatrix}1 & \alpha & \alpha^2\\ 4 & 2\alpha & \alpha^2\\ 9 & 3\alpha & \alpha^2\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 6\\ 1 & 4 & 9\end{vmatrix}=-648\alpha$$

$$\alpha^3\begin{vmatrix}1 & 1 & 1\\ 4 & 2 & 1\\ 9 & 3 & 1\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 6\\ 1 & 4 & 9\end{vmatrix}=-648\alpha$$

$-8\alpha^3=-648\alpha$

$\Rightarrow \alpha^3=81\alpha$

$\therefore\ \alpha=0,9,-9$