COMPOUND ANGLE

1. Let α and β be real numbers such that $-\frac{\pi}{4}<\beta<0<\alpha<\frac{\pi}{4}$. If $\sin(\alpha+\beta)=\frac{1}{3}$ and $\cos(\alpha-\beta)=\frac{2}{3}$, then the greatest integer less than or equal to

$$\left(\frac{\sin\alpha}{\cos\beta}+\frac{\cos\beta}{\sin\alpha}+\frac{\cos\alpha}{\sin\beta}+\frac{\sin\beta}{\cos\alpha}\right)^2$$

is _______. **[JEE(Advanced) 2022]**

2. Let α and β be nonzero real numbers such that 2(cosβ – cosα) + cosα cosβ = 1. Then which of the following is/are true ? **[JEE(Advanced) 2017]**

(A) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(B) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

(C) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

3. The value of $\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to **[JEE(Advanced) 2016]**

(A) $3-\sqrt{3}$ (B) $2(3-\sqrt{3})$ (C) $2(\sqrt{3}-1)$ (D) $2(2+\sqrt{3})$

SOLUTIONS

1. Ans. (1)

Sol. $\alpha \in \left(0, \frac{\pi}{4}\right), \beta \in \left(-\frac{\pi}{4}, 0\right) \Rightarrow \alpha + \beta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\sin(\alpha+\beta) = \frac{1}{3}, \cos(\alpha-\beta) = \frac{2}{3}$

$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\alpha}{\sin\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\sin\beta}{\cos\alpha}\right)^2$

$\left(\frac{\cos(\alpha-\beta)}{\cos\beta\sin\beta} + \frac{\cos(\beta-\alpha)}{\sin\alpha\cos\alpha}\right)^2$

$= 4\cos^2(\alpha-\beta)\left(\frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha}\right)^2$

$= 4\cos^2(\alpha-\beta)\left(\frac{2\sin(\alpha+\beta)\cos(\alpha-\beta)}{\sin 2\alpha \sin 2\beta}\right)^2 \quad \ldots(1)$

$= \frac{16\cos^4(\alpha-\beta)\sin^2(\alpha+\beta)\times 4}{\left(\cos 2(\alpha-\beta) - \cos 2(\alpha+\beta)\right)^2}$

$= \frac{64\cos^4(\alpha-\beta)\sin^2(\alpha+\beta)}{\left(2\cos^2(\alpha-\beta) - 1 - 1 + 2\sin^2(\alpha+\beta)\right)^2}$

$= 64 \times \frac{16}{81} \times \frac{1}{9} \frac{1}{\left(2\times\frac{4}{9} - 1 - 1 + \frac{2}{9}\right)^2}$

$= \frac{64\times 16}{81\times 9} \cdot \frac{81}{64} = \frac{16}{9}$

$\left[\frac{16}{9}\right] = 1$

2. Ans. (Bonus)

3. Ans. (C)

Sol. We have,

$= 2.\sum_{k=1}^{13} \frac{\sin\left(\left(\frac{k\pi}{6} + \frac{\pi}{4}\right) - \left((k-1)\frac{\pi}{6} + \frac{\pi}{4}\right)\right)}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right).\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$

$= 2\sum_{k=1}^{13}\left(\cot\left((k-1)\frac{\pi}{6} + \frac{\pi}{4}\right) - \cot\left(\frac{k\pi}{6} + \frac{\pi}{4}\right)\right)$

$= 2\left[\cot\frac{\pi}{4} - \cot\left(\frac{13\pi}{6} + \frac{\pi}{4}\right)\right]$

$= 2\left(1 - \cot\left(\frac{5\pi}{12}\right)\right)$

$= 2\left(1 - \left(2 - \sqrt{3}\right)\right) = 2\left(\sqrt{3} - 1\right)$