CIRCLE

1. Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with center at the point A = (4, 1) , where $1 < r < 3$. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If AB = $\sqrt{5}$, then the value of r^2 is **[JEE(Advanced) 2023]**

2. Let ABC be the triangle with AB = 1, AC = 3 and $\angle BAC = \frac{\pi}{2}$. If a circle of radius r > 0 touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is _______.

[JEE(Advanced) 2022]

3. Let G be a circle of radius R > 0. Let $G_1, G_2, \ldots, G_n$ be n circles of equal radius r > 0. Suppose each of the n circles $G_1, G_2, \ldots, G_n$ touches the circle G externally. Also, for i = 1, 2,…, n–1, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are TRUE ?

[JEE(Advanced) 2022]

(A) If n = 4, then $(\sqrt{2} - 1)r < R$

(B) If n = 5, then r < R

(C) If n = 8, then $(\sqrt{2} - 1)\, r < R$

(D) If n = 12, then $\sqrt{2}(\sqrt{3} + 1)\, r > R$

4. Consider a triangle Δ whose two sides lie on the x-axis and the line x + y + 1 = 0. If the orthocenter of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is

[JEE(Advanced) 2021]

(A) $x^2 + y^2 - 3x + y = 0$ (B) $x^2 + y^2 + x + 3y = 0$

(C) $x^2 + y^2 + 2y - 1 = 0$ (D) $x^2 + y^2 + x + y = 0$

Paragraph for Question No. 5 and 6

Let

$$M = \{(x,y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \le r^2\},$$

where r > 0. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}$, n = 1, 2, 3, ... Let $S_0 = 0$ and, for $n \ge 1$, let S_n denote the sum of the first n terms of this progression. For $n \ge 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

5. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then

[JEE(Advanced) 2021]

(A) $k + 2l = 22$ (B) $2k + l = 26$ (C) $2k + 3l = 34$ (D) $3k + 2l = 40$

6. Consider M with $r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

[JEE(Advanced) 2021]

(A) 198 (B) 199 (C) 200 (D) 201

7. Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is _____ **[JEE(Advanced) 2020]**

8. A line $y = mx + 1$ intersects the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ?

[JEE(Advanced) 2019]

(A) $6 \le m < 8$ (B) $2 \le m < 4$ (C) $4 \le m < 6$ (D) $-3 \le m < -1$

9. Let the point B be the reflection of the point A(2, 3) with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is _____

[JEE(Advanced) 2019]

10. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions :

(i) centre of C_3 is collinear with the centres of C_1 and C_2

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expression given in the List-I whose values are given in List-II below:

[JEE(Advanced) 2019]

	List-I		**List-II**
(I)	$2h + k$	(P)	6
(II)	$\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Which of the following is the only INCORRECT combination ?

Options :

(A) (IV), (S) (B) (IV), (U) (C) (III), (R) (D) (I), (P)

11. Answer the following by appropriately matching the lists based on the information given in the paragraph.

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions :

(i) centre of C_3 is collinear with the centres of C_1 and C_2

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expression given in the List-I whose values are given in List-II below:

[JEE(Advanced) 2019]

	List-I		**List-II**
(I)	$2h + k$	(P)	6
(II)	$\dfrac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\dfrac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\dfrac{5}{4}$
(IV)	α	(S)	$\dfrac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\dfrac{10}{3}$

Which of the following is the only CORRECT combination?

Options :

(A) (II), (T) (B) (I), (S) (C) (I), (U) (D) (II), (Q)

Paragraph for Question No. 12 and 13

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$

12. Let E_1E_2 and F_1F_2 be the chord of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slop -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents of S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3 and G_3 lie on the curve **[JEE(Advanced) 2018]**

(A) $x + y = 4$ (B) $(x-4)^2 + (y-4)^2 = 16$

(C) $(x-4)(y-4) = 4$ (D) $xy = 4$

13. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve-

[JEE(Advanced) 2018]

(A) $(x + y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

14. Let T be the line passing through the points P(–2, 7) and Q(2, –5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangents to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE ? **[JEE(Advanced) 2018]**

(A) The point (–2, 7) lies in E_1
(B) The point $\left(\frac{4}{5},\frac{7}{5}\right)$ does **NOT** lie in E_2
(C) The point $\left(\frac{1}{2},1\right)$ lies in E_2
(D) The point $\left(0,\frac{3}{2}\right)$ does **NOT** lie in E_1

15. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ? **[JEE(Advanced) 2017]**

16. The circle C_1: $x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then- **[JEE(Advanced) 2016]**

(A) $Q_2Q_3 = 12$
(B) $R_2R_3 = 4\sqrt{6}$
(C) area of the triangle OR_2R_3 is $6\sqrt{2}$
(D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

17. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1,0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. then the locus of E passes through the point(s)- **[JEE(Advanced) 2016]**

(A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$
(B) $\left(\frac{1}{4},\frac{1}{2}\right)$
(C) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$
(D) $\left(\frac{1}{4},-\frac{1}{2}\right)$

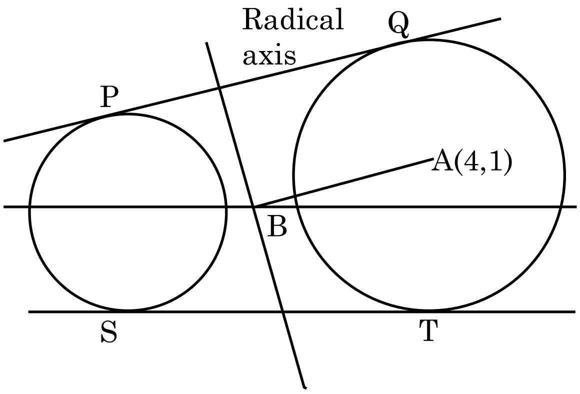
18. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then :- **[JEE(Advanced) 2014]**

(A) radius of S is 8
(B) radius of S is 7
(C) centre of S is (–7, 1)
(D) centre is S is (–8, 1)

SOLUTIONS

1. Ans. (2)

Sol.



radical axis $8x + 2y - 17 = 1 - r^2$

$8x + 2y = 18 - r^2$

$B\left(\frac{18-r^2}{8}, 0\right)$ A(4, 1)

$AB = \sqrt{5}$

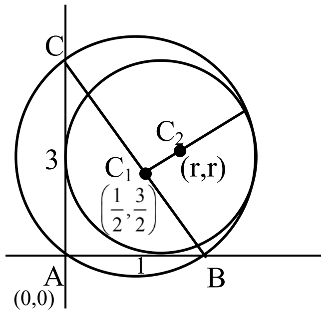
$\sqrt{\left(\frac{18-r^2}{8} - 4\right)^2 + 1} = \sqrt{5}$

$r^2 = 2$

$\Rightarrow n = \sin\alpha + \cos\alpha$

2. Ans. (0.83 or 0.84)

Sol. $4 - \sqrt{10} = 0.83$ or 0.84



$C_1\left(\frac{1}{2}, \frac{3}{2}\right)$ and $r_1 = \frac{\sqrt{10}}{2}$

$C_2 = (r, r)$

$\therefore$ circle C_2 touches C_1 internally

$\Rightarrow C_1C_2 = \left|r - \frac{\sqrt{10}}{2}\right|$

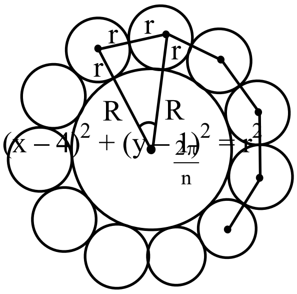
$\Rightarrow \left(r - \frac{1}{2}\right)^2 + \left(r - \frac{3}{2}\right)^2 = \left(r - \frac{\sqrt{10}}{2}\right)^2$

$r^2 - 4r + \sqrt{10}r = 0$

$r = 0$ (reject) or $r = 4 - \sqrt{10}$

3. Ans. (C, D)

Sol.



Let $C_2\ (x-4)^2 + (y-1)^2 = r^2$

$2(R + r)\sin\frac{\pi}{n} = 2r$

$\frac{R + r}{r} = \operatorname{cosec}\frac{\pi}{n}$

(A) $n = 4$, $R + r = \sqrt{2}\, r$

(B) $n = 5$, $\frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{5} < \operatorname{cosec}\frac{\pi}{6}$

$R + r < 2r \Rightarrow r > R$

(C) $n = 8$, $\frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{8} > \operatorname{cosec}\frac{\pi}{4}$

$R + r > \sqrt{2}\, r$

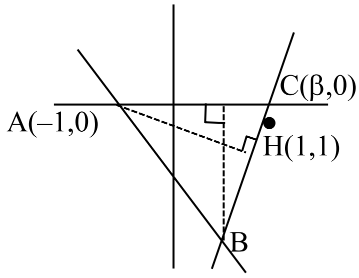
(D) $n = 12$, $\frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{12} = \sqrt{2}\left(\sqrt{3} + 1\right)$

$R + r = \sqrt{2}\left(\sqrt{3} + 1\right)r$

$\sqrt{2}\left(\sqrt{3} + 1\right)r > R$

4. Ans. (B)

Sol.



$(1,-2) = (\alpha, -\alpha-1)$

$\Rightarrow \alpha = 1$

one of the vertex is intersection of x-axis and $x + y + 1 = 0 \Rightarrow A(-1,0)$

Let vertex B be $(\alpha, -\alpha-1)$

Line AC $\perp$ BH $\Rightarrow \alpha = 1 \Rightarrow B(1,-2)$

Let vertex C be$(\beta, 0)$

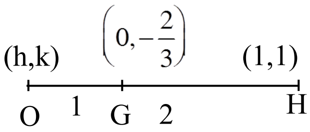
Line AH $\perp$ BC

$m_{AH}.m_{BC} = -1$

$\frac{1}{2}.\frac{2}{\beta - 1} = -1 \Rightarrow \beta = 0$

Centroid of ΔABC is $\left(0, -\frac{2}{3}\right)$

Now G(centroid) divides line joining circum centre (O) and ortho centre (H) in the ratio 1: 2

$\Rightarrow$ 

$2h + 1 = 0 \quad 2k + 1 = -2$

$h = -\frac{1}{2} \quad k = -\frac{3}{2}$

$\Rightarrow$ circum centre is $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

Equation of circum circle is

(passing through C(0,0)) is

$x^2 + y^2 + x + 3y = 0$

5. Ans. (D)

Sol. $S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{n-1}}$

$= 2\left(1 - \frac{1}{2^n}\right) = 2 - \frac{1}{2^{n-1}}$

Centre of C_n is $\left(2 - \frac{1}{2^{n-2}}, 0\right)$

and radius of C_n is $\frac{1}{2^{n-1}}$

when $r = \frac{1025}{513} < 2$

C_n will lie inside m

when $2 - \frac{1}{2^{n-2}} + \frac{1}{2^{n-1}} < \frac{1025}{513}$

$\Rightarrow k = 10$

Also $\ell = 5$

$3k + 2\ell = 30 + 10 = 40$

6. Ans. (B)

Sol. Center of D_n is (S_{n-1}, S_{n-1})

$r = \frac{1}{2^{n-1}}$

D_n will lie inside

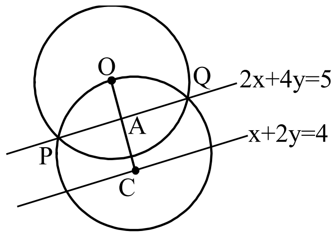
when $\sqrt{2}(S_{n-1}) + a_n < \frac{2^{199} - 1}{2^{198}}\sqrt{2}$

$\Rightarrow \frac{\sqrt{2}}{2^{n-2}} > \frac{\sqrt{2}}{2^{198}} + \frac{1}{2^{n-1}}$

$\Rightarrow n = 199$

7. Ans. (2)

Sol.



M-I

$OA = \frac{\sqrt{5}}{2} \quad OC = \frac{4}{\sqrt{5}}$

$CQ = OC = \frac{4}{\sqrt{5}}$ and $CA = \frac{3}{2\sqrt{5}}$

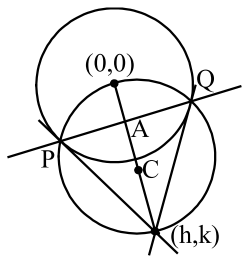
$\therefore \quad OQ = \sqrt{OA^2 + AQ^2}$

$= \sqrt{OA^2 + (CQ^2 - CA^2)}$

$\Rightarrow \quad \sqrt{\frac{5}{4} + \frac{16}{5} - \frac{9}{20}} = \sqrt{4}$

$\Rightarrow \quad 2 = r$

M-II



PQ : $hx + ky = r^2$

Given PQ $2x + 4y = 5$

$\Rightarrow \quad \frac{h}{2} = \frac{k}{4} = \frac{r^2}{5} \Rightarrow h = \frac{2r^2}{5} \quad k = \frac{4r^2}{5}$

$\therefore \quad C = \left(\frac{r^2}{5}, \frac{2r^2}{5}\right)$

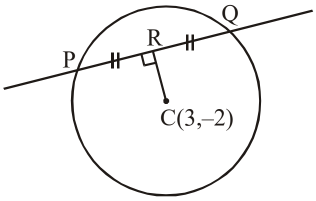
$\therefore \quad$ C lies on $x + 2y = 4$

$\Rightarrow \quad \frac{r^2}{5} + 2\left(\frac{2r^2}{5}\right) = 4$

$\Rightarrow \quad r^2 = 4 \qquad \Rightarrow r = 2$

8. Ans. (B)

Sol.



$$R \equiv \left(-\frac{3}{5}, \frac{-3m}{5}+1\right)$$

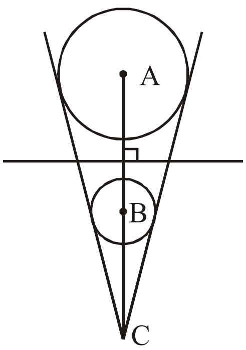
$$\text{So, } m\left(\frac{-\frac{3m}{5}+3}{-\frac{3}{5}-3}\right) = -1$$

$\Rightarrow m^2 - 5m + 6 = 0$

$\Rightarrow m = 2, 3$

9. Ans. (10.00)

Sol.



Distance of point A from given line = $\frac{5}{2}$

$$\frac{CA}{CB} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{AC}{AB} = \frac{2}{1}$$

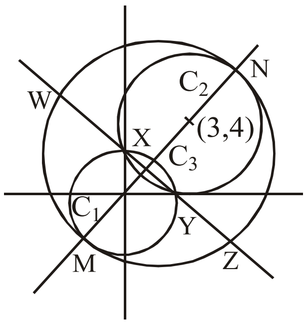
$$\Rightarrow \quad AC = 2 \times 5 = 10$$

Q.10 Ans. (A)

Q.11 Ans. (D)

Solution for Q.10 and Q.11

Sol.



$MC_1 + C_1C_2 + C_2N = 2r$

$\Rightarrow 3 + 5 + 4 = 2r \Rightarrow r = 6 \Rightarrow$ Radius of $C_3 = 6$

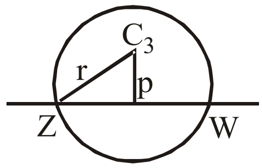
Suppose centre of C_3 be

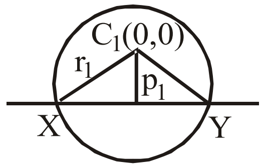
$$\left(0 + r_4\cos\theta, 0 + r_4\sin\theta\right), \left\{\begin{matrix} r_4 = C_1C_3 = 3 \\ \tan\theta = \frac{4}{3}\end{matrix}\right\}$$

$$C_3 = \left(\frac{9}{5}, \frac{12}{5}\right) = (h, k) \Rightarrow 2h + k = 6$$

Equation of ZW and XY is $3x + 4y - 9 = 0$

(common chord of circle $C_1 = 0$ and $C_2 = 0$)





$$ZW = 2\sqrt{r^2 - p^2} = \frac{24\sqrt{6}}{5}$$

(where r = 6 and $p = \frac{6}{5}$)

$$XY = 2\sqrt{r_1^2 - p_1^2} = \frac{24}{5}$$

(where $r_1 = 3$ and $p_1 = \frac{9}{5}$)

$$\frac{\text{Length of ZW}}{\text{Length of XY}} = \sqrt{6}$$

Let length of perpendicular from M to ZW be λ, $\lambda = 3 + \frac{9}{5} = \frac{24}{5}$

$$\frac{\text{Area of }\Delta MZN}{\text{Area of }\Delta ZMW} = \frac{\frac{1}{2}(MN)\times\frac{1}{2}(ZW)}{\frac{1}{2}\times ZW \times \lambda}$$

$$= \frac{1}{2}\frac{MN}{\lambda} = \frac{5}{4}$$

$$C_3 : \left(x - \frac{9}{5}\right)^2 + \left(y - \frac{12}{5}\right)^2 = 6^2$$

$C_1 : x^2 + y^2 - 9 = 0$

common tangent to C_1 and C_3 is common chord of C_1 and C_3 is $3x + 4y + 15 = 0$.

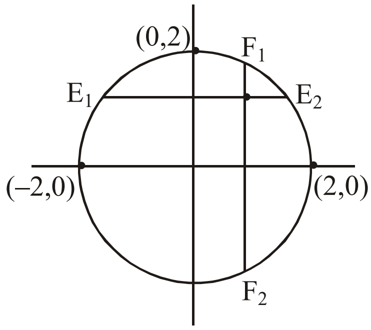
Now 3x + 4y + 15 = 0 is tangent to parabola $x^2 = 8\alpha y$.

$$x^2 = 8\alpha\left(\frac{-3x-15}{4}\right) \Rightarrow 4x^2 + 24\alpha x + 120\alpha = 0$$

$$D = 0 \Rightarrow \alpha = \frac{10}{3}$$

12. Ans. (A)

Sol.



co-ordinates of E_1 and E_2 are obtained by solving y = 1 and $x^2 + y^2 = 4$

$\therefore \quad E_1\left(-\sqrt{3},1\right)$ and $E_2\left(\sqrt{3},1\right)$

co-ordinates of F_1 and F_2 are obtained by solving x = 1 and $x^2 + y^2 = 4$

$F_1\left(1,\sqrt{3}\right)$ and $F_2\left(1,-\sqrt{3}\right)$

Tangent at E_1 : $-\sqrt{3}x + y = 4$

Tangent at E_2 : $\sqrt{3}x + y = 4$

$\therefore \quad E_3\,(0, 4)$

Tangent at F_1 : $x + \sqrt{3}y = 4$

Tangent at F_2 : $x - \sqrt{3}y = 4$

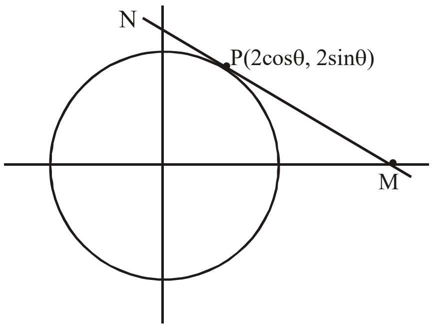
$\therefore \quad F_3(4, 0)$

and similarly $G_3(2, 2)$

(0, 4), (4, 0) and (2, 2) lies on x + y = 4

13. Ans. (D)

Sol.



Tangent at P(2cosθ, 2sinθ) is xcosθ + ysinθ = 2

M(2secθ, 0) and N(0, 2cosecθ)

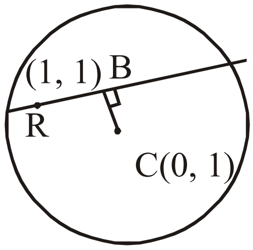
Let midpoint be (h, k)

h = secθ, k = cosecθ

$$\frac{1}{h^2} + \frac{1}{k^2} = 1$$

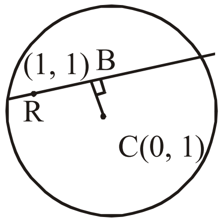
$$\frac{1}{x^2} + \frac{1}{y^2} = 1$$

14. Ans. (B, D)



AP = AQ = AM

Locus of M is a circle having PQ as its diameter



Hence, E_1 : (x – 2) (x + 2) + (y – 7)(y + 5) = 0 and $x \neq \pm 2$

Locus of B (midpoint)

is a circle having RC as its diameter

E_2 : $x(x-1) + (y-1)^2 = 0$

Now, after checking the options, we get (D)

The points $\left(\frac{4}{5},\frac{7}{5}\right)$, (1, 1), (–2, 7) are collinear therefore (B) option is also correct.

15. Ans. (2)

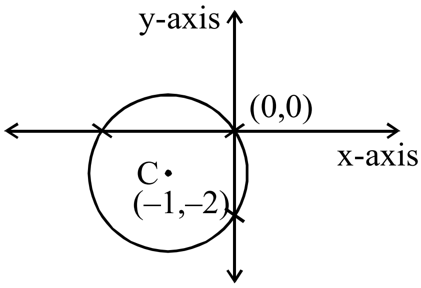
Sol. We shall consider 3 cases.

Case I : When p = 0

(i.e. circle passes through origin)

Now, equation of circle becomes

$x^2 + y^2 + 2x + 4y = 0$



Case II : When circle intersects x-axis at 2 distinct points and touches y-axis

Now $(g^2 - c) > 0$ & $f^2 - c = 0$

$\Rightarrow$ $1 - (-p) > 0$ & $4 - (-p) = 0$

$\Rightarrow$ $p = -4$

$\Rightarrow$ $p > -1$

$\therefore$ Not possible.

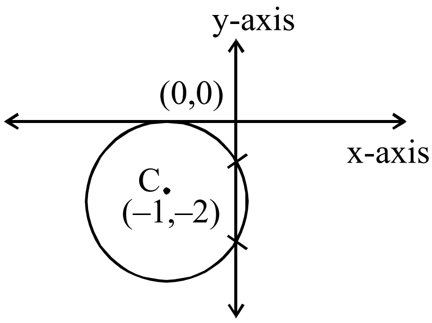
Case III : When circle intersects y-axis at 2 distinct points & touches x-axis.

Now, $g^2 - c = 0$ & $f^2 - c > 0$

$\Rightarrow$ $1 - (-p) = 0$ & $4 - (-p) > 0$

$\Rightarrow$ $p = -1$ $\Rightarrow$ $p > -4$

$\therefore$ $p = -1$ is possible.



$\therefore$ Finally we conclude that $p = 0, -1$

$\Rightarrow$ Two possible values of p.

16. Ans. (A, B, C)

Sol. On solving $x^2 + y^2 = 3$ and $x^2 = 2y$ we get point $P(\sqrt{2},1)$

Equation of tangent at P

$\sqrt{2}.x + y = 3$

Let Q_2 be (0,k) and radius is $2\sqrt{3}$

$$\therefore \left|\frac{\sqrt{2}(0)+k-3}{\sqrt{2+1}}\right| = 2\sqrt{3}$$

$\therefore$ k = 9, – 3

$Q_2(0,9)$ and $Q_3(0,-3)$

hence $Q_2Q_3 = 12$

R_2R_3 is internal common tangent of circle C_2 and C_3

$$\therefore R_2R_3 = \sqrt{(Q_2Q_3)^2 - (2\sqrt{3}+2\sqrt{3})^2}$$

$$= \sqrt{12^2 - 48} = \sqrt{96} = 4\sqrt{6}$$

Perpendicular distance of origin O from R_2R_3 is equal to radius of circle $C_1 = \sqrt{3}$

Hence area of ΔOR_2R_3

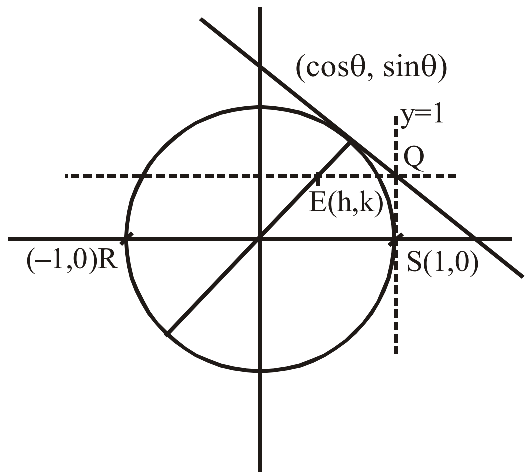
$$= \frac{1}{2}\times(R_2R_3)\sqrt{3} = \frac{1}{2}.4\sqrt{6}.\sqrt{3} = 6\sqrt{2}$$

Perpendicular Distance of P from $Q_2Q_3 = \sqrt{2}$

$$\therefore \text{Area of } \Delta PQ_2Q_3 = \frac{1}{2}\times 12\times\sqrt{2} = 6\sqrt{2}$$

17. Ans. (A, C)

Sol.



Tangent at P : x cosθ + y sinθ = 1 ....(i)

Tangent at S : x = 1 ....(ii)

$\therefore$ By (i) & (ii) : $Q\left(1, \frac{1-\cos\theta}{\sin\theta}\right)$

Line through Q parallel to RS :

$$y = \frac{1-\cos\theta}{\sin\theta} \Rightarrow y = \tan\frac{\theta}{2} \quad ....(iii)$$

Normal at P :

$$y = \frac{\sin\theta}{\cos\theta}x \Rightarrow y = \tan\theta.x \quad ....(iv)$$

Point of intersection of equation (iii) and (iv),

$$E: \ h = \frac{1-\tan^2\frac{\theta}{2}}{2}; k = \tan\frac{\theta}{2}$$

eliminating θ : $h = \frac{1-k^2}{2} \Rightarrow y^2 = 1-2x$

Options (A) and (C) satisfies the locus.

18. Ans. (B,C)

Sol. Let circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Put (0,1) $1 + 2f + c = 0$ ....(1)

orthogonal with

$x^2 + y^2 - 2x - 15 = 0$

$2g(-1) = c - 15 \Rightarrow c = 15 - 2g$ ....(2)

orthogonal with

$x^2 + y^2 - 1 = 0$

$c = 1$ ....(3)

$\Rightarrow g = 7$ & $f = -1$

centre is $(-g, -f) \equiv (-7,1)$

radius $= \sqrt{g^2 + f^2 - c} = \sqrt{49 + 1 - 1} = 7$