3D GEOMETRY

1. Let Q be the cube with the set of vertices $\left\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \{0,1\}\right\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1, \ell_2)$ denote the shortest distance between them.

Then the maximum value of $d(\ell_1, \ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

[JEE(Advanced) 2023]

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

2. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}_2 = \left(\hat{j}-\hat{k}\right)+\mu\left(\hat{i}+\hat{k}\right)$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X. **[JEE(Advanced) 2023]**

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	List-II
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H_0 is	(3) 0
(S) The distance of origin from the point of intersection of planes y = z, x = 1 and H_0 is	(4) $\sqrt{2}$
	(5) $\frac{1}{\sqrt{2}}$

The correct option is :

(A) (P) → (2) (Q) → (4) (R) → (5) (S) → (1)

(B) (P) → (5) (Q) → (4) (R) → (3) (S) → (1)

(C) (P) → (2) (Q) → (1) (R) → (3) (S) → (2)

(D) (P) → (5) (Q) → (1) (R) → (4) (S) → (2)

3. Let P_1 and P_2 be two planes given by

$$P_1: 10x + 15y + 12z - 60 = 0,$$

$$P_2: -2x + 5y + 4z - 20 = 0.$$

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2? **[JEE(Advanced) 2022]**

(A) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$ (B) $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$

(C) $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$ (D) $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

4. Let S be the reflection of a point Q with respect to the plane given by

$$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$$

where t, p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are TRUE ? **[JEE(Advanced) 2022]**

(A) $3(\alpha + \beta) = -101$ (B) $3(\beta + \gamma) = -71$

(C) $3(\gamma + \alpha) = -86$ (D) $3(\alpha + \beta + \gamma) = -121$

Question Stem for Question Nos. 5 and 6

Question Stem

Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let |M| represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the **square** of the distance of the point (0, 1, 0) from the plane P.

5. The value of |M| is _________. **[JEE(Advanced) 2021]**

6. The value of D is _________. **[JEE(Advanced) 2021]**

7. Let L_1 and L_2 be the following straight lines.

$L_1 : \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3}$ and $L_2 : \frac{x-1}{-3} = \frac{y}{-1} = \frac{z-1}{1}$

Suppose the straight line

$L : \frac{x-\alpha}{l} = \frac{y-1}{m} = \frac{z-\gamma}{-2}$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE? **[JEE(Advanced) 2020]**

(A) $\alpha - \gamma = 3$ (B) $\ell + m = 2$ (C) $\alpha - \gamma = 1$ (D) $\ell + m = 0$

8. Let α, β, γ, δ be real numbers such that $\alpha^2 + \beta^2 + \gamma^2 \neq 0$ and $\alpha + \gamma = 1$. Suppose the point (3,2,–1) is the mirror image of the point (1,0,–1) with respect to the plane $\alpha x + \beta y + \gamma z = \delta$. Then which of the following statements is/are TRUE ? **[JEE(Advanced) 2020]**

(A) $\alpha + \beta = 2$ (B) $\delta - \gamma = 3$ (C) $\delta + \beta = 4$ (D) $\alpha + \beta + \gamma = \delta$

9. Let L_1 and L_2 denotes the lines

$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$ and

$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \mathbb{R}$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ? **[JEE(Advanced) 2019]**

(A) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(B) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(C) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(D) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

10. Three lines are given by

$$\vec{r} = \lambda\hat{i}, \ \lambda \in \mathbb{R}$$

$$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in \mathbb{R} \text{ and}$$

$$\vec{r} = \nu(\hat{i} + \hat{j} + \hat{k}), \nu \in \mathbb{R}.$$

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals _____. **[JEE(Advanced) 2019]**

11. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE ? **[JEE(Advanced) 2018]**

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, –1

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point (4, 2, –2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_2 is $\frac{2}{\sqrt{3}}$

12. Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____. **[JEE(Advanced) 2018]**

13. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT.

If $\vec{p} = \overrightarrow{SP}$, $\vec{q} = \overrightarrow{SQ}$, $\vec{r} = \overrightarrow{SR}$ and $\vec{t} = \overrightarrow{ST}$, then the value of $\left|(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})\right|$ is_____.

[JEE(Advanced) 2018]

14. The equation of the plane passing through the point (1,1,1) and perpendicular to the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7, is- **[JEE(Advanced) 2017]**

(A) 14x + 2y + 15z = 31

(B) 14x + 2y – 15z = 1

(C) –14x + 2y + 15z = 3

(D) 14x – 2y + 15z = 27

15. Consider a pyramid OPQRS located in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then-

[JEE(Advanced) 2016]

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equaiton of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

16. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is **[JEE(Advanced) 2016]**

(A) $x + y - 3z = 0$ (B) $3x + z = 0$

(C) $x - 4y + 7z = 0$ (D) $2x - y = 0$

17. In $\mathbb{R}^3$, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0,1,0) from P_3 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relations is (are) true ?

[JEE(Advanced) 2015]

(A) $2\alpha + \beta + 2\gamma + 2 = 0$ (B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$ (D) $2\alpha - \beta + 2\gamma - 8 = 0$

18. In $\mathbb{R}^3$, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M ? **[JEE(Advanced) 2015]**

(A) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ (B) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$ (C) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ (D) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

19. From a point $P(\lambda,\lambda,\lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

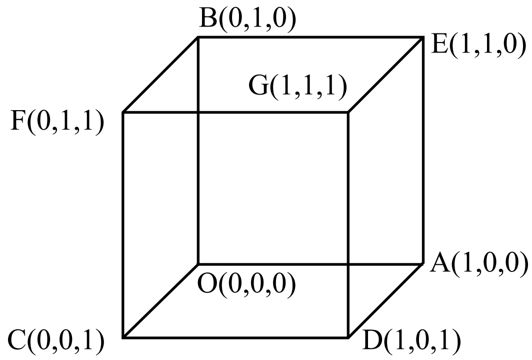
[JEE(Advanced) 2014]

(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$

SOLUTIONS

1. Ans. (A)

Sol.



DR'S of OG = 1, 1, 1

DR'S of AF = –1, 1, 1

DR'S of CE = 1, 1, –1

DR'S of BD = 1, –1, 1

Equation of OG $\Rightarrow \frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

Equation of AB $\Rightarrow \frac{x-1}{1}=\frac{y}{-1}=\frac{z}{0}$

Normal to both the line's

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix}=\hat{i}+\hat{j}-2\hat{k}$$

$\overrightarrow{OA}=\hat{i}$

$$\text{S.D.} = \frac{\left|\hat{i}.(\hat{i}+\hat{j}-2\hat{k})\right|}{\left|\hat{i}+\hat{j}-2\hat{k}\right|}=\frac{1}{\sqrt{6}}$$

2. Ans. (B)

Sol. $L_1 : \vec{r}_1=\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$

$L_2 : \vec{r}_2=\hat{j}-\hat{k}+\mu\left(\hat{i}+\hat{k}\right)$

Let system of planes are

$ax + by + cz = 0$ ....(1)

$\because$ It contain L_1

$\therefore a + b + c = 0$ ....(2)

For largest possible distance between plane (1) and L_2 the line L_2 must be parallel to plane (1)

$\therefore a + c = 0$ .... (3)

$\Rightarrow \boxed{b=0}$

$\therefore$ Plane H_0 : $\boxed{x-z=0}$

Now $d(H_0) = \perp$ distance from point (0, 1, –1) on L_2 to plane.

$$\Rightarrow d\left(H_0\right)=\left|\frac{0+1}{\sqrt{2}}\right|=\frac{1}{\sqrt{2}}$$

$\therefore P \rightarrow 5$

for 'Q' distance = $\left|\frac{2}{\sqrt{2}}\right|=\sqrt{2}$

$\therefore Q \rightarrow 4$

$\therefore$ (0, 0, 0) lies on plane

$\therefore R \rightarrow 3$

for 'S' x = z ; y = z ; x = 1

$\therefore$ point of intersection p(1, 1, 1).

$\therefore$ $OP=\sqrt{1+1+1}=\sqrt{3}$

$\therefore S \rightarrow 2$

$\therefore$ option (B) is correct

3. Ans. (A, B, D)

Sol. line of intersection is $\frac{x}{0}=\frac{y-4}{-4}=\frac{z}{5}$

(1) Any skew line with the line of intersection of given planes can be edge of tetrahedron.

(2) any intersecting line with line of intersection of given planes must lie either in plane P_1 or P_2 can be edge of tetrahedron.

4. Ans. (A, B, C)

Sol. $\vec{r}=\hat{k}+t\left(-\hat{i}+\hat{j}\right)+p\left(-\hat{i}+\hat{k}\right)$

$\vec{n}=\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow x + y + z = 1$

Q(10,15,20) and S(α,β,γ)

$$\frac{\alpha-10}{1}=\frac{\beta-15}{1}=\frac{\gamma-20}{1}=-2\left(\frac{10+15+20-1}{1+1+1}\right)$$

$$=-\frac{88}{3}$$

$$\Rightarrow\left(\alpha,\beta,\gamma\right)\equiv\left(-\frac{58}{3},-\frac{43}{3},-\frac{28}{3}\right)$$

$\Rightarrow$ A, B, C are correct options

5. Ans. (1.00)

6. Ans. (1.50)

Solutions for 5 & 6

Sol. $7x + 8y + 9z - (\gamma - 1) = A(4x + 5y + 6z - \beta) + B(x + 2y + 3z - \alpha)$

$x : 7 = 4A + B$

$y : 8 = 5A + 2B$

$A = 2, B = -1$

const. term : $-(\gamma - 1) = -A\beta - \alpha B$

$\Rightarrow -(\gamma - 1) \equiv 2\beta + \alpha$

$\alpha - 2\beta + \gamma = 1$

$$M = \begin{pmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \Rightarrow |M| = \alpha - 2\beta + \gamma = 1$$

Plane P : $x - 2y + z = 1$

Perpendicular distance

$$= \left|\frac{3}{\sqrt{6}}\right| = P \Rightarrow D = P^2 = \frac{9}{6} = 1.5$$

7. Ans. (A, B)

Sol. Point of intersection of L_1 & L_2 is (1, 0, 1)

Line L passes through (1, 0, 1)

$$\frac{1-\alpha}{\ell} = -\frac{1}{m} = \frac{1-\gamma}{-2} \qquad ...(1)$$

acute angle bisector of L_1 & L_2

$$\vec{r} = \hat{i} + \hat{k} + \lambda\left(\frac{\hat{i} - \hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} + \hat{k}}{\sqrt{11}}\right)$$

$$\vec{r} = \hat{i} + \hat{k} + t\left(\hat{i} + \hat{j} - 2\hat{k}\right)$$

$$\Rightarrow \frac{\ell}{1} = \frac{m}{1} = \frac{-2}{-2} \quad \Rightarrow \quad \ell = m = 1$$

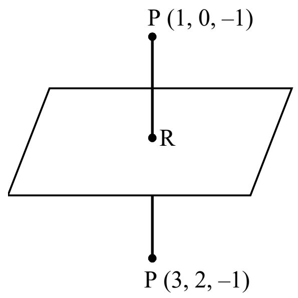
From (1)

$$\frac{1-\alpha}{1} = -1 \quad \Rightarrow \quad \alpha = 2$$

$$\& \ \frac{1-\gamma}{-2} = -1 \quad \Rightarrow \quad \gamma = -1$$

8. Ans. (A, B, C)

Sol.



R is mid point of PQ

$\therefore$ R(2,1,–1) and it lies on plane

equation of plane is $\alpha a + \beta y + \gamma z = \delta$

$\therefore 2\alpha + \beta - \gamma = \delta \qquad ...(1)$

Normal vector to plane is

$\vec{n} = 2i + 2j$

$$\therefore \frac{\alpha}{2} = \frac{\beta}{2} = \frac{\gamma}{0} = k$$

$\therefore \alpha = 2k, \beta = 2k, \gamma = 0 \qquad ...(2)$

and $\alpha + \gamma = 1$ (given) $\qquad ...(3)$

from (2) and (3)

$\therefore \alpha = 1, \beta = 1, \gamma = 0$

and from (1)

$2(1) + 1 - 0 = \delta$

$\delta = 3$

Now :

$\alpha + \beta = 2$

$\delta - \gamma = 3$

$\delta + \beta = 4$

so, A,B,C are correct.

9. Ans. (A, B, D)

Sol. Points on L_1 and L_2 are respectively

$A(1 - \lambda, 2\lambda, 2\lambda)$ and $B(2\mu, -\mu, 2\mu)$

So, $\overrightarrow{AB} = (2\mu + \lambda - 1)\hat{i} + (-\mu - 2\lambda)\hat{j} + (2\mu - 2\lambda)\hat{k}$

and vector along their shortest distance

$= 2\hat{i} + 2\hat{j} - \hat{k}$.

Hence, $\dfrac{2\mu + \lambda - 1}{2} = \dfrac{-\mu - 2\lambda}{2} = \dfrac{2\mu - 2\lambda}{-1}$

$$\Rightarrow \quad \lambda = \frac{1}{9} \ \& \ \mu = \frac{2}{9}$$

Hence, $A \equiv \left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$ and $B \equiv \left(\frac{4}{9}, -\frac{2}{9}, \frac{4}{9}\right)$

$\Rightarrow$ Mid point of AB $\equiv \left(\frac{2}{3}, 0, \frac{1}{3}\right)$

10. Ans. (0.75)

Sol. $A(1,0,0)$, $B\left(\frac{1}{2},\frac{1}{2},0\right)$ & $C\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Hence,

$$\overrightarrow{AB}=-\frac{1}{2}\hat{i}+\frac{1}{2}\hat{j} \;\&\; \overrightarrow{AC}=-\frac{2}{3}\hat{i}+\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}$$

So, $\Delta=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{\frac{1}{2}\times\frac{2}{3}-\frac{1}{4}}$

$$=\frac{1}{2\times2\sqrt{3}}$$

$\Rightarrow \quad (6\Delta)^2=\frac{3}{4}=0.75$

11. Ans. (C, D)

Sol. D.C. of line of intersection (a, b, c)

$\Rightarrow \quad 2a+b-c=0$

$a+2b+c=0$

$$\frac{a}{1+2}=\frac{b}{-1-2}=\frac{c}{4-1}$$

$\therefore$ D.C. is $(1,-1,1)$

(B) $\frac{3x-4}{9}=\frac{1-3y}{9}=\frac{z}{3}$

$\Rightarrow \quad \frac{x-4/3}{3}=\frac{y-1/3}{-3}=\frac{z}{3}$

$\Rightarrow$ lines are parallel.

(C) Acute angle between P_1 and P_2

$$=\cos^{-1}\left(\frac{2\times1+1\times2-1\times1}{\sqrt{6}\sqrt{6}}\right)$$

$$=\cos^{-1}\left(\frac{3}{6}\right)=\cos^{-1}\left(\frac{1}{2}\right)=60^\circ$$

(D) Plane is given by

$(x-4)-(y-2)+(z+2)=0$

$\Rightarrow \quad x-y+z=0$

Distance of (2, 1, 1) from plane

$$=\frac{2-1+1}{\sqrt{3}}=\frac{2}{\sqrt{3}}$$

12. Ans. (8)

Sol. Let $P(\alpha,\beta,\gamma)$

$Q(0,0,\gamma)$ &

$R(\alpha,\beta,-\gamma)$

Now, $\overline{PQ}\parallel\hat{i}+\hat{j}\Rightarrow\left(\alpha\hat{i}+\beta\hat{j}\right)\parallel\left(\hat{i}+\hat{j}\right)$

$\Rightarrow \alpha=\beta$

Also, mid point of PQ lies on the plane

$\Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}=3\Rightarrow\alpha+\beta=6\Rightarrow\alpha=3$

Now, distance of point P from X-axis is $\sqrt{\beta^2+\gamma^2}=5$

$\Rightarrow \beta^2+\gamma^2=25 \;\Rightarrow\; \gamma^2=16$

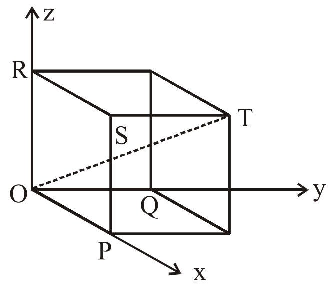
as $\beta=\alpha=3$

as $\gamma=4$

Hence, $PR=2\gamma=8$

13. Ans. (0.5)

Sol.



$$\vec{p}=\overline{SP}=\left(\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}-\hat{j}-\hat{k}\right)$$

$$\vec{q}=\overline{SQ}=\left(-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}+\hat{j}-\hat{k}\right)$$

$$\vec{r}=\overline{SR}=\left(-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}-\hat{j}+\hat{k}\right)$$

$$\vec{t}=\overline{ST}=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|=\frac{1}{4}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&-1\\-1&1&-1\end{vmatrix}\times\frac{1}{4}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&-1&1\\1&1&1\end{vmatrix}$$

$$=\frac{1}{16}\left|\left(2\hat{i}+2\hat{j}\right)\times\left(-2\hat{i}+2\hat{j}\right)\right|=\left|\frac{\hat{k}}{2}\right|=\frac{1}{2}$$

14. Ans. (A)

Sol. The normal vector of required plane is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2\hat{j} - 15\hat{k}$$

$\therefore$ The equation of required plane passing through (1, 1, 1) will be

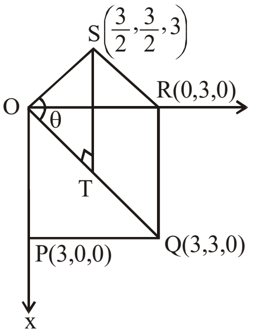
$-14(x - 1) - 2(y - 1) - 15(z - 1) = 0$

$\Rightarrow \boxed{14x + 2y + 15z = 31}$

$\therefore$ Option (A) is correct

15. Ans. (B, C, D)

Sol.



Given OP = OR = 3 and OPQR is a square

$\Rightarrow$ OQ = $3\sqrt{2}$ $\Rightarrow$ OT $= \frac{3}{\sqrt{2}}$ and ST = 3

using ΔSOT, $\tan\theta = \frac{ST}{OT} = \sqrt{2}$

$\Rightarrow$ $\theta = \tan^{-1}\sqrt{2}$

clearly, equation of plane containing triangle OQS is Y – X = 0

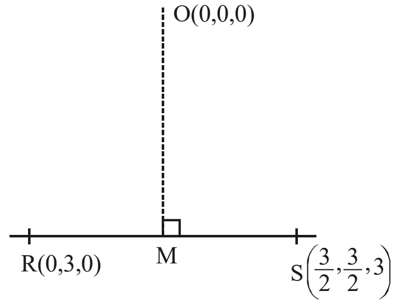
Also, length of perpendicular from P to the plane containing the triangle OQS is PT = $\frac{3}{\sqrt{2}}$

Also equation of RS is

$$\bar{r} = 3\hat{j} + t\left(\frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$$

$$= \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$$

Let co-ordinates of $M = \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$



$\because$ $\overrightarrow{OM}.\overrightarrow{RS} = 0$

$\Rightarrow \frac{9}{4}t - \frac{3}{2}\left(3 - \frac{3t}{2}\right) + 9t = 0$

$\Rightarrow \frac{9t}{2} + 9t = \frac{9}{2}$ $\Rightarrow t = \frac{1}{3}$

$\therefore$ $M = \left(\frac{1}{2}, \frac{5}{2}, 1\right)$

$\Rightarrow OM = \sqrt{\frac{1}{4} + \frac{25}{4} + 1} = \sqrt{\frac{30}{4}} = \sqrt{\frac{15}{2}}$

16. Ans. (C)

Sol. Line AP : $\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = \lambda$

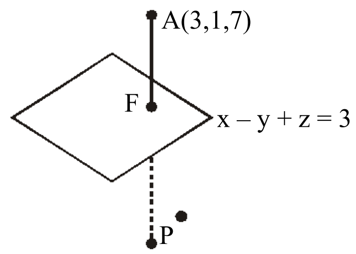
$\Rightarrow$ F(3 + λ, 1 – λ, λ + 7) lies in the plane

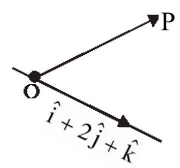
$\therefore$ 3 + λ – (1 – λ) + λ + 7 = 3

3λ = –6 ⇒ λ = –2

$\Rightarrow$ F(1,3,5)

$\Rightarrow$ P(–1,5,3)





so required plane is $\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 2 & 1 \\ -1 & 5 & 3 \end{vmatrix} = 0$

$\therefore$ x – 4y + 7z = 0

17. Ans. (B, D)

Sol. Let $P_3 : (x + z - 1) + \lambda y = 0$

$x + \lambda y + z - 1 = 0$ ...(i)

distance of (0,1,0) from P_3 is 1

$$\Rightarrow \frac{|\lambda - 1|}{\sqrt{2+\lambda^2}} = 1$$

$$\Rightarrow (\lambda - 1)^2 = 2 + \lambda^2$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

$\therefore$ P_3 is $2x - y + 2z - 2 = 0$

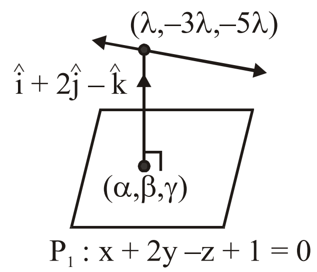
distance from (α,β,γ) is $\left|\frac{2\alpha - \beta + 2\gamma - 2}{\sqrt{9}}\right| = 2$

$\therefore$ $2\alpha - \beta + 2\gamma - 2 = 6$ or $2\alpha - \beta + 2\gamma - 2 = -6$

$2\alpha - \beta + 2\gamma - 8 = 0$ or $2\alpha - \beta + 2\gamma + 4 = 0$

18. Ans. (A, B)

Sol. Straight line 'L' is parallel to line of intersection of plane P_1 & plane P_2.



$\therefore$ Equation of line 'L' is

$$\frac{x}{1} = \frac{y}{-3} = \frac{z}{-5} = \lambda$$

$$\frac{\alpha - \lambda}{1} = \frac{\beta + 3\lambda}{2} = \frac{\gamma + 5\lambda}{-1} = k$$

$$\left.\begin{array}{l}\alpha = k + \lambda \\ \beta = 2k - 3\lambda \\ y = -k - 5\lambda\end{array}\right\} ...(1)$$

satisfying in plane P_1

$k + \lambda + 4k - 6\lambda + k + 5\lambda + 1 = 0$

$6k = -1$

putting in (1) required locus is

$$x = -\frac{1}{6} + \lambda$$

$$y = -\frac{1}{3} - 3\lambda$$

$$z = \frac{1}{6} - 5\lambda$$

Now check the options.

19. Ans. (C)

Sol. Line L_1 given by $y = x$; $z = 1$ can be expressed as

$$L_1: \frac{x}{1} = \frac{y}{1} = \frac{z-1}{0}$$

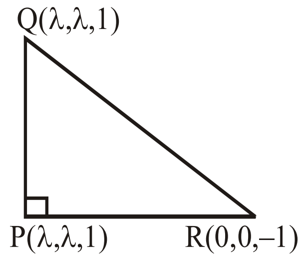
Similarly $L_2(y = -x; z = -1)$ can be expressed as

$$L_2 : \frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0}$$

Let any point Q $(\alpha,\alpha,1)$ on L_1 and $R(\beta,-\beta,-1)$ on L_2

Given that PQ is perpendicular to L_1

$\Rightarrow (\lambda - \alpha).1 + (\lambda - \alpha).1 + (\lambda - 1).0 = 0 \Rightarrow \lambda = \alpha$



$\therefore$ $Q(\lambda, \lambda, 1)$

Similarly PR is perpendicular to L_2

$(\lambda - \beta).1 + (\lambda + \beta)(-1) + (\lambda + 1).0 = 0 \Rightarrow \beta = 0$

$\therefore$ $R(0,0,-1)$

Now as given

$$\Rightarrow \overrightarrow{PR}.\overrightarrow{PQ} = 0$$

$0.\lambda + 0.\lambda + (\lambda - 1)(\lambda + 1) = 0$

$\lambda \neq 1$ as P & Q are different points $\Rightarrow \lambda = -1$