PRACTICE QUESTION PAPER - 2 (SOLUTION)

CLASS – XII

Session : 2022 – 23

Subject : Mathematics

Time : 3 Hrs. **Max. Marks : 80**

SECTION – A

1. **(B)**

Given $A = A^T$ and $B = B^T$

$(AB - BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T$

$= BA - AB$

$= -(AB - BA)$

$\therefore$ $AB - BA$ is a skew symmetric matrix.

2. **(D)**

Given, $|A| = 5, n = 3$

Let adj A = B

Now, $|B| = |\text{adj } A|$

$= |A|^{n-1}$

$= 5^2 = 25$

Now, $|\text{adj } B| = |B|^{n-1}$

$= 25^2 = 625$

$\therefore$ $|\text{adj (adj } A)| = 625$.

3. **(C)**

We have, $\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}$

and $\vec{a} - \vec{b} = 4\hat{i} + (7 - \lambda)\hat{k}$

Since $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$ are orthogonal

$\therefore (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$

$\Rightarrow \left[6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}\right] \cdot \left[4\hat{i} + (7 - \lambda)\hat{k}\right] = 0$

$\Rightarrow 24 + (7 + \lambda)(7 - \lambda) = 0$

$\Rightarrow \lambda^2 = 73$

$\Rightarrow \lambda = \pm\sqrt{73}$

4. **(D)**

We have, $f(0) = 1$

$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h)$

$= \lim_{h \to 0} \frac{|\sin h|}{h} = 1$

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$\lim_{h \to 0} \frac{|\sin(-h)|}{-h} = -1$

$\therefore \lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

Hence, $f(x)$ is discontinuous at $x = 0$

Since $f(x)$ is discontinuous at $x = 0$, so it is not differentiable at x = 0.

5. **(A)**

Let $I = \int \frac{x^3}{x+1} dx$

$I = \int \left[\left(x^2 - x + 1\right) - \frac{1}{x+1} \right] dx$

$I = \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + C$

$\therefore I = x - \frac{x^2}{2} + \frac{x^3}{3} - \log|x+1| + C$

6. **(D)**

Order = 2, degree is not defined.

7. **(B)** The co-ordinates of the corner points of the feasible region are

$A\left(\frac{18}{7}, \frac{2}{7}\right), B\left(\frac{7}{2}, \frac{3}{4}\right), C\left(\frac{3}{2}, \frac{15}{4}\right), D\left(\frac{3}{13}, \frac{24}{13}\right)$

Corner Points	$Z = 5x + 2y$
$A\left(\frac{18}{7}, \frac{2}{7}\right)$	$Z_A = \frac{94}{7}$
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	$Z_B = 19$
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	$Z_C = 15$
$D\left(\frac{3}{13}, \frac{24}{13}\right)$	$Z_D = \frac{63}{13}$

So, maximum value of $Z = 19$ at $B\left(\frac{7}{2}, \frac{3}{4}\right)$

8. (D)

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{(\hat{i}-\hat{j})\cdot(-\hat{j}+\hat{k})}{|-\hat{j}+\hat{k}|}$

$= \frac{1}{\sqrt{2}}$

9. **(B)**

Let $I = \int x^x(1+\log x)dx$

Put $x^x = t \Rightarrow x^x(1+\log x)dx = dt$

$\therefore I = \int 1\cdot dt$

$= t + C$

$I = x^x + C$

10. **(A)**

Given $B = \begin{bmatrix} 2 & a & 5 \\ -1 & 4 & b \\ c & -4 & 9 \end{bmatrix}$ is a symmetric matrix

$\therefore B^T = B$

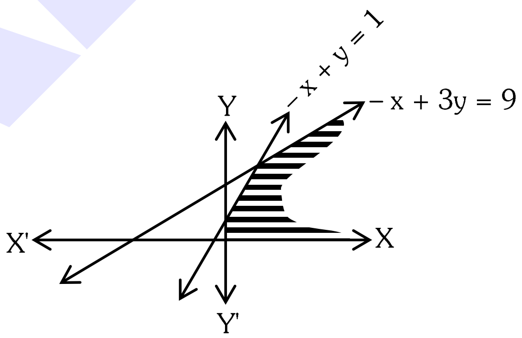
$\Rightarrow \begin{bmatrix} 2 & -1 & c \\ a & 4 & -4 \\ 5 & b & 9 \end{bmatrix} = \begin{bmatrix} 2 & a & 5 \\ -1 & 4 & b \\ c & -4 & 9 \end{bmatrix}$

$\Rightarrow a = -1, b = -4, c = 5$

$\therefore a + b + c = 0$

11. **(B)**

Unbounded feasible region.



12. **(C)**

We have, $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

$\Rightarrow 5(-2x-12) - 3(14-18) - 1(-42-9x) = 0$

$\Rightarrow x = -6$

13. (B)

$$A^2-4A=\begin{bmatrix}1&4\\2&3\end{bmatrix}\begin{bmatrix}1&4\\2&3\end{bmatrix}-4\begin{bmatrix}1&4\\2&3\end{bmatrix}$$

$$=\begin{bmatrix}9&16\\8&17\end{bmatrix}-\begin{bmatrix}4&16\\8&12\end{bmatrix}$$

$$=\begin{bmatrix}5&0\\0&5\end{bmatrix}$$

$$=5\begin{bmatrix}1&0\\0&1\end{bmatrix}=5I$$

14. (D)

We have $P(A'\cup B')=\frac{1}{4}$

$$\Rightarrow P((A\cap B)')=\frac{1}{4}$$

$$\Rightarrow P(A\cap B)=1-\frac{1}{4}=\frac{3}{4}$$

Since $P(A\cap B)\neq 0,$ so A & B are not mutually exclusive

Also $P(A)\times P(B)=\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}\neq P(A\cap B)$

So, A & B are not independent.

15. (C)

Given, $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}}dy=-\frac{1}{\sqrt{1-x^2}}dx$$

Integrating both side

$$\int\frac{1}{\sqrt{1-y^2}}dy=-\int\frac{1}{\sqrt{1-x^2}}dx$$

$$\sin^{-1}y=-\sin^{-1}x+C\Rightarrow \sin^{-1}y+\sin^{-1}x=C$$

16. (B)

We have, $y=\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

Put $x=\cot\theta\Rightarrow\theta=\cot^{-1}x$

$$\therefore y=\cos^{-1}\left(\frac{\cot^2\theta-1}{\cot^2\theta+1}\right)$$

$\Rightarrow y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \cos^{-1}(\cos 2\theta)$

$\Rightarrow y = 2\theta$

$\Rightarrow y = 2\cot^{-1} x$

differentiating w.r.t. x

$\frac{dy}{dx} = \frac{-2}{1+x^2}$

17. (B)

$|\vec{a}\times\vec{b}| = 35$

$|\vec{a}||\vec{b}|\sin\theta = 35$

$\Rightarrow \sin\theta = \frac{35}{\sqrt{26}\times 7} = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{25}{26}} = \frac{1}{\sqrt{26}}$

Now, $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta$

$= \sqrt{26}\times 7\times\frac{1}{\sqrt{26}} = 7$

18. (C)

We have, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

19. (A)

Let $\sin^{-1}x = A$ ...(i)

$\Rightarrow x = \sin A$

$\Rightarrow x = \cos\left(\frac{\pi}{2} - A\right)$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - A$ ...(ii)

(i) + (ii) $\Rightarrow \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\therefore$ Reason is true

Now, $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$

$= \sin\left(\frac{\pi}{2}\right) = 1$

$\therefore$ Assertion is true

Hence both A and R are true and R is the correct explanation of A.

20. **(D)**

We know equation of line passes through $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$\therefore$ Reason is true.

Now, equation of line passes through $A(2,-1,4)$ & $B(1,1,-2)$ is $\frac{x-2}{1-(2)}=\frac{y+1}{1-(-1)}=\frac{z-4}{-2-4}$

$$\Rightarrow \frac{x-2}{-1}=\frac{y+1}{2}=\frac{z-4}{-6}$$

$\therefore$ Assertion is false.

Hence, A is false but R is true.

SECTION – B

21. Let $\sin^{-1}\frac{3}{5}=A,\ \sin^{-1}\frac{5}{13}=B$

Then, $A,\ B\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\Rightarrow \cos A>0$ and $\cos B>0$

$$\Rightarrow \sin A=\frac{3}{5} \text{ and } \sin B=\frac{5}{13}$$

Now, $\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

$\cos B=\sqrt{1-\sin^2 B}=\sqrt{1-\frac{25}{169}}=\frac{12}{13}$ **[1]**

Now $\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)=\cos(A+B)$

$$=\cos A\cos B-\sin A\sin B$$

$$=\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}$$

$=\frac{33}{65}$ **[1]**

OR

Let $x_1,x_2\in R$, such that

If $f(x_1)=f(x_2)\Rightarrow x_1^3=x_2^3$

$$\Rightarrow x_1^3-x_2^3=0$$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

$\Rightarrow (x_1 - x_2)\left[\left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3}{4}x_2^2\right] = 0$

$\Rightarrow x_1 - x_2 = 0 \qquad \left[\because \left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3}{4}x_2^2 \neq 0\right]$

$\Rightarrow x_1 = x_2$ **[1]**

$\therefore$ f is one-one

Let $y \in R$

Put $y = f(x) \Rightarrow y = x^3$

$\Rightarrow x = y^{1/3}$

Thus, for each y in the co-domain R there exists $y^{1/3}$ in R

such that $f(y^{1/3}) = (y^{1/3})^3 = y$

$\therefore f$ is onto **[1]**

Hence f is one-one and onto

22. At any instant t, let r be the radius, v the volume and s the surface area of the balloon. Then,

$\frac{ds}{dt} = 2\,cm^2/sec$ (given)

Now $s = 4\pi r^2$

$\Rightarrow \frac{ds}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$2 = 8\pi r\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r}$ ......(i) **[1]**

Now, $v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$\Rightarrow \frac{dv}{dt} = 4\pi r^2 \cdot \left(\frac{1}{4\pi r}\right)$ [from (i)]

$\frac{dv}{dt} = r$

at r = 6 cm $\quad \frac{dv}{dt} = 6\ cm^3/sec$ **[1]**

23. Given, $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$, $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$

vector perpendicular to $\vec{a}$ & $\vec{b}$ is parallel to $\vec{a} \times \vec{b}$.

$\therefore$ Let required vector $\vec{c} = \lambda(\vec{a} \times \vec{b})$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix} = 18\hat{i} - 18\hat{j} + 9\hat{k}$ **[1]**

$\therefore \vec{c} = \lambda(18\hat{i} - 18\hat{j} + 9\hat{k})$

Also, $|\vec{c}| = 3$ (given)

$\therefore \lambda^2(18^2 + (-18)^2 + 9^2) = 3^2$

$\Rightarrow \lambda = \pm\frac{1}{9}$

$\therefore \vec{c} = \pm\frac{1}{9}(18\hat{i} - 18\hat{j} + 9\hat{k})$

$\vec{c} = \pm(2\hat{i} - 2\hat{j} + \hat{k})$ **[1]**

OR

The given lines are

$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda$ (say) .... (1)

$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu$ (say) .... (2)

Any point on (1) is $P(3\lambda+1, 2\lambda-1, 5\lambda+1)$

and on (2) is $Q(4\mu-2, 3\mu+1, -2\mu-1)$

If possible, let the given line intersect

Then, P and Q coincide for same particular values of λ & u.

In that case, we have,

$3\lambda+1 = 4\mu-2,\ 2\lambda-1 = 3\mu+1,\ 5\lambda+1 = -2\mu-1$ **[1]**

$\Rightarrow 3\lambda - 4\mu = -3$ ....(i)

$2\lambda - 3\mu = 2$ ....(ii)

$5\lambda + 2\mu = -2$ ....(iii)

Solving (i) & (ii), we get $\lambda = -17$ and $u = -12$

However, these value of λ and μ do not satisfy (iii)

Hence, the given lines do not intersect. **[1]**

24. We have, $x = 3\sin t - \sin 3t$

$\Rightarrow \dfrac{dx}{dt} = 3\cos t - 3\cos 3t$ ...(i)

Also, $y = 3\cos t - \cos 3t$

$\Rightarrow \dfrac{dy}{dt} = -3\sin t + 3\sin 3t$ ...(ii)

$\therefore \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-3\sin t + 3\sin 3t}{3\cos t - 3\cos 3t}$ **[1]**

$= \dfrac{\sin 3t - \sin t}{\cos t - \cos 3t}$

$= \dfrac{2\cos 2t \sin t}{-2\sin 2t \sin t} = -\cot 2t$

$\therefore \dfrac{d^2y}{dx^2} = 2\text{cosec}^2 2t \cdot \dfrac{dt}{dx}$

$= \dfrac{2\text{cosec}^2 2t}{3\cos t - 3\cos 3t}$

at $t = \dfrac{\pi}{3}, \dfrac{d^2y}{dx^2} = \dfrac{2\text{cosec}^2\left(2\dfrac{\pi}{3}\right)}{3\left(\cos\dfrac{\pi}{3} - \cos\pi\right)}$

$= \dfrac{16}{27}$ **[1]**

25. Position vector of A, $\overrightarrow{OA} = \hat{i} + \hat{j} + \hat{k}$

Position vector of B, $\overrightarrow{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$ and

Position vector of C, $\overrightarrow{OC} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \hat{j} + 2\hat{k}$

and $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= \hat{i} + 2\hat{j}$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix}$

$= -4\hat{i} + 2\hat{j} - \hat{k}$ **[1]**

Now, Area of $\Delta ABC = \dfrac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \dfrac{1}{2}\left|-4\hat{i} + 2\hat{j} - \hat{k}\right|$

$= \dfrac{1}{2}\sqrt{16 + 4 + 1} = \dfrac{\sqrt{21}}{2}$ sq. units **[1]**

SECTION – C

26. Let $I=\int\sqrt{2x^2+3x+4}\,dx$

$$I=\sqrt{2}\int\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{23}}{4}\right)^2}\,dx$$ **[1]**

Put $x+\frac{3}{4}=t$

$\Rightarrow dx=dt$

$$\therefore\ I=\sqrt{2}\int\sqrt{t^2+\left(\frac{\sqrt{23}}{4}\right)^2}\,dt$$

$$I=\sqrt{2}\left[\frac{t}{2}\sqrt{t^2+\frac{23}{16}}+\frac{23}{32}\log\left|t+\sqrt{t^2+\frac{23}{16}}\right|\right]+C$$ **[1]**

$$\left[\because\ \int\sqrt{t^2+a^2}\,dt=\frac{t}{2}\sqrt{t^2+a^2}+\frac{a^2}{2}\log\left|t+\sqrt{t^2+a^2}\right|+C\right]$$

$$\Rightarrow I=\frac{\sqrt{2}}{2}\left(x+\frac{3}{4}\right)\cdot\sqrt{\left(x+\frac{3}{4}\right)^2+\frac{23}{16}}+\frac{23\sqrt{2}}{32}\log\left|\left(x+\frac{3}{4}\right)+\sqrt{\left(x+\frac{3}{4}\right)^2+\frac{23}{16}}\right|+C$$

$$I=\frac{(4x+3)\sqrt{2x^2+3x+4}}{8}+\frac{23\sqrt{2}}{32}\log\left|\frac{4x+3}{4}+\frac{\sqrt{2x^2+3x+4}}{\sqrt{2}}\right|+C$$ **[1]**

27. Let E_1, E_2 and E_3 be the event that the problem is solved by three students respectively. Then,

$$P(E_1)=\frac{1}{2},\ P(E_2)=\frac{1}{3},\ P(E_3)=\frac{1}{4}$$

$$\Rightarrow\ P(\bar{E}_1)=\frac{1}{2},\ P(\bar{E}_2)=\frac{2}{3},\ P(\bar{E}_3)=\frac{3}{4}$$

P (exactly one of them solves the problem)

$$=P\left[(E_1\cap\bar{E}_2\cap\bar{E}_3)\cup(\bar{E}_1\cap E_2\cap\bar{E}_3)\cup(\bar{E}_1\cap\bar{E}_2\cap E_3)\right]$$ **[1]**

$$=P(E_1\cap\bar{E}_2\cap\bar{E}_3)+P(\bar{E}_1\cap E_2\cap\bar{E}_3)+P(\bar{E}_1\cap\bar{E}_2\cap E_3)$$

$$=P(E_1)\times P(\bar{E}_2)\times P(\bar{E}_3)+P(\bar{E}_1)\times P(E_2)\times P(\bar{E}_3)+P(\bar{E}_1)\times P(\bar{E}_2)\times P(E_3)$$ **[1]**

$$=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}+\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}$$

$$=\frac{11}{24}$$ **[1]**

OR

Here, X can take the value 0,1, 2 or 3

P(X = 0) = P (getting no red ball)

$$= \frac{{}^4C_3}{{}^7C_3} = \frac{4}{35}$$

P(X = 1) = P (getting 1 red and 2 white balls)

$$= \frac{{}^3C_1 \times {}^4C_2}{{}^7C_3} = \frac{18}{35}$$

P(X = 2) = P (getting 2 red and 1 white ball)

$$= \frac{{}^3C_2 \times {}^4C_1}{{}^7C_3} = \frac{12}{35}$$

P(x = 3) = P (getting 3 red balls) $= \frac{{}^3C_3}{{}^7C_3} = \frac{1}{35}$ **[1]**

Thus, probability distribution of X is given below.

$X = x_i$	0	1	2	3
$P(x_i)$	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

$\therefore$ Mean $\mu = \sum x_i p_i = \left(0 \times \frac{4}{35}\right) + \left(1 \times \frac{18}{35}\right) + \left(2 \times \frac{12}{35}\right) + \left(3 \times \frac{1}{35}\right) = \frac{9}{7}$ **[2]**

28. Let $I = \int_0^{\pi/2} \left(\frac{1}{3 + 2\cos x}\right) dx$

$$= \int_0^{\pi/2} \frac{1}{3 + 2\left(\frac{1 - \tan^2 x/2}{1 + \tan^2 x/2}\right)} dx$$

$$= \int_0^{\pi/2} \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 5} dx$$ **[1]**

Put $\tan\frac{x}{2} = t$

$\Rightarrow \sec^2\frac{x}{2} dx = 2dt$

also, when $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$\therefore I = 2\int_0^1 \frac{1}{t^2 + \left(\sqrt{5}\right)^2} dt$ **[1]**

$= 2\frac{1}{\sqrt{5}}\left[\tan^{-1}\frac{t}{\sqrt{5}}\right]_0^1$

$= \frac{2}{\sqrt{5}}\tan^{-1}\frac{1}{\sqrt{5}}$ **[1]**

OR

Let $I = \int_0^{\pi} x\sin^3 x\,dx$ ....(i)

$I = \int_0^{\pi}(\pi - x)\sin^3(\pi - x)dx$ $\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$ **[1]**

$I = \int_0^{\pi}(\pi - x)\sin^3 x\,dx$ ....(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi}\pi\sin^3 x\,dx$ **[1]**

$2I = \int_0^{\pi}\frac{\pi}{4}(3\sin x - \sin 3x)dx$

$\therefore\ I = \frac{\pi}{8}\int_0^{\pi}(3\sin x - \sin 3x)dx$

$I = \frac{\pi}{8}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^{\pi}$

$I = \frac{\pi}{8}\left[\left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right]$

$I = \frac{\pi}{8}\times\frac{16}{3} = \frac{2\pi}{3}$ **[1]**

29. We have, $\frac{dy}{dx} = \frac{e^x\left(\sin^2 x + \sin 2x\right)}{y(2\log y + 1)}$

$\Rightarrow y(2\log y + 1)dy = e^x\left(\sin^2 x + \sin 2x\right)dx$ **[1]**

Integrating both sides.

$\Rightarrow \int y(2\log y + 1)dy = \int e^x\left(\sin^2 x + \sin 2x\right)dx$

$\Rightarrow 2\int y\log y\,dy + \int y\,dy = \int e^x\left(\sin^2 x + \sin 2x\right)dx$

$$\Rightarrow 2\left[\log y \cdot \frac{y^2}{2} - \int \frac{1}{y} \cdot \frac{y^2}{2} dy\right] + \frac{y^2}{2} = e^x \sin^2 x + C$$ **[1]**

$$\Rightarrow y^2 \log y - \int y\, dy + \frac{y^2}{2} = e^x \sin^2 x + C$$

$$\Rightarrow y^2 \log y = e^x \sin^2 x + C$$

is the required solution. **[1]**

OR

The given differential equation may be written as

$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$, which is homogeneous differential equation.

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore\ v + x\frac{dv}{dx} = \frac{x^2 - 2v^2x^2 + vx^2}{x^2}$$ **[1]**

$$\Rightarrow v + x\frac{dv}{dx} = 1 - 2v^2 + v$$

$$\Rightarrow x\frac{dv}{dx} = 1 - 2v^2$$

$$\Rightarrow \frac{1}{1-2v^2} dv = \frac{1}{x} dx$$ **[1]**

On integrating both sides

$$\int \frac{1}{1-2v^2} dv = \int \frac{1}{x} dx$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2 - v^2} dv = \int \frac{1}{x} dx$$

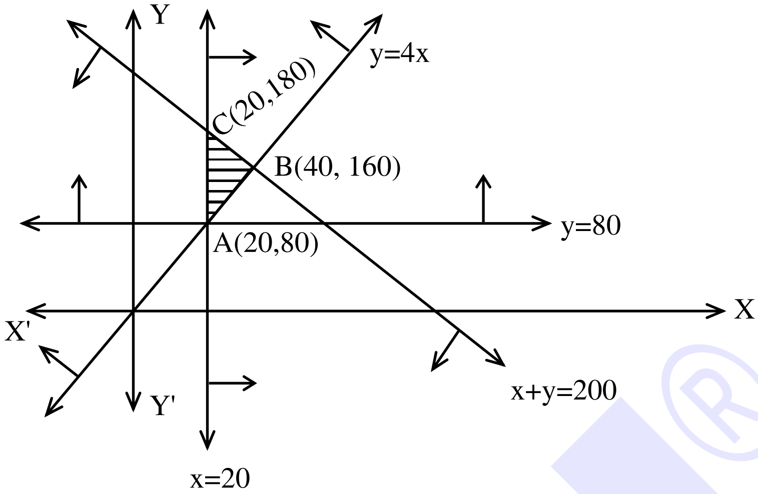
$$\Rightarrow \frac{1}{2\sqrt{2}} \log\left|\frac{\frac{1}{\sqrt{2}} + v}{\frac{1}{\sqrt{2}} - v}\right| = \log|x| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log\left|\frac{x + \sqrt{2}y}{x - \sqrt{2}y}\right| - \log|x| = C \qquad \left[\because v = \frac{y}{x}\right]$$ **[1]**

This is the required solution.

30. Maximize $Z = 400x + 300y$

Subject to constraints: $x \geq 20$, $y \geq 4x$, $y \geq 80$ and $x + y \leq 200$.



[1]

Corner points	Z = 400 x + 300 y
A (20, 80)	32000
B (40, 160)	64000
C (20, 180)	62000

$\therefore$ maximum value of Z is 64000 at x = 40 and y = 160 **[2]**

31. Let $I = \int \frac{x^3 - 1}{x^3 + x} dx$

$I = \int \left(1 - \frac{x+1}{x(x^2+1)}\right) dx$ ...(i)

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\Rightarrow x + 1 = A(x^2+1) + (Bx+C)\cdot x$ ...(ii) **[1]**

Put $x = 0 \Rightarrow A = 1$

On comparing coefficient of x in (ii), we get C = 1

On comparing coefficient of x^2 in (ii), we get B = –1

Thus, $\frac{x+1}{x(x^2+1)} = \frac{1}{x} + \frac{1-x}{x^2+1}$ **[1]**

$\therefore I = \int \left(1 - \frac{1}{x} - \frac{1-x}{x^2+1}\right) dx$

$\Rightarrow I = \int 1.dx - \int \frac{1}{x} dx - \int \frac{1}{x^2+1} dx + \int \frac{x}{x^2+1} dx$

$= x - \log|x| - \tan^{-1} x + \frac{1}{2}\log(x^2+1) + C$ **[1]**

SECTION – D

32. Consider the equations

$x^2 = y$ ...(i)

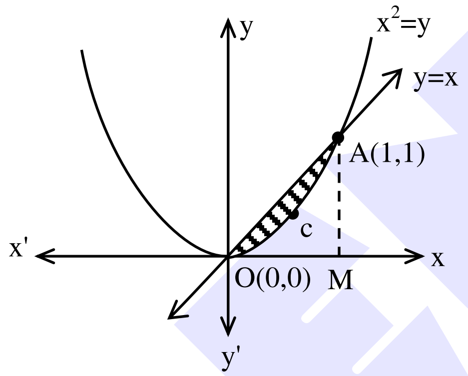
$x = y$ ...(ii)

Solving simultaneously we get

$x^2 = x$

$\Rightarrow x = 0 \text{ or } x = 1$

$\therefore\ y = 0 \text{ or } y = 1$ **[1]**



[1]

So point of intersection of (i) & (ii) are O (0, 0) and A (1, 1)
Required area = Area OAMO – Area OCAMO

$= \int_0^1 y_{line}dx - \int_0^1 y_{parabola}dx$ **[1]**

$= \int_0^1 xdx - \int_0^1 x^2dx$

$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

$= \left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{6}$ sq. units **[2]**

33. Given, $A = \{1,2,3,4,5,6,7\}$

and R = {(a, b) : both a and b are either odd or even}

(i) Reflexivity : let $a \in A$

Then, it is clear that a and a are both odd or both even

$\therefore (a,a) \in R\ \forall a \in A$, So, R is reflexive **[1½]**

(ii) Symmetry : Let $a, b \in A$

If $(a,b)\in R \Rightarrow$ a and b are both odd or both even

$\Rightarrow$ b and a are also both odd or both even

$\Rightarrow (b,a)\in R$, so, R is symmetric **[1½]**

(iii)Transitivity : Let $a,b,c\in A$

If (a, b) $\in$ R $\Rightarrow$ a and b are both even or both odd ...(1)

and $(b,c)\in R\Rightarrow$ b and c are both even or both odd ...(2)

from (1) & (2) a and c are both even or both odd

$\Rightarrow (a,c)\in R$

$\therefore$ R is transitive

Since R is reflexive, symmetric and transitive therefore R is an equivalence relation. **[2]**

OR

Given S be the set of all points in a plane,

R = { (A, B) : d (A, B) < 2 units}

(i) Reflexivity : let $A\in S$

d (A, A) = 0 < 2

$\Rightarrow (A,A)\in R\ \forall A\in S$

So R is reflexive **[1]**

(ii) Symmetry: Let $A,B\in S$

If $(A,B)\in R\Rightarrow d(A,B)<2$

$\Rightarrow d(B,A)<2$

$\Rightarrow (B,A)\in R$

So, R is symmetric **[2]**

(iii)Transitivity : Consider the $A(0,0),B(1\cdot 5,0),C(3,0)$

Then, $d(A,B)=1\cdot 5,\ d(B,C)=1\cdot 5,\ d(A,C)=3$

Since $d(A,B)<2$ and $d(B,C)<2$

$\Rightarrow (A,B)\in R$ and $(B,C)\in R$

But $d(A,C)>2$

$\Rightarrow (A,C)\notin R$

$\therefore$ R is not transitive. **[2]**

34. Given, $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$

and $\vec{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

On comparing the given equation with standard equations

$\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$, we have

$\vec{a}_1=\left(\hat{i}+2\hat{j}+\hat{k}\right),\ \vec{b}_1=\hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2=\left(2\hat{i}-\hat{j}-\hat{k}\right),\ \vec{b}_2=2\hat{i}+\hat{j}+2\hat{k}$

$\therefore\ \vec{a}_2-\vec{a}_1=\hat{i}-3\hat{j}-2\hat{k}$ **[1]**

and $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}=-3\hat{i}+3\hat{k}$ **[1]**

$\Rightarrow\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{(-3)^2+3^2}=3\sqrt{2}$

Now, $\text{S.D.}=\left|\dfrac{\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$ **[1]**

$=\left|\dfrac{\left(-3\hat{i}+3\hat{k}\right)\cdot\left(\hat{i}-3\hat{j}-2\hat{k}\right)}{3\sqrt{2}}\right|=\left|\dfrac{-3-6}{3\sqrt{2}}\right|$

$=\dfrac{3\sqrt{2}}{2}$ units **[2]**

OR

The given lines are

$L_1:\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$

$L_2:\vec{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$

On comparing L_1 & L_2 with standard equation

$\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ & $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

$\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}_1=\hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2=2\hat{i}-\hat{j}-\hat{k},\ \vec{b}_2=\hat{i}-\hat{j}+\hat{k}$ **[1]**

Now, $\vec{a}_2-\vec{a}_1=\hat{i}-3\hat{j}-4\hat{k}$

Clearly $\vec{b}_1=\vec{b}_2=\vec{b}$ (let)

$\therefore L_1$ & L_2 are parallel. **[1]**

Now, $\vec{b}\times(\vec{a}_2-\vec{a}_1)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 1 & -3 & -4\end{vmatrix}$

$=7\hat{i}+5\hat{j}-2\hat{k}$ **[1]**

Also, $|\vec{b}|=\sqrt{1^2+(-1)^2+1^2}=\sqrt{3}$ **[1]**

Shortest distances between 2 parallel lines

$=\dfrac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}$

$=\dfrac{|(7\hat{i}+5\hat{j}-2\hat{k})|}{\sqrt{3}}$

$=\dfrac{\sqrt{49+25+4}}{\sqrt{3}}$

$=\sqrt{26}$ units **[1]**

35. The given equation are

$x-y+2z=1$

$2y-3z=1$

$3x-2y+4z=2$

Let $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$ **[1]**

Then, the given system in matrix form is AX = B.

Now, $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow A\cdot\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}=I$

$\Rightarrow A^{-1}=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ **[2]**

Now, AX = B

$\Rightarrow X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 0, y = 5, z = 3$ **[2]**

SECTION – E

36. (i) Since x is the number of days after 1st July.

$\therefore$ Price = Rs. (300 – 3x)

Quantity = (80 + x) quintals. **[1]**

(ii) Revenue R(x) = Price × Quantity

R(x) = (300 – 3x) (80 + x)

$\Rightarrow R(x) = 24000 + 60x - 3x^2$ **[1]**

(iii)We have $R(x) = 24000 + 60x - 3x^2$

$R'(x) = 60 - 6x$ and $R''(x) = -6$

for R(x) to be maximum, $R'(x) = 0$ and $R''(x) < 0$

$\Rightarrow 60 - 6x = 0$

$\Rightarrow x = 10$

$\therefore$ Shyam's father should harvest the onion after 10 days of 1st July i.e. on 11th July **[2]**

OR

We have $R(x) = 24000 + 60x - 3x^2$

$R'(x) = 60 - 6x$ and $R''(x) = -6$

For R(x) to be maximum, $R'(x) = 0$ and $R''(x) < 0$

$\Rightarrow 60 - 6x = 0$

$\Rightarrow x = 10$

$\therefore$ maximum revenue = R(10)

$= 24000 + 60(10) - 3(10)^2$

$= \text{Rs.} 24300$ **[2]**

37. (i) since 'x' be the charge per bike per day and n be the number of bike rented per day

Revenue $R(x) = n \times x$

$R(x) = (2000 - 10x)x$

$\Rightarrow R(x) = 2000x - 10x^2$ **[1]**

(ii) Since $x = 260 > 200$

Hence, no bike will be rented (n = 0)

$\therefore\ R(x) = 0$ **[1]**

(iii)We have, $R(x) = 2000x - 10x^2$

$R'(x) = 2000 - 20x$ and $R''(x) = -20$

for $R(x)$ to be the maximum $R'(x) = 0$ and $R''(x) < 0$

$\Rightarrow 2000 - 20x = 0$

$x = 100$

Thus, $R(x)$ is maximum at x = 100 **[2]**

OR

We have, $R(x) = 2000x - 10x^2$

$R'(x) = 2000 - 20x$ and $R''(x) = -20$

for $R(x)$ to be maximum $R'(x) = 0$ and $R''(x) < 0$

$\Rightarrow 2000 - 20x = 0$

$\Rightarrow x = 100$

$\therefore R(x)$ is maximum at x = 100

$\therefore$ Maximum revenue $= R(100)$

$= 2000(100) - 10(100)^2$

$= \text{Rs.}1,00,000$ **[2]**

38. (i) Let E_1 be the event that it rain on chosen day

E_2 be the event that it does not rain on chosen day

and A be the event that the weatherman predict rain.

Then we have $P(E_1) = \frac{6}{366}$, $P(E_2) = \frac{360}{366}$

$P(A/E_1) = \frac{80}{100}$, $P(A/E_2) = \frac{20}{100}$

$\therefore$ Probability that it will not rain on the chosen day, if weatherman predict rain for that day

$= P(E_2/A)$

$$= \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\frac{360}{366}\times\frac{20}{100}}{\frac{6}{366}\times\frac{80}{100} + \frac{360}{366}\times\frac{20}{100}}$$

$= \frac{15}{16}$ **[2]**

(ii) Let E_1 be the event that it rain on chosen day

E_2 be the event that it does not rain on chosen day

and A be the event that the weatherman predict rain.

Then we have $P(E_1) = \frac{6}{365}, P(E_2) = \frac{359}{365}$

$P(A/E_1) = \frac{80}{100}, P(A/E_2) = \frac{20}{100}$

$\therefore$ Probability that it will rain on the chosen day, if weatherman predict rain for that day

$= P(E_1/A)$

$$= \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\frac{6}{365}\times\frac{80}{100}}{\frac{6}{365}\times\frac{80}{100}+\frac{359}{365}\times\frac{20}{100}} = \frac{24}{383}$$ **[2]**