**CLASS : XII**

**SUBJECT – MATHEMATICS**

**SOLUTIONS**

**1.** (c) Given, f(x) = x|x|

$$f(x)=\begin{cases}-x^2 & x<0\\ x^2 & x\ge 0\end{cases}$$

$$f'(x)=\begin{cases}-2x & x<0\\ 2x & x\ge 0\end{cases}$$

$\therefore$ f' (x) = 2|x|

**2.** (b) We know that f(0) = cos0 = 1 and f(2π) = cos2π = 1 So, different elements in R may have the same image. Hence, f is not one-one.

**3.** (c) We have, $\left|\hat{a}+\hat{b}\right|=1$

Let θ be the angle between $\hat{a}$ and $\hat{b}$.

Now, $\left|\hat{a}+\hat{b}\right|=1$

$\Rightarrow |\hat{a}+\hat{b}|^2=1$

$\Rightarrow (\hat{a}+\hat{b}).(\hat{a}+\hat{b})=1$

$\Rightarrow |\hat{a}|^2+|\hat{b}|^2+2\hat{a}\cdot\hat{b}=1$

$\Rightarrow 1+1+2|\hat{a}||\hat{b}|\cos\theta=1$

$[\because |\hat{a}| = |\hat{b}| = 1]$

$\Rightarrow 2\cos\theta=-1$

$\Rightarrow \cos\theta=-\dfrac{1}{2}$

$\Rightarrow \theta=\pi-\dfrac{\pi}{3}=\dfrac{2\pi}{3}$

**4.** (a) We have, $\left(\dfrac{dy}{dx}\right)^4+3x\dfrac{d^2y}{dx^2}=0$

$\therefore$ Order $=2\Rightarrow m=2$

and degree $=1\Rightarrow n=1$

$\therefore\ m-n=2-1=1$

**5.** (a) We have, $xdy+(x-1)dx=0$

$\Rightarrow\ xdy=(1-x)dx$

$\Rightarrow\ dy=\left(\dfrac{1-x}{x}\right)dx \ \ldots(i)$

On integrating both sides of Eq. (i), we get

$\Rightarrow \int dy=\int\left(\dfrac{1-x}{x}\right)dx$

$\Rightarrow \int dy=\int\dfrac{1}{x}dx-\int dx$

$\Rightarrow\ y=\log x-x+C$

**6.** (b) Given A ≡ (2, 3, 1)
and B ≡ (5, 4, 2)

$\therefore$ Direction ratios of $\overrightarrow{AB}$ are

(5 - 2), (4 - 3), (2 - 1)
i.e. 3, 1, 1

**7.** (b) Given $R=\{(a,a^3)$ : a as prime number less than 5$\}$

$\Rightarrow\ R=\{(2,8),(3,27)\}$

Hence, range of $R=\{8,27\}$

**8.** (a) Null matrix

**9.** (b) We have, $\tan\left(\dfrac{1}{2}\cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right)\right)$

Let $\cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right)=x\Rightarrow\cos x=\dfrac{\sqrt{5}}{3}$

$\Rightarrow \sin x=\sqrt{1-\left(\dfrac{\sqrt{5}}{3}\right)^2}=\sqrt{1-\dfrac{5}{9}}=\dfrac{2}{3}$

Now, $\tan\left(\dfrac{x}{2}\right)=\dfrac{1-\cos x}{\sin x}$

$\Rightarrow\ \tan\left(\dfrac{x}{2}\right)=\dfrac{1-\dfrac{\sqrt{5}}{3}}{\dfrac{2}{3}}=\dfrac{3-\sqrt{5}}{2}$

$\therefore \tan\left(\dfrac{1}{2}\cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right)\right)=\dfrac{3-\sqrt{5}}{2}$

**10.** (b) We have, f(x) = 2|x| + 3|sinx| + 6

RHD $f'(0)=\lim_{h\to0}\dfrac{f(0+h)-f(0)}{h}$

$2|0+h|+3\sin|0+h|+6-f(0)$

$=\lim_{h\to0}\dfrac{2|0+h|+3\sin|0+h|+6-(2(0)+3(0)+6)}{h}$

$\Rightarrow f'(0) = \lim_{h\to0}\frac{2|h|+3|\sinh|}{h}$

$\Rightarrow f'(0) = \lim_{h\to0}\frac{2h}{h}+\frac{3\sin h}{h}$

$\Rightarrow f'(0) = 2+3 = 5$

**11.** (c) Given, $xy^2 = ax^2+bxy+y^2$
On differentiating w.r.t. x, we get

$x2y\frac{dy}{dx}+y^2 = 2ax+b\left(x\frac{dy}{dx}+y\right)+2y\frac{dy}{dx}$

$\Rightarrow 2xx\frac{dy}{dx}+y^2 = 2ax+bx\frac{dy}{dx}+by+2y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(2xy-bx-2y) = 2ax+by-y^2$

$\Rightarrow \frac{dy}{dx} = \frac{2ax+by-y^2}{2xy-bx-2y}$

**12.** (a) Let $I = \int \sin^5\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)dx$

Put $\sin\frac{x}{2} = t \Rightarrow \frac{1}{2}\cos\frac{x}{2}dx = dt$

$\therefore I = 2\int t^5 dt$

$\Rightarrow I = 2\frac{t^6}{6}+C$

$\Rightarrow I = \frac{1}{3}\left(\sin^6\frac{x}{2}\right)+C$

**13.** (b)

Let $I = \int\frac{dx}{x(1+x^2)} \Rightarrow I = \int\frac{dx}{x^3\left(\frac{1}{x^2}+1\right)}$

Put $\frac{1}{x^2}+1 = t \Rightarrow \frac{-2}{x^3}dx = dt$

$\therefore I = -\frac{1}{2}\int\frac{dt}{t} = -\frac{1}{2}\log|t|+C$

$\Rightarrow I = -\frac{1}{2}\log\left|\frac{1}{x^2}+1\right|+C$

$\Rightarrow I = -\frac{1}{2}\log\left|\frac{x^2+1}{x^2}\right|+C$

$\Rightarrow I = \frac{1}{2}\log\left|\frac{x^2}{x^2+1}\right|+C$

**14.** (c) We have,

$\tan^{-1}\left\{\tan\frac{15\pi}{4}\right\} = \tan^{-1}\left\{\tan\left(4\pi-\frac{\pi}{4}\right)\right\}$

$= \tan^{-1}\left\{-\tan\frac{\pi}{4}\right\}$

$\left[\because \tan(4\pi-\theta) = -\tan\theta\right]$

$= \tan^{-1}(-1)$

Now, let $\tan^{-1}(-1) = \theta$

$\Rightarrow \tan\theta = -1$

$\Rightarrow \tan\theta = -\tan\frac{\pi}{4}$

$\Rightarrow \tan\theta = \tan\left(\frac{-\pi}{4}\right)$ $[\because \tan(-\theta) = -\tan\theta]$

$\Rightarrow \theta = -\frac{\pi}{4}\left[\because \frac{-\pi}{4}\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)\right]$

$\therefore \tan^{-1}\left\{\tan\frac{15\pi}{4}\right\} = \frac{-\pi}{4}$

**15.** (b) Let $I = \int_0^{3/2}\left[x^2\right]dx$

$\Rightarrow I = \int_0^1 0dx+\int_1^{\sqrt2}dx+\int_{\sqrt2}^{\frac{3}{2}}2dx$

$\Rightarrow I = 0+[x]_1^{\sqrt2}+[2x]_{\sqrt2}^{\frac{3}{2}}$

$\Rightarrow I = \sqrt2-1+3-2\sqrt2$

$\Rightarrow I = \sqrt2+2-2\sqrt2$ or $= 2-\sqrt2$

**16.** (a) (B'AB)' = (AB)'(B')' = B' A'B = B'AB [$\because$ A' = A]

**17.** (b) Let $\vec{a} = -\frac{3}{4}\hat{j}$. So, any vector in the direction opposite to $\vec{a}$ is given by $\vec{b} = -\vec{a} = \frac{3}{4}\hat{j}$

$\therefore \hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\frac{3}{4}\hat{j}}{\left(\frac{3}{4}\right)} = \hat{j}$

So, required vector is $\hat{j}$.

**18.** (b) Let $\vec{a} = 2\hat{j}$ and $\vec{b} = -3\hat{j}$

$\therefore$ Area of triangle $= \frac{1}{2}\left|\vec{a}\times\vec{b}\right| = \frac{1}{2}\left|(2\hat{i})\times(-3\hat{j})\right|$

$= \frac{1}{2}\left|-6\hat{k}\right|\left[\because \hat{i}\times\hat{j} = \hat{k}\right]$

$= \frac{1}{2}\times6 = 3$ sq unlts

**I9.** (a) The given matrices are $A = \begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}$

and $B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$

Then, $A+B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}+\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$

$=\begin{bmatrix}2+1 & 4+3\\ 3-2 & 2+5\end{bmatrix}=\begin{bmatrix}3 & 7\\ 1 & 7\end{bmatrix}$

Hence, Assertion is true. Reason ls also true and it is the correct explanation of Assertion.

**20.** (b) Assertion Consider, $x^2+1=17$

$\Rightarrow x^2=16\Rightarrow x=\pm 4$

Hence, pre-images of 17 are ±4.

Hence, both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

**21.** Given, $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$

Now, $A^2=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$

$=\begin{bmatrix}4-3 & 6+6\\ -2-2 & -3+4\end{bmatrix}=\begin{bmatrix}1 & 12\\ -4 & 1\end{bmatrix}$ (1)

$\therefore A^2-4A+I=\begin{bmatrix}1 & 12\\ -4 & 1\end{bmatrix}-4\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}1 & 12\\ -4 & 1\end{bmatrix}-\begin{bmatrix}8 & 12\\ -4 & 8\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}1-8+1 & 12-12+0\\ -4+4+0 & 1-8+1\end{bmatrix}$

$=\begin{bmatrix}-9 & 0\\ 0 & -6\end{bmatrix}=-6\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=-6I$ (1)

***OR***

We have $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$

$\therefore A^2=A.A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}$

and $8A+KI=8\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}+K\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}8+K & 0\\ -8 & 56+K\end{bmatrix}$ …(i) (1)

$\because A^2=8A+KI$

$\Rightarrow \begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}=\begin{bmatrix}8+K & 0\\ -8 & 56+K\end{bmatrix}$

$\Rightarrow 1=8+K$ and $56+K=49$

$\Rightarrow K=-7$ …(ii) (1)

**22.** At x = 2, f(2) = k(2) + 5 = 2k + 5

$=\lim_{x\to 2^+} f(x)=\lim_{h\to 0} f(2+h)$

[put x = 2 + h when $x\to 2^+$ then $h\to 0$]

$=\lim_{h\to 0}\left[(2+h)-1\right]=\lim_{h\to 0}(1+h)=1$

$=\lim_{x\to 2^-} f(x)=\lim_{h\to 0} f(2-h)$

[put x = 2 – h when $x\to 2^-$ then, $h\to 0$]

$=\lim_{h\to 0}\{k(2-h)+5\}$

$=\lim_{h\to 0}\{2k+5-kh\}=2k+5$ (1)

$\because$ f(x) is continuous at x = 2.

So, LHL = RHL = f(2)

$\Rightarrow 2k+5=1\Rightarrow k=-\frac{4}{2}=-2$ (1)

**23.** Let $I=\int\frac{1}{\sqrt{9+8x-x^2}}dx$

$=\int\frac{1}{\sqrt{-\left[x^2-8x-9\right]}}dx$

$=\int\frac{1}{\sqrt{-[(x-4)^2-16-9]}}dx$ (1)

$=\int\frac{1}{\sqrt{25-(x-4)^2}}dx$

$=\int\frac{1}{\sqrt{(5)^2-(x-4)^2}}$

$=\sin^{-1}\frac{x-4}{5}+C$ (1)

***OR***

Let $I=\int \underset{II}{x}\ \underset{I}{\log x}$

$=\log x\cdot\frac{x^2}{2}-\int\frac{x^2}{2}\cdot\frac{1}{x}dx$

[using integration by parts] (1)

$=\frac{1}{2}x^2\log x-\frac{1}{2}\int x dx$

$=\frac{1}{2}x^2\log x-\frac{x^2}{4}+C$

$=\frac{x^2}{2}\left[\log x-\frac{1}{2}\right]+C$ (1)

**24.** Let $f(x)=x+\frac{1}{x}$

$\therefore f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$ (1)

Now, we have x > 1

$\Rightarrow x^2 > 1 \quad \Rightarrow x^2 - 1 > 0$

$\Rightarrow \frac{x^2 - 1}{x^2} > 0$ [$\because$ x > 1]

$\Rightarrow f'(x) > 0$

Hence, f(x) is increasing for x > 1. (1)

**25.** We have, $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$= \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$

$= -17\hat{i} + 13\hat{j} + 7\hat{k}$ (1)

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + (13)^2 + (7)^2}$

$= \sqrt{289 + 169 + 49} = \sqrt{507}$ (1)

**26.** Given, f(x) = $\frac{3x-1}{2}, x \in R$

**For one-one** Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{3x_1 - 1}{2} = \frac{3x_2 - 1}{2}$

$\Rightarrow 3x_1 - 1 = 3x_2 - 1$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one. (1)

**For onto** Let y ∈ R, then f (x) = y

$\Rightarrow \frac{3x-1}{2} = y$

$\Rightarrow 3x - 1 = 2y$

$\Rightarrow 3x = 2y + 1$

$\Rightarrow x = \frac{2y+1}{3} \in R$ …(i) (1)

Thus, for each y ∈ Ft, there exists x = $\frac{2y+1}{3}$ ∈ R such that $f\left(\frac{2y+1}{3}\right) = y$

Hence, f is onto.

So, f is one-one and onto. **Hence proved.**

**27.** Given, $y = x^x$

On taking log both sides, we get

$\log y = \log x^x$

$\Rightarrow \log y = x \log x$ (1)

On differentiating both sides w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$

[by using product rule of derivative]

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log x \cdot 1$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (1 + \log x)$

$\Rightarrow \frac{dy}{dx} = y(1 + \log x$ ...(i) (1)

Again, differentiating both sides w.r.t. x, we get

$\frac{d^2y}{dx^2} = y\frac{d}{dx}(1+\log x) + (1+\log x)\frac{dy}{dx}$

[by using product rule of derivative]

$\Rightarrow \frac{d^2y}{dx^2} = y \times \frac{1}{x} + (1+\log x)\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + (1+\log x)\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)$ [using Eq. (i)]

$\therefore \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$ **Hence proved.** (1)

***OR***

Given, $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2} \times \frac{d}{dx}(\sin^{-1} x) - (\sin^{-1} x) \times \frac{d}{dx}\sqrt{1-x^2}}{(\sqrt{1-x^2})^2}$$

$$= \frac{\left[\sqrt{1-x^2} \times \frac{1}{\sqrt{1-x^2}} - \sin^{-1} x \cdot \frac{1}{2\sqrt{1-x^2}} \cdot \frac{d}{dx}(1-x^2)\right]}{\left(\sqrt{1-x^2}\right)^2}$$

$$= \frac{\left[\sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} - (\sin^{-1} x) \cdot \frac{-2x}{2\sqrt{1-x^2}}\right]}{1-x^2}$$

$$= \frac{1 + \frac{x \sin^{-1} x}{\sqrt{1-x^2}}}{(1-x^2)} \Rightarrow \frac{dy}{dx} = \frac{1+xy}{1-x^2} \quad \left[\because \frac{\sin^{-1} x}{\sqrt{1-x^2}} = y\right]$$

$\Rightarrow (1-x^2)\frac{dy}{dx} = 1 + xy$ (1½)

On differentiating both sides of above equation w.r.t. x, we get

$$\left(1-x^2\right)\cdot\frac{d}{dx}\left(\frac{dy}{dx}\right)+\frac{dy}{dx}\cdot\frac{d}{dx}\left(1-x^2\right)=\frac{d}{dx}(1+xy)$$

[by using product rule of derivative]

$$\Rightarrow\left(1-x^2\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}(-2x)=x\frac{dy}{dx}+y\cdot1$$

$$\Rightarrow\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-x\frac{dy}{dx}-y=0$$

$$\therefore\left(1-x^2\right)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0\ \ (1½)$$

**28.** Let

$$I=\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx=\int\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$

$$\left[\because \tan\theta=\frac{\sin\theta}{\cos\theta}\text{ and }\cot\theta=\frac{\cos\theta}{\sin\theta}\right]$$

$$=\int\frac{\sin x+\cos x}{\sqrt{\sin x}\sqrt{\cos x}}dx=\int\frac{\sin x+\cos x}{\sqrt{\sin x\cos x}}dx$$

$$=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{2\sin x\cos x}}dx$$

$$=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{1-(\sin x-\cos x)^2}}dx\ \ (1)$$

Put sinx - cosx = t

$\Rightarrow$ (cos x + sin x)dx = dt

$$\therefore\ I=\sqrt{2}\int\frac{1}{\sqrt{1-t^2}}dt$$

$$=\sqrt{2}\sin^{-1}(t)+C\left[\because\int\frac{1}{\sqrt{1-x^2}}dx=\sin^{-1}x+C\right]$$

$$=\sqrt{2}\sin^{-1}(\sin x-\cos x)+C\cdot[\because t=\sin x-\cos x]$$

***OR***

$$\text{Let }I=\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

$$=\int e^x\left[\frac{1+2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\right]dx$$

$$\left[\because\sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\text{ and }1+\cos\theta=2\cos^2\frac{\theta}{2}\right]$$

$$=\int e^x\left[\frac{1}{2\cos^2\left(\frac{x}{2}\right)}+\frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\right]dx$$

$$=\int e^x\left[\frac{1}{2}\sec^2\left(\frac{x}{2}\right)+\tan\frac{x}{2}\right]dx\ \ (1)$$

Thus, we have integration of the form

$$\int e^x\left[f(x)+f'(x)\right]dx$$

Here,

$$f(x)=\tan\frac{x}{2}\Rightarrow f'(x)=\sec^2\frac{x}{2}\cdot\frac{1}{2}=\frac{1}{2}\sec^2\cdot\frac{x}{2}$$

$$\therefore\ I=e^xf(x)+C=e^x\tan\frac{x}{2}+C$$

$$\left[\because\int e^x\left[f(x)+f'(x)\right]dx=e^xf(x)+C\right]$$

**29.** Given, differential equation is

$(1 + e^{2x})dy + (1 + y^2)e^xdx = 0$

$\Rightarrow (1 + e^{2x})dy = -(1 + y^2)\, e^xdx$

$$\Rightarrow\frac{dy}{1+y^2}=-\frac{e^x}{1+e^{2x}}dx\ \ (1)$$

On integrating both sides, we get

$$\int\frac{dy}{1+y^2}=-\int\frac{e^x}{1+e^{2x}}dx$$

Put $e^x = t \Rightarrow e^xdx = dt$

$$\therefore\int\frac{dy}{1+y^2}=-\int\frac{dt}{1+t^2}$$

$\Rightarrow \tan^{-1}y = -\tan^{-1}t + C$

$\Rightarrow \tan^{-1}y + \tan^{-1}e^x = C\ [\because t = e^x]$ (1)

Now, it is given that y = 1, when x = 0.

$\therefore \tan^{-1}(1) + \tan^{-1}(e^0) = C$

$\Rightarrow \tan^{-1}(1) + \tan^{-1}(1) = C\ [\because e^0 = 1]$

$$\Rightarrow\ \frac{\pi}{4}+\frac{\pi}{4}=C$$

$$\Rightarrow C=2\times\frac{\pi}{4}=\frac{\pi}{2}$$

Hence, the required particular solution is

$$\tan^{-1}y+\tan^{-1}e^x=\frac{\pi}{2}$$

***OR***

We have, $y^2dx + (x^2 - xy + y^2)dy = 0$

$$\Rightarrow\frac{dy}{dx}=\frac{-y^2}{x^2-xy+y^2}\ \ ...(i)$$

This is homogeneous differential equation.

Now, on putting

$$y=vx\Rightarrow\frac{d'y}{dx}=v+x\frac{dv}{dx}\ \text{ in}$$

Eq. (i), we get (1)

$$v+x\frac{dv}{dx}=\frac{-v^2x^2}{x^2-vx^2+v^2x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{-v^2}{1-v+v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^2}{1-v+v^2} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v-v^3}{1-v+v^2}$$

$$\therefore \frac{1-v+v^2}{v\left(1+v^2\right)}dv = -\frac{1}{x}dx \ \ (1)$$

On integrating both sides, we get

$$\int \frac{1+v^2}{v\left(1+v^2\right)}dv - \int \frac{v}{v\left(1+v^2\right)}dv = -\int \frac{1}{x}dx$$

$$\Rightarrow \int \frac{1}{v}dv - \int \frac{1}{1+v^2}dv = -\int \frac{1}{x}dx$$

$$\Rightarrow \log|v| - \tan^{-1} v = -\log|x| + \log C$$

$$\Rightarrow \log\left|\frac{vx}{C}\right| = \tan^{-1} v$$

$$\Rightarrow \left|\frac{vx}{C}\right| = e^{\tan^{-1} v}$$

$$\Rightarrow \left|\frac{y}{C}\right| = e^{\tan^{-1}\left(\frac{y}{x}\right)} \ [\because vx = y]$$

$$\therefore |y| = Ce^{\tan^{-1}\left(\frac{y}{x}\right)}$$

which is the required solution.

**30.** Given, $f(x) = \begin{cases} x, & x < 1 \\ 2-x, & 1 \le x \le 2 \\ -2+3x-x^2, & x > 2 \end{cases}$

**Differentiability at x = 1**

$$\text{LHD} = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$= \lim_{h\to 0}\frac{(1-h)-\left[2-(1)\right]}{-h} = \lim_{h\to 0}\frac{-h}{-h} = 1$$

$$\text{RHD} = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{2-(1+h)-(2-1)}{h} = \lim_{h\to 0}\frac{-h}{h} = -1$$

$\because$ LHD $\neq$ RHD

So, f(x) is not differentiable at x = 1. (1)

**Differentiability at x = 2**

$$\text{LHD} = \lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}$$

$$= \lim_{h\to 0}\frac{2-(2-h)-(2-2)}{-h}$$

$$= \lim_{h\to 0}\frac{h}{-h} = -1$$

$$\text{RHD} = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0}\frac{-2+3(2+h)-(2+h)^2-(2-2)}{h}$$

$$= \lim_{h\to 0}\frac{-2+6+3h-\left(4+h^2+4h\right)-0}{h}$$

$$= \lim_{h\to 0}\frac{-h^2-h}{h}$$

$$= \lim_{h\to 0}\frac{-h(h+1)}{h} = -(0+1) = -1 \ \ (1)$$

$\because$ LHD = RHD

So, f(x) is differentiable at x = 2.

Hence, f(x) is not differentiable at x = 1, but it differentiable at x = 2. (1)

**31.** Let θ be the angle between the vectors a and b.

Then, $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

For θ to be an obtuse angle, we must have

$\Rightarrow \cos\theta < 0 \ \forall\, x \in (0, \infty)$

$$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} < 0 \forall x \in (0,\infty)$$

$$\Rightarrow \vec{a}\cdot\vec{b} < 0 \forall x \in (0,\infty)$$

$\Rightarrow c\,(\log_2 x)^2 - 12 + 6c\,(\log_2 x) < 0 \ \forall\, x \in (0, \infty)$

$\Rightarrow cy^2 + 6cy - 12 < 0 \ \forall\, y \in R$, (1)

where $y = \log_2 x$ [$\because x > 0 \Rightarrow y = \log_2 x \in R$]

$\Rightarrow c < 0$ and $36c^2 + 48c < 0$

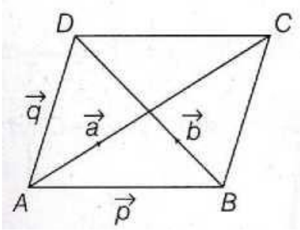
[$\because$ $ax^2 + bx + c < 0 \ \forall\, x \Rightarrow a < 0$ and discriminant < 0]

$\Rightarrow c < 0$ and $c(3c + 4) < 0$ 4

$\Rightarrow c < 0$ and $-\frac{4}{3} < c < 0$ 3

$$\Rightarrow c \in \left(\frac{-4}{3}, 0\right) \ (2)$$

**32.** Let ABCD be a parallelogram such that $\overrightarrow{AC} = \vec{a}$ and $\overrightarrow{BD} = \vec{b}$. Also, let $\overrightarrow{AB} = \vec{p}$ and $\overrightarrow{AD} = \vec{q}$, then $\overrightarrow{BC} = \vec{q}$ and $\overrightarrow{DC} = \vec{p}$.




Now, by triangle law of addition, we get

$\overrightarrow{AC} = \vec{p} + \vec{q} \Rightarrow \vec{a} = \vec{p} + \vec{q}$ ...(i)

Similarly,

$\overrightarrow{BD} = -\vec{p} + \vec{q} \Rightarrow \vec{b} = -\vec{p} + \vec{q}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$\vec{a} + \vec{b} = 2\vec{q} \Rightarrow \vec{q} = \frac{1}{2}(\vec{a} + \vec{b})$ ...(iii)

On subtracting Eq. (ii) from Eq, (i), we get

$\vec{a} - \vec{b} = 2\vec{p} \Rightarrow \vec{p} = \frac{1}{2}(\vec{a} - \vec{b})$ ...(iv) (1)

Now, $\vec{p} \times \vec{q} = \frac{1}{4}\left\{(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right\}$

[using Eqs. (iii) and (iv)]

$= \frac{1}{4}\left\{\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}\right\}$

$= \frac{1}{4}\left\{\vec{a} \times \vec{b} + \vec{a} \times \vec{b}\right\}$

$\left[\begin{array}{l} \because \vec{a} \times \vec{a} = \vec{b} \times \vec{b} = \vec{0} \\ \text{and } \vec{a} \times \vec{b} = -\vec{b} \times \vec{a} \end{array}\right]$

$= \frac{1}{4} \times 2(\vec{a} \times \vec{b}) \;\; = \frac{1}{2}(\vec{a} \times \vec{b})$

So, area of a parallelogram ABCD = $\frac{1}{2} |\vec{a} \times \vec{b}|$

Hence proved.

Now, area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + k$ and $\hat{i} + 3\hat{j} - \hat{k}$

$= \frac{1}{2} |(2\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} + 3\hat{j} - \hat{k})|$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$$

$$= \frac{1}{2}\left|\hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1)\right|$$

$$= \frac{1}{2}\left|-2\hat{i} + 3\hat{j} + 7\hat{k}\right| = \frac{1}{2}\sqrt{(-2)^2 + (3)^2 + (7)^2}$$

$$= \frac{1}{2}\sqrt{4+9+49} = \frac{1}{2}\sqrt{62} \text{ sq units } (2)$$

**33.** LHS = I + A

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$$\text{RHS} = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right\}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1-0 & 0+\tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1-0 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \tan\frac{\alpha}{2}\sin\alpha & -\sin\alpha + \tan\frac{\alpha}{2}\cos\alpha \\ -\tan\frac{\alpha}{2}\cos\alpha + \sin\alpha & \tan\frac{\alpha}{2}\sin\alpha + \cos\alpha \end{bmatrix}$$

[multiplying rows by columns]

$$= \begin{bmatrix} \dfrac{\cos\alpha\cdot\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\sin\alpha}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\alpha\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\cos\alpha}{\cos\frac{\alpha}{2}} \\ \dfrac{-\sin\frac{\alpha}{2}\cos\alpha + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\sin\frac{\alpha}{2}\sin\alpha + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \dfrac{\sin\left(\frac{\alpha}{2} - \alpha\right)}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$\left[\begin{array}{l} \because \cos(A-B) = \cos A\cos B + \sin A\sin B \\ \text{and } \sin(A-B) = \sin A\cos B - \cos A\sin B \end{array}\right]$$

$$= \begin{bmatrix} \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = \text{LHS}$$

**Hence proved.** (2)

***OR***

We have the following system of equations

x + y + z = 7000 ...(i)

⇒ 10x + 16y + 17z = 110000 ...(ii)

and x - y = 0 ...(iii)

This system of equations can be written in matrix from as AX = B

where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$$

$$\text{Here, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{vmatrix}$$

⇒ |A| = 1(0 +17)-1(0-17)+1(-10 -16)

= 17+17-26

= 8 $\neq$ 0 (1)

So, A is non-singular matrix and its inverse exists.

Now, cofactors of elements of |A| are

$$A_{11} = (-1)^2 \begin{vmatrix} 16 & 17 \\ -1 & 0 \end{vmatrix} = 1(0+17) = 17$$

$$A_{12} = (-1)^3 \begin{vmatrix} 10 & 17 \\ 1 & 0 \end{vmatrix} = -1(0-17) = 17$$

$$A_{13} = (-1)^4 \begin{vmatrix} 10 & 16 \\ 1 & -1 \end{vmatrix} = 1(-10-16) = -26$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -1(0+1) = -1$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1(0-1) = -1$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1(-1-1) = 2$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 16 & 17 \end{vmatrix} = 1(-17-16) = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 10 & 17 \end{vmatrix} = -1(17-10) = -7$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 10 & 16 \end{vmatrix} = 1(16-10) = 6 \ (1)$$

$$\therefore \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 17 & 17 & -26 \\ -1 & -1 & 2 \\ 1 & -7 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}$$

and the solution of given system is given by X = $A^{-1}$ B.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}\begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 119000 - 110000 + 0 \\ 119000 - 110000 + 0 \\ -182000 + 220000 + 0 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 9000 \\ 9000 \\ 38000 \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix}$$

On comparing the corresponding elements, we get

x = 1125, y = 1125, z = 4750. (2)

**34.** We have the following LPP,

Maximise Z = 8x + 9y

Subject to the constraints

2x + 3y $\le$ 6

3x - 2y $\le$ 6

y $\le$ 1 and x, y $\ge$ 0

Now, considering the inequations as equations, we get

2x + 3y = 6 …(i)

3x - 2y = 6 ...(ii)

and y = 1 ...(iii)

Table for line 2x + 3y = 6 is

| | | |
|---|---|---|
| **x** | 3 | 0 |
| **y** | 0 | 2 |

So, it passes through the points, (3,0) and (0,2).

On putting (0,0) in the inequality 2x + 3y < 6, we get $0 \leq 6$ [which is true]

So, the half plane is towards the origin.

Table for line 3x - 2y = 6 is

| | | |
|---|---|---|
| **x** | 2 | 0 |
| **y** | 0 | -3 |

So, it passes through the points (2,0) and (0-3).

On putting (0,0) in the inequality $3x - 2y \leq 6$,

we get $0 \leq 6$ [which is true]

So, the half plane is towards the origin.

The line y = 1 is perpendicular to Y-axis.

On putting (0,0) in the inequality $y \leq 1$, we get

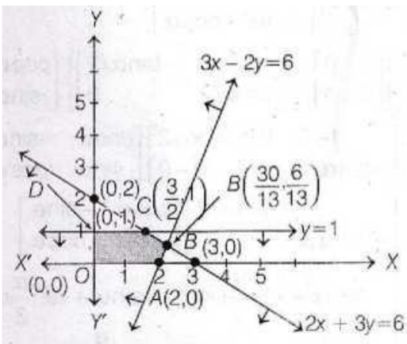
$0 \leq 1$ [which is true]

So, the half plane is towards the origin.

Also, $x \geq 0$, $y \geq 0$, so the feasible region lies in the first quadrant.

The point of intersection of Eqs. (i) and (ii) is $\left(\frac{30}{13},\frac{6}{13}\right)$, Eqs. (ii) and (iii) is $\left(\frac{8}{3},1\right)$ and Eqs.(i) and (iii) is $\left(\frac{3}{2},1\right)$. (1)

The graphical representation of the above system of inequations is given below




Clearly, feasible region is OABCDO, whose corner pdints are O(0,0), A(2,0), B $\left(\frac{30}{13},\frac{6}{13}\right)$, C $\left(\frac{3}{2},1\right)$ and D(0,1).

The values of Z at corner points are as follows

| Corner points | Z = 8x + 9y |
|---|---|
| 0(0,0) | Z = 0 + 0 = 0 |
| A (2,0) | Z = 8 × 2 + 0 = 16 |
| B $\left(\frac{30}{13},\frac{6}{13}\right)$ | $Z = 8 \times \frac{30}{13} + \frac{9\times 6}{13}$ $= \frac{294}{13} = 22.62$ (maximum) |
| C $\left(\frac{3}{2},1\right)$ | $Z = 8 \times \frac{3}{2} + 9 = 21$ |
| D(0, 1) | Z = 0 + 9 × 1 = 9 |

In the table, we find that maximum value of Z is 22.62, when $x = \frac{30}{13}$ and $y = \frac{6}{13}$.

***OR***

To solve the LPP graphically, we first convert the inequations into equations to obtain the following lines.

2x + y = 8,

x + 2y = 10,

x = 0, y = 0

The Iine2x + y = 8 meets the coordinate axes at $A_1(4, 0)$ and $B_1(0, 8)$, join these points to obtain the line represented by 2x + y = 8.

Clearly, O(0,0) does not satisfy the inequation $2x + y \geq 8$.

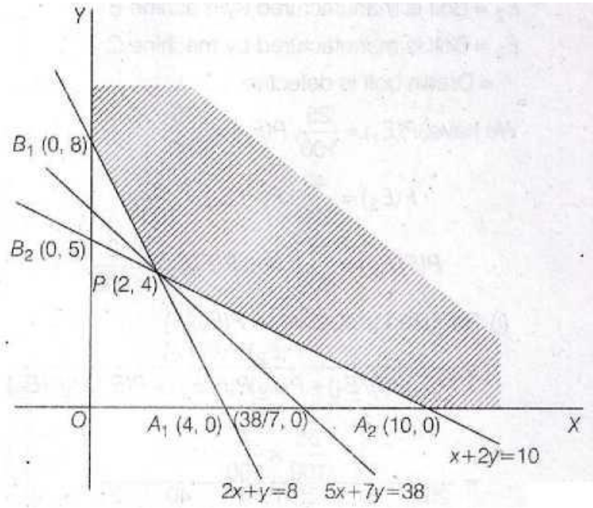
So, the region not containing the origin is represented by this inequation. (1)

The line x +2y = 10 meets the coordinate axes at $A_2$ (10, 0) and $B_2(0, 5)$. Join these points to obtain the line represented by x + 2y = 10.

Clearly, O(0, 0) does not satisfy the inequation $x + 2y \geq 10$.

So, the region not containing the origin is represented by this inequation. Clearly, $x \geq 0$ and $y \geq 0$ respresent the first quadrant. (1)

Thus, the shaded region in figure is the feasible region of the LPR The coordinates of the corner points of this region are $A_2(10, 0)$, P(2, 4) and $B_1(0, 8)$.




The point P(2, 4) is obtained by solving 2x + y = 8 and x + 2y = 10 simultaneously. The values of the objective function Z = 5x + 7y at the corner points of the feasible region are given in the following table

| **Point (x,y)** | **Value of the objective function Z = 5x + 7y** |
|---|---|
| $A_2(10, 0)$ | Z = 5 × 10 + 7 × 0 = 50 |
| P(2, 4) | Z = 5 × 2 +7 × 4 = 38 |
| $B_1(0, 8)$ | Z = 5 × 0 + 7 × 8 = 56 |

Clearly, Z is minimum at x = 2 and y = 4

**35.** Let $I = \int_0^{2\pi} \frac{x\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x} dx$ ...(i)

Then, $I = \int_0^{2\pi} \frac{(2\pi - x)\sin^{2n}(2\pi - x)}{\sin^{2n}(2\pi - x) + \cos^{2n}(2\pi - x)} dx$

$\Rightarrow 1 = \int_0^{2\pi} \frac{(2\pi - x)\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x} dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{2\pi} \frac{2\pi\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x} dx$$

$$\Rightarrow I = \pi\int_0^{2\pi} \frac{\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x} dx$$

$$\Rightarrow I = 2\pi\int_0^{\pi} \frac{\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x} dx \quad (1)$$

$$\left[\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x),\ \text{if } f(2a - x) = f(x)\right]$$

$$\Rightarrow I = 4\pi\int_0^{\frac{\pi}{2}} \frac{\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x} dx \quad ...(iii)$$

$$\Rightarrow I = 4\pi\int_0^{\frac{\pi}{2}} \frac{\sin^{2n}\left(\frac{\pi}{2} - x\right)}{\sin^{2n}\left(\frac{\pi}{2} - x\right) + \cos^{2n}\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow I = 4\pi\int_0^{\frac{\pi}{2}} \frac{\cos^{2n}x}{\cos^{2n}x + \sin^{2n}x} dx \quad ...(iv)\ (1)$$

On adding Eqs. (iii) and (iv), we get

$$2I = 4\pi\int_0^{\frac{\pi}{2}} \left[\frac{\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x} + \frac{\cos^{2n}x}{\sin^{2n}x + \cos^{2n}x}\right] dx$$

$$= 4\pi\int_0^{\frac{\pi}{2}} 1dx = 4\pi \times \frac{\pi}{2}$$

$\therefore I = \pi^2$ (2)

**36.** Let $E_1$, $E_2$, $E_3$ and A be the events defined as follows

$E_1$ : You guess the answer
$E_2$ : You copy the answer
$E_3$ : You know the answer
A : You answer correctly

$P(E_1) = \frac{1}{3} P(E_2) = \frac{1}{6}$,

$P\left(\frac{A}{E_1}\right) = \frac{1}{8}$ and $P\left(\frac{A}{E_2}\right) = \frac{1}{4}$

Since, $E_1$, $E_2$, $E_3$ are mutually exclusive and exhaustive events

$\therefore P(E_1) + P(E_2) + P(E_3) = 1$
$\Rightarrow P(E_3) = 1 - P(E_1) - P(E_2)$

$= 1 - \frac{1}{3} - \frac{1}{6}$

$= \frac{3}{6} = \frac{1}{2}$

(i) Required probability $= P\left(\frac{A}{E_1}\right) = \frac{1}{8}$

(ii) Required probability $= P\left(\frac{A}{E_3}\right) = 1$

(iii) Required probability $= P\left(\frac{E_3}{A}\right)$

$$= \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$=\frac{1}{\frac{1}{24}+\frac{1}{24}+\frac{1}{2}}=\frac{12}{14}=\frac{6}{7}$$

***OR***

Required probability = P(A)

$$=\sum_{i=1}^{3}P(E_i)P\left(\frac{A}{E_i}\right)$$

$$=P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)$$

$$=\frac{1}{3}\times\frac{1}{8}+\frac{1}{6}\times\frac{1}{4}+\frac{1}{2}\times 1 \quad =\frac{1}{24}+\frac{1}{24}+\frac{1}{2}$$

$$=\frac{1+1+12}{24} \quad =\frac{14}{24}=\frac{7}{12}$$

**37.** (i) Let $I=\int\sqrt{1-x^2}\,dx$

$$I=\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x+C$$

(ii) Required expression is given by

$$\pi(1)^2-\left[\int_0^1\sqrt{1-x^2}\,dx-\int_0^1(1-x)\,dx\right]$$

(iii) We have,

$x^2+y^2=1$ and $x+y=1$

$\Rightarrow x^2+(1-x)^2=1$

$\Rightarrow x^2+1+x^2-2x=1$

$\Rightarrow 2x^2-2x=0$

$\Rightarrow 2x(x-1)=0$

$\Rightarrow x=0, 1$

when $x = 0$

$\Rightarrow y=1$

and $x = 1$

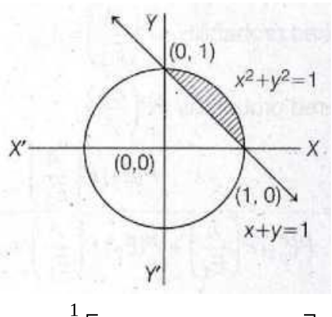
$\Rightarrow y=0$

$\therefore$ Point of intersection are (1, 0) and (0, 1).

Now,

$$\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$$

***OR***




Required area $=\int_0^1\left[\sqrt{1-x^2}-(1-x)\right]dx$

$$=\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x-x+\frac{x^2}{2}\right]_0^1$$

$$=0+\frac{1}{2}\sin^{-1}1-1+\frac{1}{2}-0$$

$$=\frac{\pi}{4}-\frac{1}{2}\text{ sq units}$$

**38.** (i) At x = 3,

$P(3) = -6(3)^2 + 120(3) + 25000$

$= -54 + 360 + 25000 =$ Rs. 25306

(ii) $P'(x) = -12x + 120$

$P'(5) = -12\times 5 + 120 = -60 + 120 = 60$

(iii) For strictly increasing, we must put $P'(x) > 0$

$\Rightarrow -12x + 120 > 0$

$\Rightarrow 120 > 12x$

$\Rightarrow x < 10$

$\therefore x \in (0,10)$

***OR***

$P(x) = -6x^2 + 120x + 25000$

$\Rightarrow P'(x) = -12x + 120$

For maximum profit, put $P'(x) = 0$

$\Rightarrow x = 10$

Now $P''(x) = -12 < 0$

$\therefore$ At x = 10, profit function is maximum.