Physics

Physics

1. (b)

2. (a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{d^2}$

3. (d) Violet (Higher the frequency, greater is stopping potential)

4. (c) most of the part of an atom is empty space

5. (b) Along $-z$ axis

6. (b) ferromagnetic material becomes paramagnetic

7. (d) 120 Ω

8. (a) eL/2m

9. (d) Primary coil is made up of a very thick copper wire.

10. (c) electric energy density is equal to the magnetic energy density.

11. (c) resistance (r)

12. (b) As from relation, $E_n = \dfrac{13.6}{n^2}\,eV$

    The negative sign of energy indicates that electrostatic force is attractive in nature that's why electron is bound to the nucleus.

13. (a) Both (A) and (R) are true and (R) is the correct explanation of (A)

14. (a) Both (A) and (R) are true and (R) is the correct explanation of (A)

15. (c) (A) is true but (R) is false

16. (a) Both (A) and (R) are true and (R) is the correct explanation of (A)

17. (a) Rectifier.

    (b) Circuit diagram of full wave rectifier :

18.
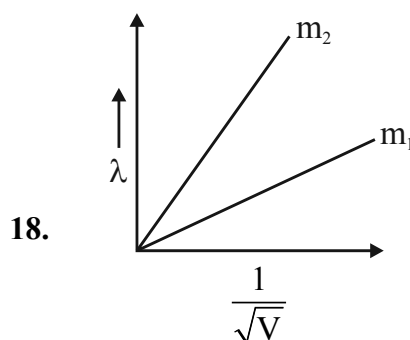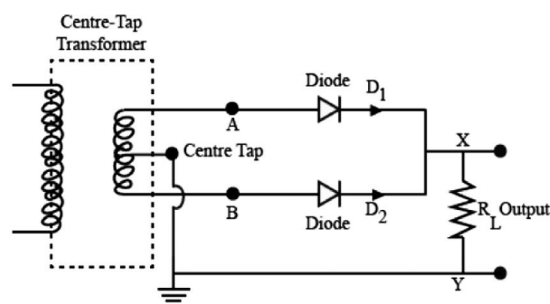
As, $\lambda = \dfrac{h}{\sqrt{2mqV}}$ or $\lambda = \left(\dfrac{h}{\sqrt{2q}}\cdot\dfrac{1}{\sqrt{m}}\right)\dfrac{1}{\sqrt{V}}$

or $\dfrac{\lambda}{\dfrac{1}{\sqrt{V}}} = \dfrac{h}{\sqrt{2q}}\cdot\dfrac{1}{\sqrt{m}}$

As the charge on two particles is same, we get

Slope $\propto \dfrac{1}{\sqrt{m}}$

19. (a) Focal length increases with increase of wavelength.

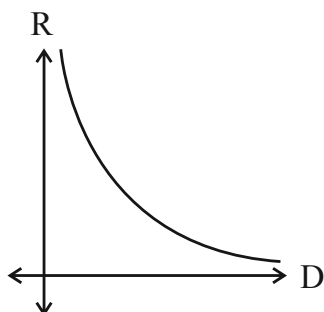$\dfrac{1}{f} = \left(\dfrac{n_2}{n_1}-1\right)\dfrac{2}{R}$ as wavelength increase, $\dfrac{n_2}{n_1}$

decreases hence focal length increases.

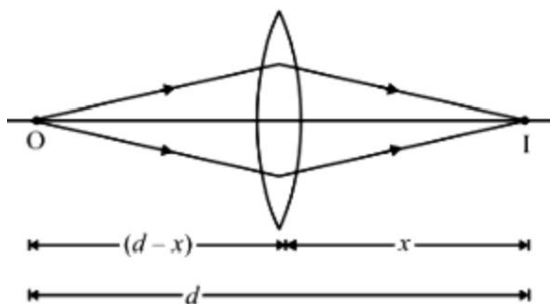(b) $n_2 > n_1$, $\dfrac{n_2}{n_1} - 1$ decrases so, focal length increase.

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\frac{2}{R}$$

20. $R = \rho\dfrac{1}{A} \quad\Rightarrow\quad \rho\dfrac{\ell}{\pi r^2} = \rho\dfrac{4\ell}{\pi D^2}$

i.e. $\quad R \propto \dfrac{1}{D^2}$

21. Let d be the least distance between object and image for a real iamge formation.

Using Lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad\Rightarrow\quad \frac{1}{f} = \frac{1}{x} + \frac{1}{d-x} = \frac{d}{x(d-x)}$$

$\Rightarrow fd = xd - x^2$

$\Rightarrow x^2 - dx + fd = 0 \quad\Rightarrow\quad x = \dfrac{d \pm \sqrt{d^2 - 4fd}}{2}$

For real roots of x, $d^2 - 4fd \geq 0$

Hence $d \geq 4f$.

OR

Let f_0 and f_e be the focal length of the objective and eyepiece respectively. For normal adjustment, the distance from objective to eyepiece, $L = f_0 + f_e$. Taking the line on the objective as object and eyepiece as lens.

$u = -(f_0 + f_e)$ and $f = f_e$

Using Lens formla,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{\left[-(f_0 + f_e)\right]} = \frac{1}{f_e}$$

$$\Rightarrow v = \left(\frac{f_0 + f_e}{f_0}\right)f_e$$

Now,

Linear magnification (eyepiece) =

$$\frac{v}{u} = \frac{\text{Image size}}{\text{Object Size}} = \frac{f_e}{f_0} = \frac{\ell}{L}$$

$\therefore$ Angular magnification of telescope,

$$M = \frac{f_0}{f_e} = \frac{L}{\ell}$$

22. (a) $n + {}^{235}_{92}U \rightarrow {}^{144}_{Z}Ba + {}^{A}_{36}X + 3n$

From law of conservation of atomic number

$0 + 92 = Z + 36$

$\Rightarrow Z = 92 - 36 = 56$

From law of conservation of mass number,

$1 + 235 = 144 + A + 3 \times 1$

$A = 236 - 147 = 89$

(b) (i) B.E. of ${}^{235}_{92}U <$ BE of $\left({}^{144}_{56}Ba + {}^{89}_{36}X\right)$ and due to difference in BE of the nuclides. A large

amount of the energy will released in the fission of $_{92}^{235}U$.

(ii) Mass number of the reactant and product nuclides are same but there is an actual mass defect. This difference in the total mass of the nuclei on both sides, gets converted into energy, i.e. $\Delta E = \Delta mc^2$.

23. (i) Electric flux through a Gaussian surface,

$$\phi = \frac{\text{Total enclosed charge}}{\varepsilon_0}$$

Net charge enclosed inside the shell, q = 0

$\therefore$ Electric flux through the shell, $\phi = \frac{q}{\varepsilon_0} = 0$

(ii) **Gauss's Law-** Electric flux through a Gaussian surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed within it.

Mathematically, $\oint \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0} \times q$

(iii) We know that electric field or net charge inside the spherical conducting shell is zero.

Hence, the force on charge $\frac{Q}{2}$ is zero.

Force on charge at A,

$$F_A = \frac{1}{4\pi\varepsilon_0} \frac{2Q\left(Q+\dfrac{Q}{2}\right)}{x^2} = \frac{1}{4\pi\varepsilon_0} \frac{3Q^2}{x^2}$$

24. Transition C and E belong to the Lyman series.

Reason : In the Lyman series, the electron jumps to the lowest energy level from any higher energy levels.

Transition B and D belong to the Balmer series.

Reason: The electron jumps from any higher energy level to the level just above the ground energy level.

The wavelength associated with the transition is given by
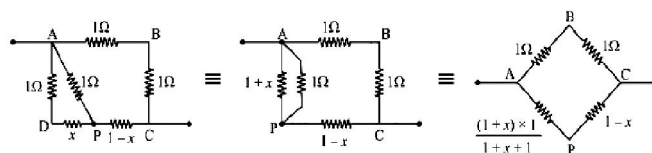
$$\lambda = \frac{hc}{\Delta E}$$

Ratio of the shortest wavelength

$$\lambda_L : \lambda_B = \frac{hc}{\Delta E_L} : \frac{hc}{\Delta E_B}$$

$$= \frac{1}{0-(-10)} : \frac{1}{0-(-3)} = 3:10$$

25. As from given, resistance of each side, R = 1 Ω

Let the resistance of DP part is x ohm.

As from circuit shown,

Two parallel resistance between AP,

$$R_1 = \frac{(1+x)(1)}{(1+x+1)} = \frac{x+1}{x+2}\Omega$$

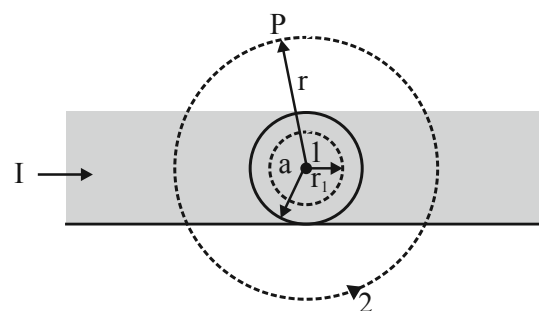As the points B and P are at the same potential,

$$\frac{1}{1} = \frac{\dfrac{1+x}{2+x}}{\dfrac{1-x}{}} \Rightarrow (1-x)(2+x) = 1+x$$

$$\Rightarrow x^2 + 2x - 1 = 0$$

From solving this equation, we get

$$x = \left(\sqrt{2}-1\right)\Omega$$

26.

(a) Consider the case r > a. The Amperian loop, labelled 2, is a circle concentric with the cross-section.

For this loop, $L = 2\pi r$

Using Ampere's circuital Law, we can write,

$$B(2\pi r) = \mu_0 I, \quad B = \frac{\mu_0 I}{2\pi r},$$

i.e. $\qquad B \propto \dfrac{1}{r} \qquad\qquad (r > a)$

(b) Consider the case r < a. The Amperian loop is a circle labelled 1. For this loop, taking the radius of the circle to be r, $L = 2\pi r$

Now the current enclosed I_e is not I, but is less than this value. Since the current distribution is uniform, the current enclosed is,

$$I_e = I\left(\frac{\pi r^2}{\pi a^2}\right) = \frac{I r^2}{a^2}$$

Using Ampere's circuital law,

$$B(2\pi r) = \mu_0 \frac{I r^2}{a^2} \Rightarrow B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r$$

i.e. $\qquad B \propto r \qquad\qquad (r > a)$

27. During charging, electric flux between the plates of capacitors keeps on charging; this results in the production of a displacement current between the plates.

Mathematically, $I_d = \varepsilon_0 \left(\dfrac{d\phi_E}{dt}\right)$

Conduction current is established by actual movement of free electrons through a metallic conductor while displacement current is established by polarisation of molecules of a dielectric under the influence of an external electric field.

28. When current flowing in one of two nearby coils is changed, the magnetic flux linked with the other coil changes; due to which an emf is induced in it (other coil). This phenomenon of electromagnetic induction is called the mutual induction. The coil, in which current is changed is called the primary coil and the coil in which emf is induced is called the secondary coil. **The SI unit of mutual inductance is henry.**

Mutual inductance is numerically equal to the magnetic flux linked with one coil (secondary coil) when unit current flows through the other coil (primary coil).
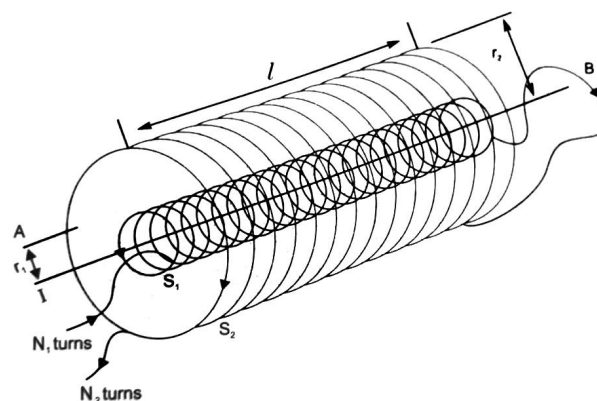
Consider two long co-axial solenoids, each of length *l*. Let n_1 be the number of turns per unit length of the inner solenoid S_1 of radius r_1, n_2 be the number of turns per unit length of the outer solenoid S_2 of radius r_2.

Imagine a time varying current I_2 through S_2 which sets up a time varying magnetic flux ϕ_1 through S_1.

$\therefore \qquad \phi = M_{12}(I_2) \qquad\qquad ....(i)$

Where M_{12} = Coefficient of mutual inductance of solenoid S_1 with respect to solenoid S_2 Magnetic field due to the current I_2 in S_2 is

$B_2 = \mu_0 n_2 I_2$

$\therefore$ Magnetic flux through S_1 is

$\phi_1 = B_2 A_1 N_1$

where, $N_1 = n_1 l$ and l = length of the solenoid

$\phi_1 = (\mu_0 n_2 I_2)\left(\pi r_1^2\right)(n_1 l)$

$\phi_1 = \mu_0 n_1 n_2 \pi r_1^2 \, l I_2$ ....(ii)

From equations (i) and (ii), we get

$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 \, l$ ....(iii)

Let us consider the reverse case.

A time varying current I_1 through S_1 develops a flux ϕ_2 through S_2.

∴ $\phi_2 = M_{21}(I_1)$ ....(iv)

where, M_{21} = Coefficient of mutual inductance of solenoid S_2 with respect to solenoid S_1 Magnetic flux due to I_1 in S_1 is confined solely inside S_1 as the solenoids are assumed to be very long.

There is no magnetic field outside S_1 due to current I_1 in S_1.

The magnetic flux linked with S_2 is

∴ $\phi_2 = B_1 A_1 N_2 \, (\mu_0 n_1 I_1)(\pi r_1^2)(n_2 l)$ ....(v)

$\phi_2 = \mu_0 n_1 n_2 \pi r_1^2 \, l I_1$

From equations (iv) and (v), we get

$M_{21} = \mu_0 n_1 n_2 \pi r_1^2$ ....(vi)

From equations (iii) and (vi), we get
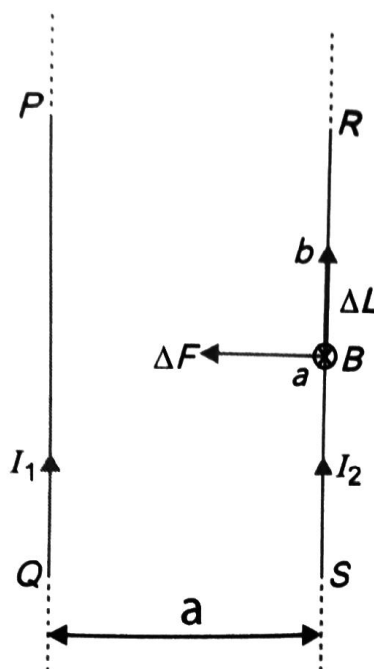
$M_{12} = M_{21} = M = \mu_0 n_1 n_2 \pi r_1^2 \, l$

We can write the above equation as

$$M = \mu_0 \left(\frac{N_1}{l}\right)\left(\frac{N_2}{l}\right)\pi r_1^2 \times l$$

$$M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l}$$

OR

Suppose two long thin straight conductors (or wires) PQ and RS are placed parallel to each other in vacuum (or air) carrying currents I_1 and I_2 respectively. It has been observed experimentally that when the currents in the wire are in the same direction, they experience an attractive force and when they carry currents in opposite directions, they experience a repulsive force.

Let the conductors PQ and RS carry currents I_1 and I_2 in same direction 1, and placed at separation a.

Consider a current-element 'ab' of length ΔL of wire RS. The magnetic field Q produced by current-carrying conductor PQ at the location of other wire RS.

$B_1 = \dfrac{\mu_0 I_1}{2\pi a}$

According to Maxwell's right hand rule or right hand palm rule number 1, the direction of B_1 will be perpendicular to the plane of paper and directed downward. Due to this magnetic field, each element of other wire experiences a force. The direction of current element is perpendicular

to the magnetic field; therefore the magnetic force on element ab of length ΔL

$$\Delta F = B_1 I_2 \, \Delta L \sin 90° = \frac{\mu_0 I_1}{2\pi a} I_2 \Delta L$$

$\therefore$ The total force on conductor of length L will be

$$F = \frac{\mu_0 I_1 I_2}{2\pi a} \sum \Delta L = \frac{\mu_0 I_1 I_2}{2\pi a} L$$

$\therefore$ Force acting per unit length of conductor

$$f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi a} \quad N/m$$

According to Fleming's left hand rule, the direction of magnetic force will be towards PQ i.e., the force will be attractive.

Definition of SI unit of Current (ampere): In SI system of fundamental unit of current 'ampere is defined assuming the force between the two current carrying wires as standard.

The force between two parallel current carrying conductors of separation a is

$$f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi a} \quad N/m$$

If $I_1 = I_2 = 1A$, $a = 1$ m, then

$$f = \frac{\mu_0}{2\pi} = 2 \times 10^{-7} \, N/m$$

Thus 1 ampere is the current which when flowing in each of parallel conductors placed at separation 1 m in vacuum exert a force of 2×10^{-7} on 1 m length of either wire.

29. Explanations

(i) (b) Gallium being trivalent atom makes p-type semiconductor.

(ii) (a) The given semiconductor is n-type semiconductor as e$^-$ concentration > hole concentration.

(iii) (a) In forward biased p-n-junction, external voltage decreases the potential barrier, so current is maximum. While in reversed biased p-n-junction, external voltage increases the potential barrier, so the current is very small of the order μA.

(iv) (a) When p-n junction is reversed biased, the width of the depletion layer becomes large and so, the electric field $\left(E = \dfrac{v}{d} \right)$ becomes very small, nearly zero.

OR

(b) diffusion in forward bias, drift in reverse bias.

30. (i) (b) As from lens maker's formula,

$$\frac{1}{f} \propto (n-1) \text{ and } P = \frac{1}{f}$$

$$\Rightarrow P \propto (n-1)$$

Also, $n \propto \dfrac{1}{\lambda}$, Hence, $P \propto \dfrac{1}{\lambda}$

Thus, if wavelength (λ) of light increases then power of lens decreases.

(ii) (b) From Lens maker's formula,

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{f} = (n-1) \left(\frac{2}{f} \right) \qquad (\because f = R)$$

$$\Rightarrow \frac{1}{2} = (n-1)$$

$\therefore n = 1.5$

(iii) (d) When the refractive index of the object is same as that of the media around it, then the rays pass undeflected and the object appears to be invisible.

(iv) (a) For virtual and erect image,

$$m = \frac{v}{u} \quad \Rightarrow 3 = \frac{u}{v} \quad \Rightarrow v = 3u,$$

and $f = \dfrac{1}{P} = f = \dfrac{1}{5} = 0.2$ m $= 20$ cm

Using lens formula, $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

$\dfrac{1}{20} = \dfrac{1}{3u} - \dfrac{1}{u} \quad \Rightarrow \dfrac{1-3}{3u} = \dfrac{1}{20}$

$\Rightarrow u = -\dfrac{40}{3}$ cm

And $v = 3u = -3 \times \dfrac{40}{3} = -40$ cm

OR

(d) Using lengs maker's formula,

$\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$\dfrac{1}{20} = (1.5 - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) \qquad \text{...(i)}$

and, $\dfrac{1}{f_w} = \left(\dfrac{1.5}{\dfrac{4}{3}} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) \qquad \text{...(ii)}$

For equation (i) and (ii), we get,

$\dfrac{\dfrac{1}{20}}{\dfrac{1}{f_w}} = \dfrac{0.5}{\dfrac{1}{8}} = 4 \quad \Rightarrow \dfrac{f_w}{20} = 4 \quad \Rightarrow f_w = 80$ cm

31. **Formula for Refraction at Spherical Surface**

Concave Spherical Surface: Let SPS' be a spherical refracting surface, which separates media '1' and '2'. Medium '1' is rarer and medium '2' is denser. The refractive indices of media 'l' and '2' are n_1 and n_2 respectively $(n_1 < n_2)$. Let P be the pole and C the centre of curvature and PC the principal axis of spherical refracting surface.
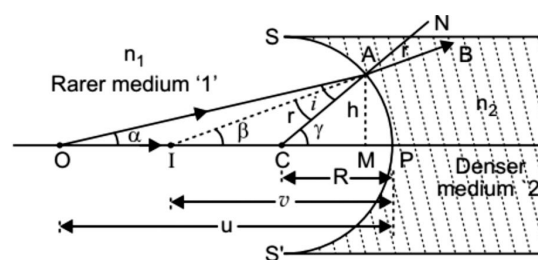
O is a point-object on the principal axis. An incident ray OA, after refraction at A on the spherical surface bends towards the normal CAN and moves along AB. Another incident ray OP falls on the surface normally and hence passes undeviated after refraction. These two rays, when produced backward meet at point I on principal axis. Thus I is the virtual image of O. Let angle of incidence of ray OA be i and angle of refraction be r i.e.,

$\angle OAC = i$ and $\angle NAB = r$

Let $\angle AOP = \alpha$, $\angle AIP = \beta$ and $\angle ACP = \gamma$

In triangle OAC $\gamma = \alpha + i$ or $i = \gamma - \alpha$ ...(i)

In triangle AIC, $\gamma = B + r$ or $r = \gamma - \beta$ ...(ii)

From Snell's law $\dfrac{\sin i}{\sin r} = \dfrac{n_2}{n_1}$ ...(iii)

If point A is very near to P, then angles i, r, α, β, γ will be very small, therefore $\sin i = i$ and $\sin r = r$

Substituting values of i and r from (i) and (ii) we get

$\dfrac{\gamma - \alpha}{\gamma - \beta} = \dfrac{n_2}{n_1}$ or $n_1(\gamma - \alpha) = n_2(\gamma - \beta)$ ...(iv)

The length of perpendicular AM dropped from A on the principal axis is h i.e., AM = h. As angles α, β and γ are very small, therefore

$\tan \alpha = \alpha$, $\tan \beta = \beta$, $\tan \gamma = \gamma$

Substituting these values in equation (iv)

$n_1(\tan \gamma - \tan \alpha) = n_2(\tan \gamma - \tan \beta)$ ...(v)

As point A is very close to P, point M is coincident with P

$$\tan\alpha = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AM}{MO} = \frac{h}{PO}$$

$$\tan\beta = \frac{AM}{MI} = \frac{h}{PI}, \quad \tan\gamma = \frac{AM}{MC} = \frac{h}{PC}$$

Substituting this value in (v), we get

$$n_1\left(\frac{h}{PC} - \frac{h}{PO}\right) = n_2\left(\frac{h}{PC} - \frac{h}{PI}\right)$$

or $\quad \dfrac{n_1}{PC} - \dfrac{n_1}{PO} = \dfrac{n_2}{PC} - \dfrac{n_2}{PI} \qquad$ ....(iv)

Let u, v and R be the distances of object 0, image I and centre of curvature C from pole P. By sign convention PO, PI and PC are negative, i.e., u = –PO, v = – PI and R = –PC

Substituting these values in (vi), we get

$$\frac{n_1}{(-R)} - \frac{n_1}{(-u)} = \frac{n_2}{(-R)} - \frac{n_2}{(-v)}$$

or $\quad \dfrac{n_2}{v} - \dfrac{n_1}{u} = \dfrac{n_2 - n_1}{R}$

sign Conventions:

(i) All the distances are measured from optical centre (P) of the lens.

(ii) Distances measured in the direction of incident ray of light are taken positive and vice-versa. As we know

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

When convex lens is immersed in water, refractive index n decreases and hence foal length will increase i.e., the focal length of a convex lens increases when it is immersed in water.

OR

(i) A wavefront is defined as a locus of all particles of medium vibrating in the same phase.

(a) The ray indicates the direction of propagation of wave while the wavefront is the surface of constant phase.

(b) The ray at each point of a wavefront is normal to the wavefront at that point.

(ii) We assume a plane wavefront AB propagating in denser medium incident on the interface PP' at angle i as shown in Fig. Let t be the time taken by the wave front to travel a distance BC. If v_1 is the speed of the light in medium I.
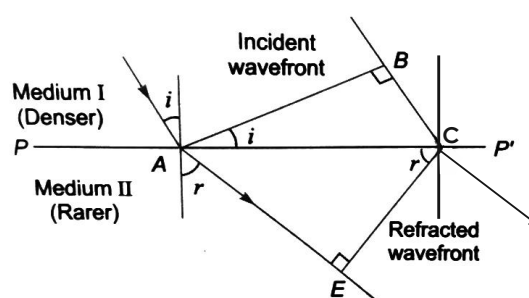
So, $BC = v_1 t$

In order to find the shape of the refracted wavefront, we draw a sphere of radius $AE = v_2 t$ where $v_2 t$ is the speed of light in medium II (rarer medium). The tangent plane CE represents the refracted wavefront.

In $\triangle ABC$, $\sin i = \dfrac{BC}{AC} = \dfrac{v_1 t}{AC}$

and in $\triangle ACE$, $\sin r = \dfrac{AE}{AC} = \dfrac{v_2 t}{AC}$

$\therefore \quad \dfrac{\sin i}{\sin r} = \dfrac{BC}{AE} = \dfrac{v_1 t}{v_2 t} = \dfrac{V_1}{V_2} \qquad$ ...(i)

Let c be the speed of light in vacuum

So, $\quad n_1 = n_1 = \dfrac{c}{v_1}$ and $n_2 = \dfrac{c}{V_2}$

$$\frac{n_2}{n_1} = \frac{v_1}{v_2} \qquad \text{...(ii)}$$

From equations (i) and (ii), we have

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

or $\qquad n_1 \sin i = n_2 \sin r$

It is known as Snell's law.

(iii) For small angle θ, $\sin \theta \simeq \theta$,

Angular width,

$$\theta = \frac{\lambda}{a}$$

$$a = \frac{\lambda}{\theta} = \frac{6 \times 10^{-7}}{0.1 \times \dfrac{\pi}{180}} = 3.4 \times 10^{-4} \text{ m}$$

(iv) Differences between interference and diffraction

Interference	Diffraction
(a) It is due to the super- position of two waves coming from two coherent sources.	(a) It is due to the superposition of secondary wavelets originating from different parts of same favefront.
(b) The width of the interference bands is equal.	(b) The width of the diffraction bands is not the same.
(c) The intensity of all maxima (fringes) is same.	(c) The intensity of central maximum is maximum and goes on decreasing rapidly with increase in order of maxima.

32. **Parallel Plate Capacitor-** Consider a parallel plate capacitor having two plane metallic plates A and B, placed parallel to each other. The plates carry equal and opposite charges +Q and –Q respectively.

In general, the electric field between the plates due to charges +Q and –Q remains uniform, but at the edges, the electric field lines deviate outward. If the separation between the plates is much smaller than the size of plates, the electric field strength between the plates may be assumed uniform.
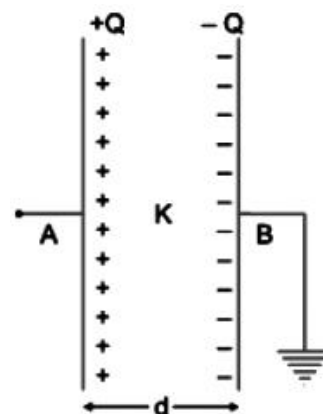
Let A be the area of each plate, 'd' the separation between the plates, K the dielectric constant of medium between the plates. If Q is the magnitude of charge density of plates, then

$$\sigma = \frac{Q}{A}$$

The electric field strength between the plates

$$E = \frac{\sigma}{k\varepsilon_0} \quad \text{where } \varepsilon_0 = \text{permittivity of free space.}$$

The potential difference between the plates,

$$V_{AB} = Ed = \frac{\sigma d}{k\varepsilon_0}$$

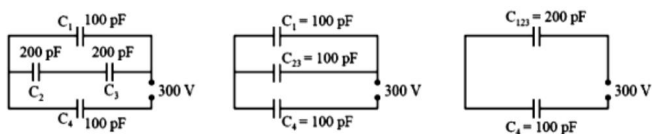Putting the value of σ and for air, $K = 1$ we get

$$V_{AB} = \frac{(Q/A)d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 A}$$

$\therefore$ Capacitance of capacitor

$$C = \frac{Q}{V_{AB}} = \frac{Q}{(Qd/\varepsilon_0 A)}$$

or $C = \dfrac{\varepsilon_0 A}{d}$

(ii) As from circuit shown,

Here C_2 and C_3 in series,

$$\therefore \ C_{23} = \frac{C_2}{2} = \frac{200}{2} = 100 \text{ pF}$$

[Where $C_2 = C_3 = 200$ pF]

Again, C_1 and C_{23} in Parallel.

Then, $C_{123} = C_1 + C_{23} = 100 + 100 = 200$ pF

The equivalent capacitance,

$$C_{eq} = \frac{200 \times 100}{(200 + 100)} = \frac{200}{3} \text{ pF}$$

Now, Charge on C_4

$$Q_4 = C_4 V = \frac{200}{3} \times 10^{-12} \times 300$$

$$= 2 \times 10^{-8} \text{ C}$$

Potential difference across C_4,

$$V_4 = \frac{200 \times 10^{-12} \times 300}{3 \times 100 \times 10^{-12}} = 200 \text{ V}$$

Potential difference across C_1,

$V_1 = 300 - 200 = 100$ V

Then, charge on C_1,

$Q_1 = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8}$ C

Potential difference across C_2 and C_3 in series combination = 100 V

Potential difference across C_2 and C_3 each
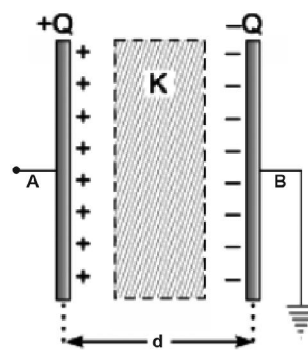
$$= \frac{100}{2} = 50 \text{ V}$$

Charge on C_2 and C_3,

$Q_2 = Q_3 = 200 \times 10^{-12} \times 50$

$\qquad = 1 \times 10^{-8}$ C

OR

(i) Consider a parallel plate capacitor, area of each plate being A, the separation between the plates being d. Let a dielectric slab of dielectric constant K and thickness t < d be placed between the plates. The thickness of air between the plates is (d–t). If charges on plates are +Q and –Q,

then surface charge density $\sigma = \dfrac{Q}{A}$

The electric field between the plates in air,

$$E_1 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The electric field between the plates in slab,

$$E_2 = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{K\varepsilon_0 A}$$

∴ The potential difference between the plates

V_{AB} = work done in carrying unit positive charge from one plate to another

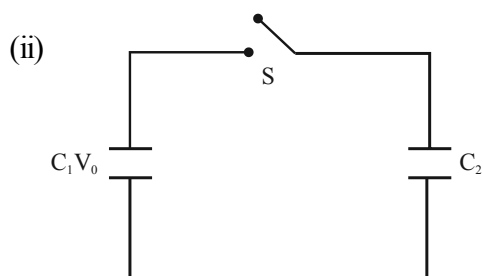$$= E_1(d-t) + E_2 t = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{K\varepsilon_0 A}t$$

$$\therefore \ V_{AB} = \frac{Q}{\varepsilon_0 A}\left[d - t + \frac{t}{K}\right]$$

∴ Capacitance of capacitor,

$$C = \frac{Q}{V_{AB}} = \frac{Q}{\frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{K}\right)}$$

or, $$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

(ii)

Before the connection of switch S,

Initial energy $U_i = \frac{1}{2}C_1 V_0^2 + 0 = \frac{1}{2}C_1 V_0^2$

After the connection of switch S
Common potential,

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{C_1 V_0}{C_1 + C_2}$$

Final energy,

$$U_f = \frac{1}{2}\left(C_1 + C_2\right)\frac{\left(C_1 V_0\right)^2}{\left(C_1 + C_2\right)^2} = \frac{1}{2}\frac{C_1^2 V_0^2}{\left(C_1 + C_2\right)}$$

Hence, Ratio, $U_f : U_i = \dfrac{C_1}{\left(C_1 + C_2\right)}$

33.   (a) The device 'X' is a capacitor.
(b) Curve B : Voltage
Curve C : Current
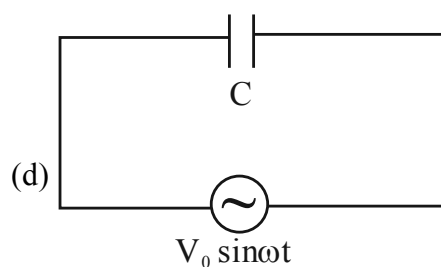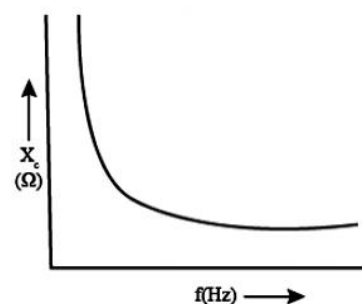Curve A : Power consumed in the circuit
Reason : This is because current leads the

voltage in phase by $\dfrac{\pi}{2}$ for a capacitor

(c) Impedance :

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$\Rightarrow X_C \propto \frac{1}{\nu}$$

(d)

Voltage applied to the circuit is
$V = V_0 \sin\omega t$
Due to this voltage, a charge will be produced which will charge the plates of the capacitor with positive and negative charges.

$$V = \frac{Q}{C} \quad \Rightarrow \quad Q = CV$$

Therefore, the instantaneous value of the current in the circuit is

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} = \frac{d}{dt}\left(CV_0 \sin\omega t\right)$$

$$\therefore \quad I = \omega CV_0 \cos\omega t = \frac{V_0}{\frac{1}{\omega C}}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where , $I_0 = \dfrac{V_0}{\dfrac{1}{\omega C}}$ = Peak value of current

Hence, current leads the voltage in phase by $\dfrac{\pi}{2}$.

OR

Expression for Impedance in LCR series circuit- Suppose resistance R, inductance L and capacitance C are connected in series and an alternating source of voltage $V = V_0 \sin \omega t$ is applied across it (fig. a). On account of being in series, the current (1) flowing through all of them is the same.

(a)

(b)
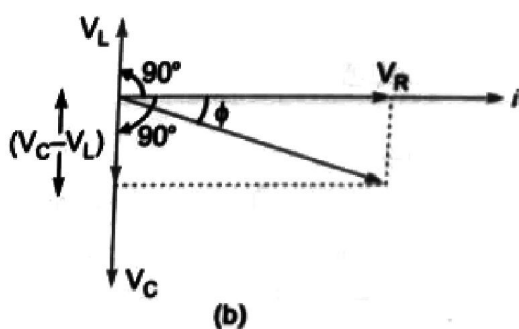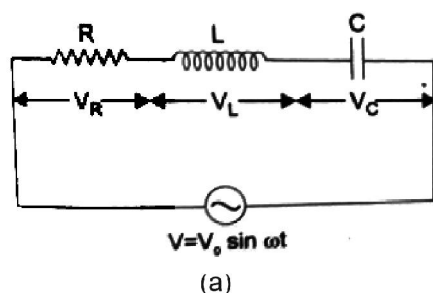
Suppose the voltage across resistance R is V_R voltage across inductance L is V_L and voltage across capacitance C is V_C. The voltage V_R and current i are in the same phase, the voltage V_L will lead the current by angle 90° while the voltage V_C will lag behind the current by angle 90° (fig.

b). Clearly V_C and V_L are in opposite directions, therefore their resultant potential difference $= V_C - V_L$ (if $V_C > V_L$).

Thus V_R and $(V_C - V_L)$ are mutually perpendicular and the phase difference between them is 90°. As applied voltage across the circuit is V, the resultant of V_R and $(V_C - V_L)$ will also be V. From fig.

$$V^2 = V_R^2 + (V_C - V_L)^2$$

$$\Rightarrow V = \sqrt{V_R^2 + \left(V_C - V_L\right)^2} \qquad ....(i)$$

But $V_R = Ri$, $V_C = X_C i$ and $V_L = X_L i$ ....(ii)

where $X_C = \dfrac{1}{\omega C}$ = capacitance reactance and

$X_L = \omega L$ = inductive reactance

$$V = \sqrt{\left(Ri\right)^2 + \left(X_C i - X_L i\right)^2}$$

Impedance of circuit,

$$Z = \dfrac{V}{i} = \sqrt{R^2 + \left(X_C - X_L\right)^2}$$

$$= \sqrt{R^2 + \left(\dfrac{1}{\omega C} - \omega L\right)^2}$$

The phase difference (ϕ) between current and

voltage is given by, $\tan \phi = \dfrac{X_C - X_L}{R}$

(ii) The curve (i) is for R_1 and the curve (ii) is for R_2.